Thermodynamic Database for the NdO$_{1.5}$-YO$_{1.5}$-YbO$_{1.5}$-ScO$_{1.5}$-ZrO$_2$ System

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September 2001
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THERMODYNAMIC DATABASE FOR THE
NdO$_{1.5}$-YO$_{1.5}$-YbO$_{1.5}$-ScO$_{1.5}$-ZrO$_2$ SYSTEM

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SUMMARY

A database for YO$_{1.5}$-NdO$_{1.5}$-YbO$_{1.5}$-ScO$_{1.5}$-ZrO$_2$ for ThermoCalc* has been developed. The basis of this work is the YO$_{1.5}$-ZrO$_2$ assessment by Du et al. (ref. 1). Experimentally only the YO$_{1.5}$-ZrO$_2$ system has been well-studied. All other systems are only approximately known. The major simplification in this work is the treatment of each single cation unit as a component. The pure liquid oxides are taken as reference states and two term lattice stability descriptions are used for each of the components. The limited experimental phase diagrams are reproduced.

1. INTRODUCTION

The rare earth oxide stabilized zirconias are of considerable technological importance. Applications range from thermal barrier coatings for turbine blades to electrodes for sensors and fuel cells. The most widely used system is Y$_2$O$_3$-ZrO$_2$. The phase diagram for this system has been extensively studied (refs. 2 to 11) Further, there are several thermodynamic assessments and calculated phase diagrams in the literature for this and related systems (refs. 1, 12 to 19).

In this report we also consider the stabilizers: Yb$_2$O$_3$, Sc$_2$O$_3$, and Nd$_2$O$_3$. These different stabilizers alone and in combinations may offer improved properties. However, both experimental phase diagrams and thermodynamic data for these systems are very limited (refs. 20 to 30). Reliable phase and thermodynamic information over a range of compositions and temperatures is essential to understand the processing and properties of zirconia with Y$_2$O$_3$, Yb$_2$O$_3$, Sc$_2$O$_3$, and Nd$_2$O$_3$ stabilizers.

Figures 1 to 10 give the available pseudo-binary phase diagrams for the systems (refs. 22 to 30) listed in table I. The most complete and most recent phase diagrams are given. In the case of Y$_2$O$_3$-ZrO$_2$, two phase diagrams are shown in figures 1(a) and (b). The first diagram from Stubican et al. (refs. 11 and 22) includes one intermediate compound—Zr$_3$Y$_2$O$_7$. The second diagram (ref. 23) also includes an ordered hexagonal phase—ZrY$_2$O$_4$. However, the existence of the second compound remains controversial and will not be considered here. In addition, the primary interest is in the ZrO$_2$-rich portion of the diagram and therefore its existence is not important in the present study. Figures 2(a) and (b) are the available diagrams for the Yb$_2$O$_3$-ZrO$_2$ system. Figure 2(a) primarily shows the liquidus and figure 2(b) shows the solid phases. Note that these are similar to the Y$_2$O$_3$-ZrO$_2$ diagram and shows an analogous intermediate phase—Zr$_3$Yb$_4$O$_{17}$. However Zr$_3$Yb$_4$O$_{17}$ is stable to 1612 °C and Zr$_3$Y$_2$O$_7$ is stable to only to 1250 °C (fig. 1(a)) or 1330 °C (fig. 1(b)). Figures 3 to 10 are the remaining available binaries. Most of these diagrams are incomplete and have many approximate boundaries.

The addition of two (or more) stabilizing rare earth oxides to zirconia may lead to some interesting properties. Yet only a portion of the Y$_2$O$_3$-Yb$_2$O$_3$-ZrO$_2$ ternary diagram has been determined experimentally (ref. 31). Three isothermal sections are shown in figures 11(a) to (c). The major features of these systems are a continuous solid solution of tetragonal zirconia and cubic zirconia with both Y$_2$O$_3$ and Yb$_2$O$_3$. Note also that the two intermediate compounds—Zr$_3$Y$_2$O$_7$ and Zr$_3$Yb$_4$O$_{17}$ connect at lower temperatures. At higher temperatures Zr$_3$Y$_2$O$_7$ becomes unstable and only Zr$_3$Yb$_4$O$_{17}$ protrudes into the ternary. At the highest temperatures, both compounds are unstable. No other ZrO$_2$ rare earth oxide experimental ternaries were found.

The first calculations of these types of refractory oxide phase diagrams were done in 1988 by Kaufman (ref. 14). Kaufman treats each oxide unit as a component. He sets the liquid oxide as a reference state and uses two-term lattice stabilities to describe each end point oxide. Thus the lattice stability of a particular polymorph becomes

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*ThermoCalc AB, Stockholm, Sweden.
the energy to form that polymorph from the liquid. Du et al., (refs. 1 and 15) have done two assessments of the Y₃O₇-ZrO₂, using the Lukas programs (ref. 32) to derive phase descriptions that best fit all the available experimental data. Their first assessment (ref. 1) also uses the liquid as the reference state. Their second assessment (ref. 15) invokes the standard element reference state (SER) and uses Gibbs energy expressions for each phase.

This report discusses the development of a Thermo-Calc database for the multicomponent system: Nd₂O₃-Sc₂O₃-Yb₂O₃-Y₂O₃-ZrO₂. The quantities of Du et al., for ZrO₂-Y₂O₃ (ref. 1) are used directly and other binaries are developed based on this. Du et al., (ref. 1) perform two optimizations for the ZrO₂-Y₂O₃ system—using a different set of liquidus data for each optimization. Their second optimization values are used here. The rare earth oxides are normalized to one cation: NdO₁.₅-ScO₁.₅-YbO₁.₅-YO₁.₅-ZrO₂. Each of these one cation oxides units are treated as a component. Two-term lattice stability phase descriptions referenced to the liquid state are used, which are less complex than the Gibbs energy phase descriptions based on the standard element reference state.

The entire database in standard Thermo-Calc * .tdb format is given in appendix I. In the following report, we discuss this database and the calculation of binary and higher order diagrams in the Nd₂O₃-Sc₂O₃-Yb₂O₃-Y₂O₃-ZrO₂ system.

II. LATTICE STABILITIES FOR END-POINTS

In order to estimate the lattice stabilities with respect to the liquid state, we need the enthalpies and entropies for the polymorphic transformations. Only limited information is available, particularly on the solid/solid transformations. These data (refs. 33 to 35) are shown in table II. As noted, each oxide is normalized to one mole of cation. The oxide abbreviations are given in table III.

It is important to discuss the compositional difference between the calculations and the measured values in table II and figures 1 to 11. The experimental phase diagrams are for RE₂O₃-ZrO₂ (RE = Rare Earth Element) and will be referred to as such; the calculated diagrams are for REO₁.₅-ZrO₂, and will be referred to as such. In order to compare the two, we need to convert from mole fraction x(RE₂O₃), in RE₂O₃-ZrO₂ to mole fraction x(REO₁.₅), in REO₁.₅-ZrO₂. Note that

\[
x(RE_2O_3 \text{in } RE_2O_3-ZrO_2) = \frac{n(RE_2O_3 \text{in } RE_2O_3-ZrO_2)}{n(RE_2O_3 \text{in } RE_2O_3-ZrO_2) + n(ZrO_2 \text{in } RE_2O_3-ZrO_2)}
\]

(1)

Here n is the number of moles. The composition for REO₁.₅-ZrO₂ is:

\[
x(REO_{1.5} \text{in } REO_{1.5}-ZrO_2) = \frac{n(REO_{1.5} \text{in } REO_{1.5}-ZrO_2)}{n(REO_{1.5} \text{in } REO_{1.5}-ZrO_2) + n(ZrO_2 \text{in } REO_{1.5}-ZrO_2)}
\]

(2)

Note that for each composition n(ZrO₂) the same and that n (RE₂O₃) = \( \frac{1}{2} \) n (REO₁.₅). So now:

\[
x(REO_{1.5}) = \frac{2x(RE_2O_3)}{2x(RE_2O_3) + (1-x(RE_2O_3))}
\]

(3)

Table IV shows the equivalent compositions.

Now consider the derivation of the lattice stabilities for each oxide. The most information is available for ZrO₂, which undergoes three transformations (ref. 36):

\[
\text{ZrO}_2: \text{Monoclinic} \rightarrow \text{Tetragonal} \rightarrow \text{Cubic} \rightarrow \text{Liquid}
\]

(4)

The approach of Kaufman (ref. 14) and ThermoCalc notation are used in the following discussion.

The liquid phase is taken as the reference state and thus:

\[
G(L, ZM; 0) = 0.0
\]

(5)
For the cubic phase:

\[ G(C,ZM;0) = \Delta H^{c} - T \Delta S^{c} = -87027 + 29.471T \] (6)

This is the value from Kubachewski and Alcock (ref. 33) with a melting point of 2953 K. More recent investigations have given the melting point of cubic zirconia as 2983 K. Du and Jin (ref. 1) adjust equation (6) to reflect this:

\[ G(C, ZM; 0) = -87986.6 + 29.496T \] (7)

The tetragonal to cubic transformation temperature is given as 2568 K (33) and thus:

\[ \Delta H^{T} - T \Delta S^{T} = -5941 + 2.313T \] (8)

Again Du and Jin (ref. 1) adjust this for a more recent transformation temperature of 2642 K:

\[ \Delta H^{T} - T \Delta S^{T} = -5941 + 2.249T \] (9)

In order to obtain the free energy of the tetragonal phase referenced to the liquid, equations (7) and (9) are added:

\[ G(T, ZM; 0) = \Delta H^{T} - T \Delta S^{T}VL = -93927 + 31.745T \] (10)

From equation (10) a 'melting point' for the tetragonal phase can be calculated to be 2959 K. A metastable melting point must always be less than the melting point of the stable phase.

Table II indicates some disagreement on the monoclinic to tetragonal transformation temperature and heat. Here we take the temperature as 1454 K and the heat as 6000 joules/mol:

\[ \Delta H^{M} - T \Delta S^{M} = -6000 + 4.127T \] (11)

The free energy of the monoclinic phase referenced to the liquid is obtained by adding equations (10) and (11):

\[ G(M, ZM; 0) = \Delta H^{M} - T \Delta S^{M} = -99927 + 35.872T \] (12)

For the other oxides—Y\textsubscript{2}O\textsubscript{3}, YbO\textsubscript{3}, NdO\textsubscript{3}, ScO\textsubscript{3}—a similar approach was followed. However, less data is available. Of these the most data is available for YO\textsubscript{1.5}. YO\textsubscript{1.5} undergoes two phase transformations:

YO\textsubscript{1.5}: Cubic \rightarrow Hexagonal \rightarrow Liquid (13)

Again, the liquid phase is taken as the reference state:

\[ G(L, YM; 0) = 0.0 \] (14)

The melting point of the hexagonal phase is accepted as 2712 K. Kaufman (ref. 14) estimates the lattice stability of the hexagonal phase as:

\[ G(H, YM; 0) = \Delta H^{H} - T \Delta S^{H} = -56735 + 20.92T \] (15)

This enthalpy is somewhat more than half the IVTAN (ref. 34) data for melting of Y\textsubscript{2}O\textsubscript{3}, but more in line with Kubachewski’s (ref. 34) values for alumina heat of melting (54 kJ/mol Al\textsubscript{2}O\textsubscript{3}).

The only measured transition heat for the polymorphs of a rare earth oxide was that for Y\textsubscript{2}O\textsubscript{3} cubic to hexagonal, as reported in IVTAN to be 54 kJ/mol. However, this number seems much too high for a solid/solid transition such as this. Kaufman (ref. 14) estimates the heat to be about 10 kJ/mol and it seems more reasonable to use this value. Accepting the cubic to hexagonal transformation temperature of 2550 K the lattice stability for the cubic phase becomes:

\[ G(C, YM; 0) = \Delta H^{C} - T \Delta S^{C} = -67419 + 25.105T \] (16)
The behavior of YbO₁₅ is controversial. The two published phase diagrams with YbO₁₅ do not agree. The YbO₁₅-ZrO₂ diagram (fig. 2) indicates that the hexagonal phase melts; whereas the YbO₁₅-NdO₂ diagram (fig. 9) indicates that the cubic phase melts. It is assumed that the hexagonal phase of YbO₁₅ melts, in analogy with YO₁₅. Therefore the phase sequence with temperature is taken as:

\[ \text{YbO₁₅: Cubic} \rightarrow \text{Hexagonal} \rightarrow \text{Liquid} \]

For NdO₁₅, the phase sequence with temperature appears to be:

\[ \text{NdO₁₅: N phase} \rightarrow \text{Hexagonal} \rightarrow \text{Cubic} \rightarrow \text{Liquid} \]

Little information is available for ScO₁₅, and it is assumed that the cubic phase melts. From the melting points and estimated heats of fusion, the lattice stabilities are derived as shown in the database (appendix I).

III. INTERACTION PARAMETERS: CUBIC, MONOCLINIC, TETRAGONAL, N PHASE, NM PHASE

In the binary case, each of the above solution phases are described by the following expression for the free energy:

\[ G_m(x,T) = \sum_{i=1}^{2} x_i \cdot G_i(T) + R \sum_{i=1}^{2} x_i \ln x_i + \varepsilon_G \varepsilon_m \]  

(17)

The first term is the sum of the lattice stabilities for each of two components. These are discussed in the previous section. The Calphad approach requires a lattice stability for every component in each phase—even if a particular component does not form the phase described. For example, a lattice stability is necessary for hexagonal ZrO₂, monoclinic YO₁₅, and tetragonal YO₁₅. The lattice stabilities are chosen so that these phases are always unstable (Appendix I).

The second term in equation (17) is the random mixing term for an ideal solution. The third term is the excess Gibbs energy and accounts for the deviations from ideality. Many solution models have been developed for this term (refs. 37 to 39). In the Calphad approach, the excess Gibbs energy is expanded as a Redlich-Kister polynomial (ref. 37):

\[ \varepsilon_G \varepsilon_m = x_1 x_2 \sum_{j=0}^{n} \varepsilon_j (x_1 - x_2)^j \]

(18)

Here only the first two terms are considered:

\[ \varepsilon_G \varepsilon_m = x_1 x_2 \varepsilon_0 L + x_1 x_2 \varepsilon_1 L(x_1 - x_2) = x_1 x_2 \varepsilon_0 L + x_1 x_2 \varepsilon_1 L(1 - 2x_2) \]

(19)

where \( \varepsilon \) and \( \varepsilon_1 \) are the zero and first order interaction parameters, respectively. In assigning mole fraction designations, ThermoCalc takes the components in alphabetical order. Thus in this case \( x_1 = x_{\text{ZrO}} \) and the sign of the second term in equation (19) is always the opposite of the sign of \( \varepsilon_1 \) for \( x_1 > 0.5 \).

The shape of the Gibbs energy curves versus composition and temperature determine the stable phase fields in a classical temperature-composition phase diagram. The first two terms in equation (17) are fixed and lead to an ideal solution, as discussed in many textbooks (refs. 38 to 39). The variables in the excess term (\( \varepsilon_0 \) and \( \varepsilon_1 \)) are adjusted to fit experimental observations for a real solution, which generally show deviations from ideality.

The YO₁₅-ZrO₂ system description can be simplified by introducing a miscibility gap in the cubic phase field. This will be discussed in the next section. It follows that the \( \varepsilon_0 \) and \( \varepsilon_1 \) parameters need to be adjusted to create this feature. This is illustrated in figure 12. A single zero order term (regular solution) can only yield a symmetric miscibility gap. The \( \varepsilon_1 \) term is also needed to produce an asymmetric miscibility gap. The effect of the sign of \( \varepsilon_1 \) on the location of the miscibility gap is shown in figure 12 and table V. For the cubic phase in a REO₁₅-ZrO₂ phase diagram, we want the miscibility gap on the ZrO₂-rich side. This requires a positive \( \varepsilon_1 \) and a negative second term in equation (19), which gives a second minima on the right, but less than the first minima on the left, as shown in figure 12.
The interaction parameters for the five solid solutions in this system are thus determined by (a) Using the assessment of Du (1) as a guide and (b) Estimating numbers which give a $G_m$ which will lead to phase regions similar to those observed in the experimental phase diagram.

III.A. Cubic Solid Solution (Cubic Zr$_2$O$_3$ and Cubic REO$_{1.5}$ss)

A major feature of the ZrO$_2$-Rare Earth Oxide phase diagrams is the large cubic phase field. This is evident in each of the experimental phase diagrams (figs. 1 to 4).

As pointed out by Degtyarev and Voronin (refs. 12 and 13), most of the Y$_2$O$_3$-ZrO$_2$ phase diagram is composed of cubic solid solutions of YO$_{1.5}$ and ZrO$_2$. Cubic zirconia has a classical face-centered cubic lattice CaF$_2$ structure. The cubic phase for Y$_2$O$_3$ has a body-centered cubic lattice like Mn$_2$O$_3$. Degtyarev and Voronin (refs. 12 and 13) further point out that this Mn$_2$O$_3$ structure can be derived from the CaF$_2$ structure by removing one quarter of the oxygen anions. Thus the two cubic phases can be derived from one G-X curve with a miscibility gap.

The above assumption simplifies the phase diagram calculation procedure. The miscibility gap option in Thermo-Calc is used which has the program search for a second free energy minimum. Further, the experimental phase diagrams indicate that the miscibility gap is shifted to the right.

The interaction parameters for the YO$_{1.5}$-ZrO$_2$ are taken from the assessment of Du et al. (ref. 1):

\begin{align}
L(C,YM,ZM; 0) &= -12060 + 11.156T \\
L(C,YM,ZM; 1) &= +13784 + 5.379T
\end{align}

The first order interaction parameter is always positive and thus shifts the miscibility gap to the right (table V) as observed experimentally.

As shown in the database (appendix I), similar interaction parameters were taken for the YbO$_{1.5}$-ZrO$_2$ and ScO$_{1.5}$-ZrO$_2$ systems. For the NdO$_{1.5}$-ZrO$_2$ system, the following interaction parameters are estimated:

\begin{align}
L(C, NM, ZM; 0) &= -52400 + 27T \\
L(C, NM, ZM; 1) &= +24400 +1.5T
\end{align}

Again the first order interaction parameter is always positive and hence the miscibility gap is shifted to the right, as the experimental phase diagram shows. Note that equation (23) yields the similar values to equation (21) at high temperatures.

Consider the remaining interaction parameters between the various rare earth oxides. Now there is no miscibility gap and in most instances a continuous or nearly continuous cubic solution (figs. 5 to 10). Only the Yb$_2$O$_3$-Nd$_2$O$_3$ and Y$_2$O$_3$-Nd$_2$O$_3$ systems have been established with any certainty. These interaction parameters are estimated as:

\begin{align}
L(C, BM, NM; 0) &= +21425 + 2.5T \\
L(C, BM, NM; 1) &= -19600 + 8T \\
L(C, NM, YM; 0) &= +21425 + 2.5T \\
L(C, NM, YM; 1) &= -19600 + 8T
\end{align}

The other rare earth oxide phase diagrams (Sc$_2$O$_3$-Y$_2$O$_3$, Sc$_2$O$_3$-Nd$_2$O$_3$, Yb$_2$O$_3$-Sc$_2$O$_3$, Yb$_2$O$_3$-Y$_2$O$_3$) are only very approximately known (figs. 5, 6, 8, and 10, respectively). In this case only the zero order interaction parameter is estimated to be +20,000 J/mol.

III.B. Hexagonal Solid Solution (Hex REO$_{1.5}$ss)

As shown in figures 1(a) and (b), the hexagonal solid solution only occupies a small region of the Y$_2$O$_3$-ZrO$_2$ phase diagram. The assessment of Du et al., gives a single Redlich-Kister term:
L(H, YM, ZM; 0) = 50420 \tag{28}

As shown in figure 12, a large positive value of $\Delta L$ gives a symmetric free energy of mixing with two small minima on either side. The minima on the YO$_{1.5}$ side gives the small hexagonal region in the phase diagram.

Most of the other hexagonal $\Delta L$ solution terms were estimated to 20000 or 35000 J/mol. However the phase diagrams for the Nd$_{0.5}$Y$_{0.5}$O$_{2}$ and Nd$_{0.5}$Yb$_{0.5}$O$_{2}$ have been reported (figs. 7 and 9, respectively). In order to obtain the hexagonal solid solutions across these phase diagrams, the following expressions are used:

\begin{align*}
L(H, BM, NM; 0) &= 12750 + 1.5T \tag{29} \\
L(H, BM, NM; 1) &= -1000 - T \tag{30} \\
L(H, YM, NM; 0) &= 11265 + 1.5T \tag{31} \\
L(H, YM, NM; 1) &= 1000 - T \tag{32}
\end{align*}

Both these sets of values for the interaction parameters give a free energy with a single, broad minima at the temperatures of interest.

III.C. Liquid Solution

The liquid solution parameters for the YO$_{1.5}$-ZrO$_2$ system have been determined by Du et al., (ref. 1) to be:

\begin{align*}
L(LIQ, YM, ZM; 0) &= -183751 + 72.4T \tag{33} \\
L(LIQ, YM, ZM; 1) &= 48733 - 9.476T \tag{34}
\end{align*}

Due to similarity between the liquid regions in the YO$_{1.5}$-ZrO$_2$ and YbO$_{1.5}$-ZrO$_2$ systems, these same parameters are taken:

\begin{align*}
L(LIQ, BM, ZM; 0) &= -183750 + 72.4T \tag{35} \\
L(LIQ, BM, ZM; 1) &= 48700 - 9.48T \tag{36}
\end{align*}

For the NdO$_{1.5}$-ZrO$_2$ system, a single term expression is used:

\begin{align*}
L(LIQ, NM, ZM; 0) &= -20000 \tag{37} \\
L(LIQ, NM, ZM; 1) &= -15000 \tag{38}
\end{align*}

All of the ScO$_{1.5}$ parameters were estimated to 20000 J/mol. As shown in figures 3, 6, 8, and 10, virtually nothing is known about the liquid phase in the scandia systems. Thus, it is reasonable to use the above approximations.

For the other systems, zero order parameters are estimated as:

\begin{align*}
L(LIQ, BM, NM; 0) &= -200 \tag{39} \\
L(LIQ, BM, YM; 0) &= -10000 \tag{40} \\
L(LIQ, NM, YM; 0) &= -200 \tag{41}
\end{align*}

III.D. Monoclinic Phase Solid Solution (Mon ZrO$_2$ ss)

The monoclinic phases show very limited solubility for a second oxide, except for the NdO$_{1.5}$-YbO$_{1.5}$ and NdO$_{1.5}$-YO$_{1.5}$ systems. Again, beginning with the data from the assessment of Du et al., (ref. 1):

\[ L(M, YM, ZM; 0) = -58223 + 98.126T \tag{42} \]
The other interaction parameters are estimated. Only two terms are used for the NdO$_{1.5}$-YbO$_{1.5}$ and NdO$_{1.5}$-YO$_{1.5}$ systems:

\[ L(M, BM, NM; 0) = -5000 \]  \hspace{1cm} (43)

\[ L(M, BM, NM; 1) = 100 \] \hspace{1cm} (44)

\[ L(M, NM, YM; 0) = -50000 \] \hspace{1cm} (45)

\[ L(M, NM, YM; 1) = 1000 \] \hspace{1cm} (46)

The remaining zero order interaction parameters are all estimated to be 20000 J/mol.

III.E. Phase MN Solid Solution (MN ss)

Again, since only the YbO$_{1.5}$-NdO$_{1.5}$ and YO$_{1.5}$-NdO$_{1.5}$ phase diagrams are known with any certainty, these interaction parameters are taken as:

\[ L(MN, BM, NM; 0) = -160000 + 48T \] \hspace{1cm} (47)

\[ L(MN, BM, NM; 1) = 120000 - 15T \] \hspace{1cm} (48)

\[ L(MN, NM, YM; 0) = -160000 + 50T \] \hspace{1cm} (49)

\[ L(MN, NM, YM; 1) = 120000 + 15T \] \hspace{1cm} (50)

All other interaction parameters are taken as 20000 J/mol.

III.F. N Phase Solid Solution (N ss)

As above, since only the YbO$_{1.5}$-NdO$_{1.5}$ and YO$_{1.5}$-NdO$_{1.5}$ phase diagrams are known with any certainty, these interaction parameters are taken as:

\[ L(N, BM, NM; 0) = -15200 - 7T \] \hspace{1cm} (51)

\[ L(N, BM, NM; 1) = -13000 - 6.7T \] \hspace{1cm} (52)

\[ L(N, NM, YM; 0) = -15000 - 7T \] \hspace{1cm} (53)

\[ L(N, NM, YM; 1) = 13000 + 5T \] \hspace{1cm} (54)

All other interaction parameters are taken as 20000 J/mol, except for:

\[ L(N, NM, ZM; 0) = 50000 \] \hspace{1cm} (55)

III.G. Tetragonal Phase Solid Solution (Tet ZrO$_{2}$ ss)

This phase shows a triangular appearance in the Y$_2$O$_3$-ZrO$_2$ phase diagram (figs. 1(a) and (b)). There is a suggestion on the other RE$_2$O$_3$-ZrO$_2$ phase diagrams (figs. 2 to 4) of similar behavior. The assessment of Du et al., (1) gives:

\[ L(T, YM, ZM; 0) = -25800 \] \hspace{1cm} (56)
The same value is adopted for YbO$_{1.5}$-ZrO$_{1.5}$:

$$L(T, BM, ZM; 0) = -25800$$ \hspace{1cm} (57)

All other interaction parameters are estimated to be 20000 J/mol.

IV. INTERMEDIATE COMPOUNDS

In the binary NdO$_{1.5}$-ZrO$_{1.5}$, ScO$_{1.5}$-ZrO$_{1.5}$, and YbO$_{1.5}$-ZrO$_{1.5}$ systems there are several intermediate compounds. The Zr$_Y$O$_{1.5}$ phase has been reported by several investigators. These line compounds are treated with two sublattice model—the ZrO$_2$ component is on one sublattice and the YO$_{1.5}$ component is one the other sublattice. Following Du et al.:

$$G(\frac{1}{7}Zr$_3$Y$_4$O$_{12}) = 3 \frac{3}{7} 0G^{C\rightarrow Y} + 4 \frac{4}{7} 0G^{C\rightarrow ZrO_2} - 3284.3 - 2.26749T = -79536 + 24.72T$$ \hspace{1cm} (58)

As mentioned, there is an analogous Zr$_Y$b$_{1.5}$O$_{1.5}$ phase. This is treated as:

$$G(\frac{1}{7}Zr$_3$Yb$_4$O$_{12}) = -79350 + 25T$$ \hspace{1cm} (59)

To account for interchange of Y and Yb in this compound, the following solution parameter is introduced:

$$L(\Delta, BM, YM; ZM; 0) = 10200$$ \hspace{1cm} (60)

The free energy of each of the compounds are chosen so that they protrude into the cubic phase. For NdO$_{1.5}$-ZrO$_{1.5}$, there exists a binary with the 1:1 composition

$$G(P, NM; ZM; 0) = -180450 + 57T$$ \hspace{1cm} (61)

For the ScO$_{1.5}$-ZrO$_{1.5}$, there are several intermediate compounds, each with a range of solution according to the experimental diagram in figure 3. However for the purpose of this approximation, we shall treat them as line compounds.

For 4(ScO$_{1.5}$) . 3(ZrO$_{1.5}$), Phase RB:

$$G(RB, SM; ZM; 0) = -557200 + 171.5T$$ \hspace{1cm} (62)

For 2(ScO$_{1.5}$) . 5(ZrO$_{1.5}$), Phase RH:

$$G(RH, SM; ZM; 0) = -620000 + 185T$$ \hspace{1cm} (63)

For 2(ScO$_{1.5}$) . 7(ZrO$_{1.5}$), Phase RM:

$$G(RM, SM; ZM; 0) = -822800 + 258T$$ \hspace{1cm} (64)

V. RESULTS: BINARY PHASE DIAGRAMS

Having described the database, we now turn to the results for the binaries and compare them to the limited experimental data. Figures 13 to 22 give the calculated binaries. Recall again that the experimental diagrams are for RE$_{1.5}$ZrO$_{2}$ whereas the calculated diagrams are for RE$_{1.5}$ZrO$_{2}$.

The calculated diagrams show the same general features as the experimental diagrams. The calculated YO$_{1.5}$-ZrO$_{2}$ phase diagram (fig. 13) is, of course, the same as that obtained by Du et al., (ref. 1) and is in good agreement with the experimental diagram of Stubican et al., (fig. 1(a)).

All other RE$_{1.5}$ZrO$_{2}$ diagrams show similar behavior to the YO$_{1.5}$-ZrO$_{2}$ phase diagram on the ZrO$_{2}$-rich side, where most of the interest is. The NdO$_{1.5}$ZrO$_{2}$, ScO$_{1.5}$ZrO$_{2}$, and YbO$_{1.5}$ZrO$_{2}$ diagrams are not so definitive, as much
less experimental data is available. Nonetheless, the general features of the experimental diagrams are reproduced on the calculated diagrams.

The Yb₂O₃-ZrO₂ diagram is also only approximately known (fig. 2) and consists of a large cubic phase field and one intermediate compound. This is reproduced in the calculated diagram (fig. 14).

The Nd₂O₃-ZrO₂ experimental diagram shows a eutectic at -2115 °C and the Nd₀.₅-ZrO₂ calculated diagram shows the same eutectic at 2100 °C. The Nd₂O₃-ZrO₂ experimental diagram shows a euctectoid at 1440 °C, whereas the calculated Nd₀.₅-ZrO₂ phase diagram shows this eutectoid at -1660 °C. But this part of the diagram is not well-known.

Only a limited portion of the Sc₂O₃-ZrO₂ phase diagram is known (fig. 3). For the purpose of this approximation, the intermediate compounds (Sc₂Zr₆O₁₇, Sc₂Zr₃O₁₅, Sc₂Zr₂O₁₃) are treated as line compounds rather than solid solutions. Even with this approximation, the general appearance is reproduced.

For the binaries among the rare earth oxides (Nd₂O₃, Sm₂O₃, Yb₂O₃, Y₂O₃), only the Nd₂O₃-Y₂O₃ (fig. 7) and Nd₂O₃-Yb₂O₃ (fig. 9) systems are known to any extent. The major features of these diagrams are reproduced in the calculated diagrams (figs. 19 and 21, respectively). Both show a substantial monoclinic solid solution. The remaining rare earth oxide binaries show large cubic solid solution regions.

There are some limited activity measurements for the Y₂O₃-ZrO₂ system at 2773 K (ref. 40) and the Sc₂O₃-ZrO₂ system at 2600 K (ref. 41) obtained via mass spectrometry. In order to compare these data to calculations from the database described here, the mole fraction must be converted according to table IV and the activity also must be converted to appropriate units. Belov et al. (refs. 40 and 41) show that the rare earth oxides vaporize as:

\[
\text{RE}_2\text{O}_3(s) = 2 \text{REO(g)} + \text{O(g)}
\]  

Thus the activity of \(\text{RE}_2\text{O}_3\) \((a(\text{RE}_2\text{O}_3))\) is defined as:

\[
a(\text{RE}_2\text{O}_3) = \frac{[P(\text{REO})]^{1/2}[P(O)]}{[P^0(\text{REO})]^{1/2}[P^0(O)]}
\]  

Here the quantities in the numerator are the partial pressures over the oxide in solution and the quantities in the denominator are the partial pressures over the pure oxide—solid \(\text{Y}_2\text{O}_3\) (presumably hexagonal) and cubic \(\text{Sc}_2\text{O}_3\).

Next consider \(\text{REO}_{1.5}\), which should vaporize as:

\[
\text{REO}_{1.5}(s) = \text{REO(g)} + \frac{1}{2} \text{O(g)}
\]  

Thus the experimental measurements need only be converted with a square root for comparison to the calculated values.

Figures 23(a and b) and 24(a and b) show the comparison of these measured activities (refs. 40 and 41) to the calculated values from this database. For both systems the data indicate measurements were taken in a single phase region across the diagram; whereas the phase diagrams—both experiment and calculated indicate some two phase regions. There appears to be a conflict in the phase boundary locations. Agreement with \(a(\text{ZrO}_2)\) is good, as these data were used in the Du et al. (ref. 1) assessment. However, agreement with \(a(\text{REO}_{1.5})\) is only fair—these data were not used in the assessment.

VI. RESULTS: TERNARY SECTIONS

This database can be used to approximate ternary sections. One would expect the following features in a ternary diagram:

1. The cubic phase occupies a large region in each binary. Therefore it seems reasonable that the cubic phase would stretch across the ternary as well.
2. The intermediate compounds should protrude into the ternary.
Calculations of the isothermal cuts led to some convergence problems in ThermoCalc and additional steps were necessary to force convergence. These are illustrated in the macro in appendix II. This macro also allows the variation of the ternary interaction parameter.

Figures 25(a) and (b) are isothermal cuts for the YO\textsubscript{1.5}−YbO\textsubscript{1.5}−ZrO\textsubscript{2} system at 1600 K, illustrating the effects of two different ternary interaction parameters:

Figure 24(a): \( L(C, BM, YM; 0) = +10000 \), insufficient for a continuous two phase region.

Figure 24(b): \( L(C, BM, YM; 0) = +100000 \), continuous two phase region.

In order to address issue (2) above, an interaction parameter was added for the intermediate phase. Figure 25 shows the effect of

\[
L(C, BM, YM: ZM) = 70000
\]

\[
L(DELTA, BM, YM: ZM) = 10200
\]

Figures 26(a) to (c) are the calculated ternary for 1473, 1673, and 2000 K. These can be compared to experimental sections in figure 11. The protrusion behavior as a function of temperature for the delta phase is reproduced by these calculations. Further work is necessary to reproduce the phase boundaries of the solution phases correctly.

VI. RESULTS: HIGHER ORDER SYSTEMS

There are no reported data on higher order phase diagrams. However they can be approximated with this database. Phase data for systems with more than three components can be presented with a phase fraction diagram or an isoplethal section with one component varying over a small range. An example of a phase fraction diagram for ScO\textsubscript{1.5}−YO\textsubscript{1.5}−YbO\textsubscript{1.5}−ZrO\textsubscript{2} is shown in figure 27. Figure 28 shows an isoplethal section for ScO\textsubscript{1.5}−YO\textsubscript{1.5}−YbO\textsubscript{1.5}−ZrO\textsubscript{2} with a small variation in YO\textsubscript{1.5}.

VII. SUMMARY AND CONCLUSIONS

An approximate database for ScO\textsubscript{1.5}−NdO\textsubscript{1.5}−YbO\textsubscript{1.5}−ZrO\textsubscript{2} has been discussed. It is based on the assessment of Du et al., for the YO\textsubscript{1.5}−ZrO\textsubscript{2} system (ref. 1) and uses a two-term lattice stability description of the phases with the liquid as the reference state. Binary phase diagrams for the other systems agree reasonably well with the limited experimental data. Ternary and higher order diagrams require the introduction of a ternary interaction parameter and an interaction parameter for the intermediate compounds to allow protrusion of the intermediate compounds into the multi-component composition space. The binary systems beyond Y\textsubscript{2}O\textsubscript{3}−ZrO\textsubscript{2} require additional experimental and modeling efforts. This will provide a stronger basis for further development of the multicomponent diagrams.
$ Database file written 99-12-23
$

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ELEMENT BM LIQ
ELEMENT NM LIQ
ELEMENT SM LIQ
ELEMENT YM LIQ
ELEMENT ZM LIQ

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NASA/TM—2001-210753

11
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<td>$5.00000E+02$</td>
<td>$-45000+30*T; 6.00000E+03$ N REF:0</td>
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<tr>
<td>$G(MN, ZM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$-45000+30*T; 6.00000E+03$ N REF:0</td>
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<tr>
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<tr>
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<td>$20000; 6.00000E+03$ N REF:0</td>
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<td>$G(MN, BM, ZM; 0)$</td>
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<td>$20000; 6.00000E+03$ N REF:0</td>
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<tr>
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<td>$20000; 6.00000E+03$ N REF:0</td>
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<tr>
<td>$G(MN, NM, YM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$-160000+50*T; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(MN, NM, YM; 1)$</td>
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<td>$-120000+15*T; 6.00000E+03$ N REF:0</td>
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<tr>
<td>$G(MN, NM, ZM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$20000; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(MN, SM, SM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$20000; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(MN, SM, YM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$20000; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(MN, YM, ZM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$20000; 6.00000E+03$ N REF:0</td>
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</tbody>
</table>

**Phase N % 1 1.0**

**Constituent N :BM,NM,SM,YM,ZM:**

<table>
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<th>Parameter Description</th>
<th>Value</th>
<th>Note</th>
</tr>
</thead>
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<td>$G(N, BM; 0)$</td>
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<td>$-5000+30*T; 6.00000E+03$ N REF:0</td>
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</tr>
<tr>
<td>$G(N, YM; 0)$</td>
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<td>$-50000+30*T; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(N, ZM; 0)$</td>
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<td>$-50000+30*T; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(N, BM, NM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$-15200-7*T; 6000 N REF:0</td>
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<tr>
<td>$G(N, BM, NM; 1)$</td>
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<td>$-13000+6.7*T; 6000 N REF:0</td>
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<tr>
<td>$G(N, BM, SM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$20000; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(N, BM, YM; 0)$</td>
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<td>$20000; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(N, BM, ZM; 0)$</td>
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<tr>
<td>$G(N, NM, YM; 0)$</td>
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<tr>
<td>$G(N, NM, YM; 1)$</td>
<td>$500$</td>
<td>$13000+5*T; 6000 N REF:0</td>
</tr>
<tr>
<td>$G(N, NM, ZM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$50000; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(N, SM, YM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$20000; 6.00000E+03$ N REF:0</td>
</tr>
<tr>
<td>$G(N, YM, ZM; 0)$</td>
<td>$5.00000E+02$</td>
<td>$20000; 6.00000E+03$ N REF:0</td>
</tr>
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</table>
PHASE P % 2 1 1 !
CONSTITUENT P : NM : ZM : !
PARAMETER G(P,NM,ZM;0) 5.00000E+02 -180450+57*T; 6.00000E+03 N REF:0 !

PHASE RB % 2 4 3 !
CONSTITUENT RB : SM : ZM : !
PARAMETER G(RB,SM,ZM;0) 5.00000E+02 -557200+171.5*T; 6.00000E+03 N REF:0 !

PHASE RH % 2 2 5 !
CONSTITUENT RH : SM : ZM : !
PARAMETER G(RH,SM,ZM;0) 5.00000E+02 -620000+185*T; 6.00000E+03 N REF:0 !

PHASE RM % 2 2 7 !
CONSTITUENT RM : SM : ZM : !
PARAMETER G(RM,SM,ZM;0) 5.00000E+02 -822800+258*T; 6.00000E+03 N REF:0 !

PHASE T % 1 1.0 !
CONSTITUENT T : BM,NM,SM,YM,ZM : !
PARAMETER G(T,BM;0) 5.00000E+02 -35000+26.462*T; 6.00000E+03 N REF:0 !
PARAMETER G(T,NM;0) 5.00000E+02 -35000+26.462*T; 6.00000E+03 N REF:0 !
PARAMETER G(T,SM;0) 5.00000E+02 -35000+26.462*T; 6.00000E+03 N REF:0 !
PARAMETER G(T,YM;0) 5.00000E+02 -35618+26.462*T; 6.00000E+03 N REF:0 !
PARAMETER G(T,ZM;0) 5.00000E+02 -93955+31.755*T; 6.00000E+03 N REF:0 !
PARAMETER G(T,BM,NM;0) 5.00000E+02 20000; 6.00000E+03 N REF:0 !
PARAMETER G(T,BM,SM;0) 5.00000E+02 20000; 6.00000E+03 N REF:0 !
PARAMETER G(T,BM,ZM;0) 5.00000E+02 -25800; 6.00000E+03 N REF:0 !
PARAMETER G(T,NM,SM;0) 5.00000E+02 20000; 6.00000E+03 N REF:0 !
PARAMETER G(T,NM,ZM;0) 5.00000E+02 20000; 6.00000E+03 N REF:0 !
PARAMETER G(T,SM,YM;0) 5.00000E+02 20000; 6.00000E+03 N REF:0 !
PARAMETER G(T,SM,ZM;0) 5.00000E+02 20000; 6.00000E+03 N REF:0 !
PARAMETER G(T,YM,ZM;0) 5.00000E+02 -25800; 6.00000E+03 N REF:0 !

LIST OF REFERENCES
NUMBER SOURCE

APPENDIX II

$ MACRO bmymzm-3.tcm GENERATED ON PC/WINDOWS NT DATE 0-3-8
$ Make the BM-YM-ZM ternary with a L(C,BM,YM,ZM;0) term directly $

. go da
. sw user nasa5rl
. def-sys bm ym zm
. get

. go gib
. l-p-d
. c
. ent-par

. L
. C
. bm
. ym
. zm

. 0
. 500
. @? Enter Value

. 6000
. N

. l-p-d
. c

. go p-3
. sp-op
. SET_MISCIBILITY_GAP
. c
. 2
. bm ym

. s-c n=1, t=1600, p=1e5, x(ym)=.78, x(zm)=.219
. l-c
. c-e

. s-a-s
. N
. Y
. zm
. Y
. bm ym
. N
. N
. bm
. N
. bm
. N
bm
N
bm
N
bm
N
bm

s-c n=1, t=1600, p=1e5, x(ym)=.78, x(zm)=.219
l-c
c-e *
l-e
SCREEN
VWCS
s-a-v 1 x(ym)
 0
 1
  .025
s-a-v 2 x(zm)
 0
 1
  .025
add
-1
li-in-eq
save
bmymzm-3
y
map
post
s-d-t
y
Y
n
pl
SCREEN
ba
read
bmyzmzm-3.PL3
l-c
s-c x(ym)=1e-3, x(zm)=.21
l-c
c-e *
c-e
l-e
SCREEN
VWCS
add
2
li-in-eq
s-c x(ym)=.05, x(zm)=.9
l-c
c-e
c-e
l-e
SCREEN
VWCS
add
-2
li-in-eq
save
bmymzm-3.PL3
y
map
post
s-t-s
4
s-c-t L_L
YbO1.5
s-c-t l-r
YO1.5
s-c-t t
ZrO2
s-f
4
0.3
s-a-t-s
x
N
X(YO1.5)
S-a-t-s
y
N
x(ZrO2)
s-tit
YbO1.5-YO1.5-ZrO2 1600K
s-l-c
n
pl
SCREEN
dump
bmp
ba
read bmymzm-3.PL3
li-in-eq
s-c x(ym)=0.78, x(zm)=0.21
l-c
c-e *
l-e

add
-1
save
bmymzm-3.PL3
y
map
post
pl
SCREEN
set-int
ba
REFERENCES

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26. ibid., Figure 4426.
27. ibid., Figure 347.
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29. ibid., Figure 5234.
30. ibid., Figure 5235.

NASA/TM—2001-210753
34. L.V. Gurvich, V.S. Iorish, D.V. Chekhovskoi, and V.S. Yungman, IVTANTHERMO-A Thermodynamic Database and Software System for the Personal Computer, NIST Special Database 5, NIST, Gaithersburg, MD, 1993.


| TABLE I.—AVAILABLE BINARY PHASE DIAGRAMS |
|---------------------|---------------------|---------------------|---------------------|---------------------|
| ZrO, Y2O3, Yb2O3, Sc2O3, Nd2O3 | ZrO, Y2O3, Yb2O3, Sc2O3, Nd2O3 |
| ZrO, Y2O3, Yb2O3, Sc2O3, Nd2O3 | ZrO, Y2O3, Yb2O3, Sc2O3, Nd2O3 |
| ZrO, Y2O3, Yb2O3, Sc2O3, Nd2O3 | ZrO, Y2O3, Yb2O3, Sc2O3, Nd2O3 |
| ZrO, Y2O3, Yb2O3, Sc2O3, Nd2O3 | ZrO, Y2O3, Yb2O3, Sc2O3, Nd2O3 |

<p>| TABLE II.—ZIRCONIA AND RARE EARTH OXIDES TRANSITION TEMPERATURES AND HEATS OF TRANSFORMATION |
|---------------------|---------------------|---------------------|---------------------|---------------------|</p>
<table>
<thead>
<tr>
<th>Oxide</th>
<th>Phase Change</th>
<th>T(trans) K</th>
<th>ΔH (kJ/mol)</th>
<th>ΔS (J/mol·K)</th>
<th>Reference</th>
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</thead>
<tbody>
<tr>
<td>ZrO</td>
<td>Monoclinic → Tetragonal</td>
<td>1448</td>
<td>5.941</td>
<td>4.103</td>
<td>Kubachevski (33)</td>
</tr>
<tr>
<td>Monoclinic → Tetragonal</td>
<td>1478</td>
<td>8.075</td>
<td>5.464</td>
<td>Pankratz (35)</td>
<td></td>
</tr>
<tr>
<td>Monoclinic → Tetragonal</td>
<td>1445</td>
<td>8.4</td>
<td>5.813</td>
<td>IVTAN (34)</td>
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<tr>
<td>Tetragonal → Cubic</td>
<td>2568</td>
<td>5.9</td>
<td>2.3</td>
<td>Kubachevski (33)</td>
<td></td>
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<tr>
<td>Tetragonal → Cubic</td>
<td>2620</td>
<td>13.001</td>
<td>4.962</td>
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<tr>
<td>Cubic → Liquid</td>
<td>2953</td>
<td>87</td>
<td>29.492</td>
<td>Kubachevski (33)</td>
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</tr>
<tr>
<td>Cubic → Liquid</td>
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<td>Y2O3</td>
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<td>2550</td>
<td>54</td>
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<td>2712</td>
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<td>Hexagonal → Liquid</td>
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<td>48.006</td>
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<td>Nd2O3</td>
<td>N phase → Hexagonal</td>
<td>2333</td>
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<tr>
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<td>48.207</td>
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### TABLE III—OXIDES AND ABBREVIATIONS

<table>
<thead>
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<th>Oxide</th>
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<td>ZrO₂</td>
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<td>YO₃</td>
<td>YM</td>
</tr>
<tr>
<td>YbO₃</td>
<td>BM</td>
</tr>
<tr>
<td>NdO₁₅</td>
<td>NM</td>
</tr>
<tr>
<td>ScO₁₇</td>
<td>SM</td>
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### TABLE IV—MOLE FRACTION OF RE₂O₃ IN RE₂O₃-ZRO₂ AND EQUIVALENT MOLE FRACTION OF REO₂ IN RE₂O₃-ZRO₂

<table>
<thead>
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<th>Mole Fraction RE₂O₃</th>
<th>Equivalent Mole Fraction REO₂</th>
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</thead>
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<td>x(RE₂O₃)</td>
<td>x(REO₂)</td>
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<tr>
<td>0.1</td>
<td>0.182</td>
</tr>
<tr>
<td>0.2</td>
<td>0.333</td>
</tr>
<tr>
<td>0.3</td>
<td>0.462</td>
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<td>0.5</td>
<td>0.667</td>
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<td>0.75</td>
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<td>0.824</td>
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<td>0.9</td>
<td>0.947</td>
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### TABLE V—EFFECT OF FIRST ORDER INTERACTION PARAMETER ON THE LOCATION OF THE MISCIBILITY GAP

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<tr>
<td>Shifted to Right</td>
<td>Positive</td>
</tr>
<tr>
<td>Shifted to Left</td>
<td>Negative</td>
</tr>
</tbody>
</table>
Figure 1.—Two experimentally determined $\text{Y}_2\text{O}_3$-$\text{ZrO}_2$ phase diagrams. (a) Stubican et al. 1981 (22, 7). (b) Pascual and Duran, 1983 (23, 9). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1984] by the American Ceramic Society. All rights reserved.
Figure 2.—Experimentally determined Yb₂O₃-ZrO₂ phase diagrams. (a) Rouanet (24), (b) Stubican et al. (11). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1985] by the American Ceramic Society. All rights reserved.
Figure 3.—Experimentally determined Sc$_2$O$_3$-ZrO$_2$ phase diagram (25). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.
Figure 4.—Experimentally determined Nd₂O₃-ZrO₂ phase diagram (26).
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Figure 5.—Experimentally determined Yb$_2$O$_3$-Y$_2$O$_3$ phase diagram (27). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.

Figure 6.—Experimentally determined Sc$_2$O$_3$-Y$_2$O$_3$ phase diagram (28). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.
Figure 7.—Experimentally determined Nd$_2$O$_3$-Y$_2$O$_3$ phase diagram (29). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.

Figure 8.—Experimentally determined Sc$_2$O$_3$-Yb$_2$O$_3$ phase diagram (28). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.
Figure 9.—Experimentally determined Nd$_2$O$_3$-Yb$_2$O$_3$ phase diagram (30). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.

Figure 10.—Experimentally determined Nd$_2$O$_3$-Sc$_2$O$_3$ phase diagram (28). Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.
Figure 11.—Experimental determined Y$_2$O$_3$-Yb$_2$O$_3$-ZrO$_2$ phase diagram (31) at (a) 1200 °C, (b) 1400 °C, and (c) 1650 °C. Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.
Figure 11.—Concluded. (c) 1650 °C. Reprinted with permission of The American Ceramic Society, Post Office Box 6136, Westerville, OH 43086-6136. Copyright [1993] by the American Ceramic Society. All rights reserved.

Figure 12.—Values of $G_{\text{config}} + G_{\text{excess}}$ versus $T$ illustrating the effect of the interaction parameters on the symmetry and location of the miscibility gap.
Figure 13.—Calculated YO$_{1.5}$-ZrO$_2$ phase diagram. Compare to Figure 1(a). The starting point for the calculation is x(YM) = 0.5, T = 3500 K.
Figure 14.—Calculated YbO$_{1.5}$-ZrO$_2$ phase diagram. Compare to Figure 2. The starting point for the calculation is $x(ZM) = 0.5$, $T = 3500$ K, followed $x(ZM) = 0.2$, $T = 2000$ K.
Figure 15.—Calculated ScO$_{1.5}$-ZrO$_2$ phase diagram. Compare to Figure 3.
The starting point for the calculation is $x(ZM) = 0.5$, $T = 3500$ K.
Figure 16.—Calculated NdO_{1.5}-ZrO_{2} phase diagram. Compare to Figure 4. The starting point for the calculation is x(ZM) = 0.1, T = 1800 K, initially mapping up with temperature, followed by x(ZM) = 0.99, T = 1000 K, initially mapping up with temperature, followed by x(ZM) = 0.99, T = 3100 K, initially mapping up with temperature, and finally x(ZM) = 0.1, T = 2300 K.
Figure 17.—Calculated YbO$_{1.5}$-YO$_{1.5}$ phase diagram. Compare to Figure 5. The starting point for this calculation is $x(YM) = 0.001$, $T = 2500$ K, map with temperature increasing first.
Figure 18.—Calculated ScO$_{1.5}$-YO$_{1.5}$ phase diagram. Compare to Figure 6. The starting point for this calculation is x(SM) = 0.001, T = 2500 K, map with temperature increasing first.
Figure 19.—Calculated NdO$_{1.5}$-YO$_{1.5}$ phase diagram. Compare to Figure 7. The starting point for this calculation is $x$(YM) = 0.01, $T$ = 3000 K, followed by $x$(YM) = 0.99, $T$ = 1500 K.
Figure 20.—Calculated ScO$_{1.5}$-YO$_{1.5}$ phase diagram. Compare to Figure 8. The starting point for this calculation is $x(SM) = 0.001$, $T = 2500$ K, map with temperature increasing first.
Figure 21 — Calculated NdO$_{1.5}$-YbO$_{1.5}$ phase diagram. Compare to Figure 9. The starting point for this calculation is $x$(BM) = 0.99, $T = 3000$ K, followed by $x$(BM) = 0.1, $T = 1500$ K.
Figure 22.—Calculated ScO$_{1.5}$-NdO$_{1.5}$ phase diagram. Compare to Figure 10. The starting point for this calculation is $x$(SM) = 0.001, $T$ = 3000 K, followed by $x$(SM) = 0.001, $T$ = 1000 K.
Figure 23.—Calculated activities across the YO$_{1.5}$-ZrO$_2$ phase diagram at 2773 K (line) and experimental data (points) of Belov and Semenov et al. (39). This line was generated using a starting point of $x$(ZM) = 0.8 and then a step with options command. (a) $a$ (YO$_{1.5}$). (b) $a$ (ZrO$_2$).
Figure 24.—Calculated activities across the ScO$_{1.5}$-ZrO$_2$ phase diagram at 2600 K (line) and experimental data (points) of Belov and Semenov et al. (40). This line was generated using a starting point of $x$(ZM) = 0.8 and then a step with options command. (a) a (ScO$_{1.5}$). (b) a (ZrO$_2$).
Figure 25.—Calculated ternary section for the YbO$_{1.5}$-YO$_{1.5}$-ZrO$_2$ system at 1600 K. (a) $L(C, BM, YM, ZM; 0) = +10\ 000$ insufficient to form a continuous two phase region. (b) $L(C, BM, YM, ZM; 0) = +100\ 000$ continuous two phase region.
Figure 26.—Calculated ternary diagram for the YbO$_{1.5}$-YO$_{1.5}$-ZrO$_2$ system at (a) 1473 K, (b) 1673 K, and (c) 2000 K.
Figure 27.—Calculated phase fraction diagram for 0.05YbO$_{1.5}$-0.04YO$_{1.5}$-0.01ScO$_{1.5}$-0.9ZrO$_2$.

Figure 28.—Calculated isopleth for YbO$_{1.5}$-YO$_{1.5}$-NdO$_{1.5}$-ZrO$_2$ with $x$(YbO$_{1.5}$) = $x$(ScO$_{1.5}$) = 0.02.
### Abstract (Maximum 200 words)

A database for \( \text{Y}_{0.5}\text{Nd}_{0.5}\text{YbO}_{1.5}\text{ScO}_{1.5}\text{ZrO}_2 \) for ThermoCalc (ThermoCalc AB, Stockholm, Sweden) has been developed. The basis of this work is the \( \text{YO}_{1.5}\text{ZrO}_2 \) assessment by Y. Du, Z. Jin, and P. Huang, “Thermodynamic Assessment of the \( \text{ZrO}_2\text{YO}_{1.5} \) System,” J. Am. Ceram. Soc. 74, [7], pp. 1569-77 (1991). Experimentally only the \( \text{YO}_{1.5}\text{ZrO}_2 \) system has been well-studied. All other systems are only approximately known. The major simplification in this work is the treatment of each single cation unit as a component. The pure liquid oxides are taken as reference states and two term lattice stability descriptions are used for each of the components. The limited experimental phase diagrams are reproduced.