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Modeling and Prediction of the Noise From Non-Axisymmetric Jets

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Overview of the Effort

The objective of this task was the development of source and directivity models to predict noise from non-axisymmetric jets. The technical approach is based on an acoustic analogy (Refs. 1 and 2) for which sound propagation is described by a Green's function for an appropriate set of linear inhomogeneous equations, and models are constructed for the nominal sound sources. Near-term goals were to develop reduced-order models that can represent three-dimensional effects associated sound generation and propagation in an exhaust jet emanating from a rectangular nozzle. Longer-term goals were to extend the initial models developed to include additional non-axisymmetric geometries, and improve their fidelity through the use of improved approximate methods and more direct numerical methods of solving the relevant governing equations.

The goals of the task directly supported a number of milestones of the NASA Fundamental Aeronautics Program Supersonics (now High-Speed) Project for the development of physics-based noise prediction tools for use in assessing potential noise-reduction concepts as part of the design process for the next generation civilian aircraft.

A literature survey and an assessment of the (then) current state of the art in jet noise prediction for non-axisymmetric jets were carried out as the first task under this contract. An informal written report was provided as the first deliverable and this report served as a guide for the overall direction of the research. This informal report is incorporated here as Appendix A.

The first technical task of this contract was to develop source and propagation models that extend existing methods for axisymmetric jets to include three-dimensional effects associated with sound propagation from a rectangular nozzle. The nozzles of interest included generic rectangular nozzles of various aspect ratios, as well as ones with internal noise sources, such as from mixers, and those which consist of a series of smaller nozzles.

A model was developed for sound propagation in a rectangular jet based on approximating the mean flow as a set of concentric ellipses in the cross flow plane, and mapping the equation governing the sound propagation to a computational domain where it can be solved with about the same computational effort as required for a round jet. A computer code was written to solve the resulting equations. The code was validated by comparing results with existing high- and low-frequency asymptotic solutions.

An existing source model, which had been used by the contractor for noise predictions in round jets, was integrated with the propagation model into a jet noise prediction code for rectangular jets. Preliminary testing of this code was carried out on a rectangular nozzle which consists of a series of smaller nozzles. A Reynolds-averaged Navier-Stokes solution was provided to the contractor for use as input to the code to provide the mean flow and turbulence characteristics of this jet. The prediction results were compared with data provided by NASA. The predictions were in qualitative agreement with overall experimentally measured azimuthal directivity patterns, but the initial model did not capture salient details of the radiated sound field, including high-frequency jet-jet shielding effects.

A model was formulated to treat the shielding effects created when the exhaust stream of a propulsive nozzle consists of a series of smaller nozzles in close proximity to one another.

An improved sound source model was developed to better represent features observed in newly available experimental data. Extensive testing of this model was carried out for round jets over a range of flow speeds and it was found to provide better prediction results than previous models. Details of the model and the results of the round jet predictions were reported in a journal publication (Ref. 3).

The new source model was combined with the original sound propagation model developed for rectangular jets to produce a new version of the rectangular jet noise prediction code. This code was validated using a set of rectangular nozzles whose geometries were specified by NASA. Nozzles of aspect ratios two, four and eight were studied at jet exit Mach numbers of 0.5, 0.7 and 0.9, for a total of nine cases. Reynolds-averaged Navier-Stokes solutions for these jets were provided to the contractor for use as input to the code. Quantitative comparisons of the predicted azimuthal and polar directivity of the acoustic spectrum were made with experimental data provided by NASA. The results of these comparisons, along with a documentation of the propagation and source models, were reported in a journal article publication (Ref. 4). The complete set of computer codes and computational modules that make up the prediction scheme, along with a user's guide describing their use and example test cases, was provided to NASA as a deliverable of this task.

The use of conformal mapping, along with simplified modeling of the mean flow field, for noise propagation modeling was explored for other nozzle geometries, to support the task milestone of developing methods which are applicable to other geometries and flow conditions of interest to NASA. A model to represent twin round jets using this approach was formulated and implemented.

A general approach to solving the equations governing sound propagation in a locally parallel non-axisymmetric jet was developed and implemented, in aid of the tasks and milestones charged with selecting more exact numerical methods for modeling sound propagation, and developing methods that have application to other nozzle geometries. The method is based on expansion of both the mean-flow-dependent coefficients in the governing equation and the Green's function in series of orthogonal functions. The method was coded and tested on two analytically prescribed mean flows which were meant to represent noise reduction concepts being considered by NASA. Testing (Ref. 5) showed that the method was feasible for the types of mean flows of interest in jet noise applications. Subsequently, this method was further developed to allow use of mean flow profiles obtained from a Reynolds-averaged Navier-Stokes (RANS) solution of the flow. Preliminary testing of the generalized code was among the last tasks completed under this contract.

The stringent noise-reduction goals of NASA's Fundamental Aeronautics Program suggest that, in addition to potentially complex exhaust nozzle geometries, next generation aircraft will also involve tighter integration of the engine with the airframe. Therefore, noise generated and propagated by jet flows in the vicinity of solid surfaces is expected to be quite significant, and reduced-order noise prediction tools will be needed that can deal with such geometries. One important source of noise is that generated by the interaction of a turbulent jet with the edge of a solid surface (edge noise). Such noise is generated, for example, by the passing of the engine exhaust over a shielding surface, such as a wing.

Work under this task supported an effort to develop a RANS-based prediction code for edge noise based on an extension of the classical Rapid Distortion Theory (RDT) to transversely sheared base flows (Refs. 6 and 7). The RDT-based theoretical analysis was applied to the generic problem of a turbulent jet interacting with the trailing edge of a flat plate. A code was written to evaluate the formula derived for the spectrum of the noise produced by this interaction and results were compared with data taken at NASA Glenn for a variety of jet/plate configurations and flow conditions (Ref. 8).

A longer-term goal of this task was to work toward the development of a high-fidelity model of sound propagation in spatially developing non-axisymmetric jets using direct numerical methods for solving the relevant equations. Working with NASA Glenn Acoustics Branch personnel, numerical methods and boundary conditions appropriate for use in a high-resolution calculation of the full equations governing sound propagation in a steady base flow were identified. Computer codes were then written (by NASA) and tested (by OAI) for an increasingly complex set of flow conditions to validate the methods. The NASA-supplied codes were ported to the High-End Computing resources of the NASA Advanced Supercomputing facility for testing and validation against analytical (where possible) and independent numerical solutions. The cases which were completed during the course of this contract were solutions of the two-dimensional linearized Euler equations with no mean flow, a uniform mean flow and a non-uniform mean flow representative of a parallel flow jet.

Detail of Work Expected and Performed Under the Task Order by Task/Subtask

Task 1.0—Assessment of State of the Art For Non-Axisymmetric Jet Noise Prediction

A literature survey and an assessment of the current state of the art of noise prediction for non-axisymmetric jets were carried out as the first task under this contract. An informal written report was provided as the first deliverable. This report is incorporated herein as Appendix A.

Task 1.1—Development of a Sound Propagation Model For Rectangular Jets

The objective of this task was to develop a simplified mean flow model which allows a solution for the Green's function for a rectangular jet to be obtained with minimal increase in computational resources relative those required for a circular jet. The method was to be coded, tested and validated against available limiting forms of the solution (i.e., high- and low- frequency limits). Further, an interface was to be developed so that the code could be used with a RANS solution of the mean flow.

To this end, the mean flow was modeled as having mean velocity and temperature contours in the cross flow plane that are concentric and coincident with level curves of a cylindrical elliptic coordinate system. The problem for the Green's function was then transformed from Cartesian coordinates to cylindrical elliptical coordinates using the conformal mapping

$$z = C \cosh W,$$

which maps the physical, $z = y_2 + iy_3$, plane into the strip $0 \leq u < \infty, -\pi \leq v \leq \pi$, in the $W = u + iv$ plane such that lines $u = \text{constant}$ correspond to concentric ellipses and $v = \text{constant}$ to confocal hyperbolas. The solution for the Green's function is then periodic in v and can be expanded in a Fourier series to reduce the partial differential equation to a system of coupled ordinary differential equations. Figure 1 shows the corresponding mapping.

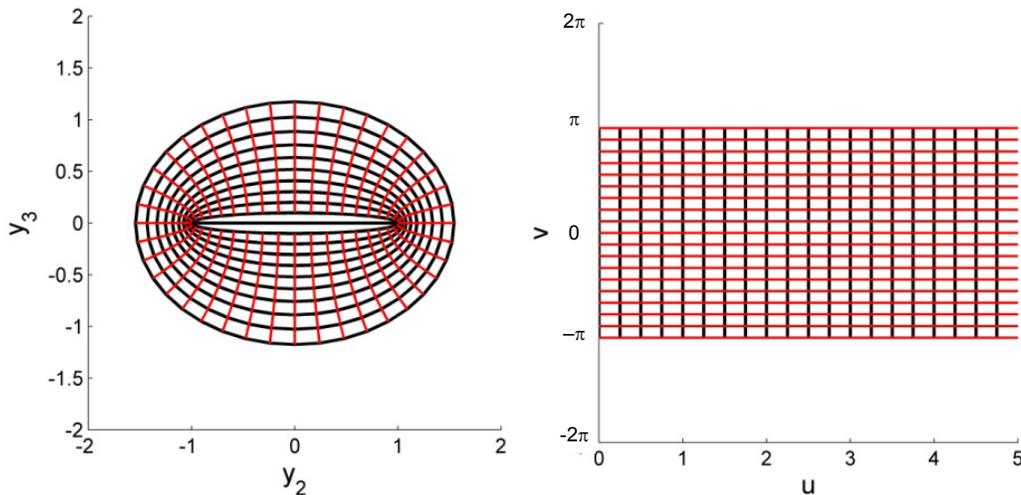


Figure 1.—Mapping from Cartesian to elliptical coordinates. Lines of constant u (black), lines of constant v (red).

The resulting coupled set of ordinary differential equations was solved using a hybrid spectral-finite difference method. A second-order central difference approximation was used to represent derivatives in the u coordinate in the both the differential equations and boundary conditions. This yields a highly banded system of algebraic equations which was solved using a standard library routine.

The code was tested for sensitivity to mesh size, domain boundaries and the number of Fourier modes using analytic mean velocity and temperature profiles and validated by comparing with asymptotic limiting forms of the problem. Figure 2 and Figure 3 show results from the code compared with the low-frequency and high-frequency asymptotic solutions, respectively.

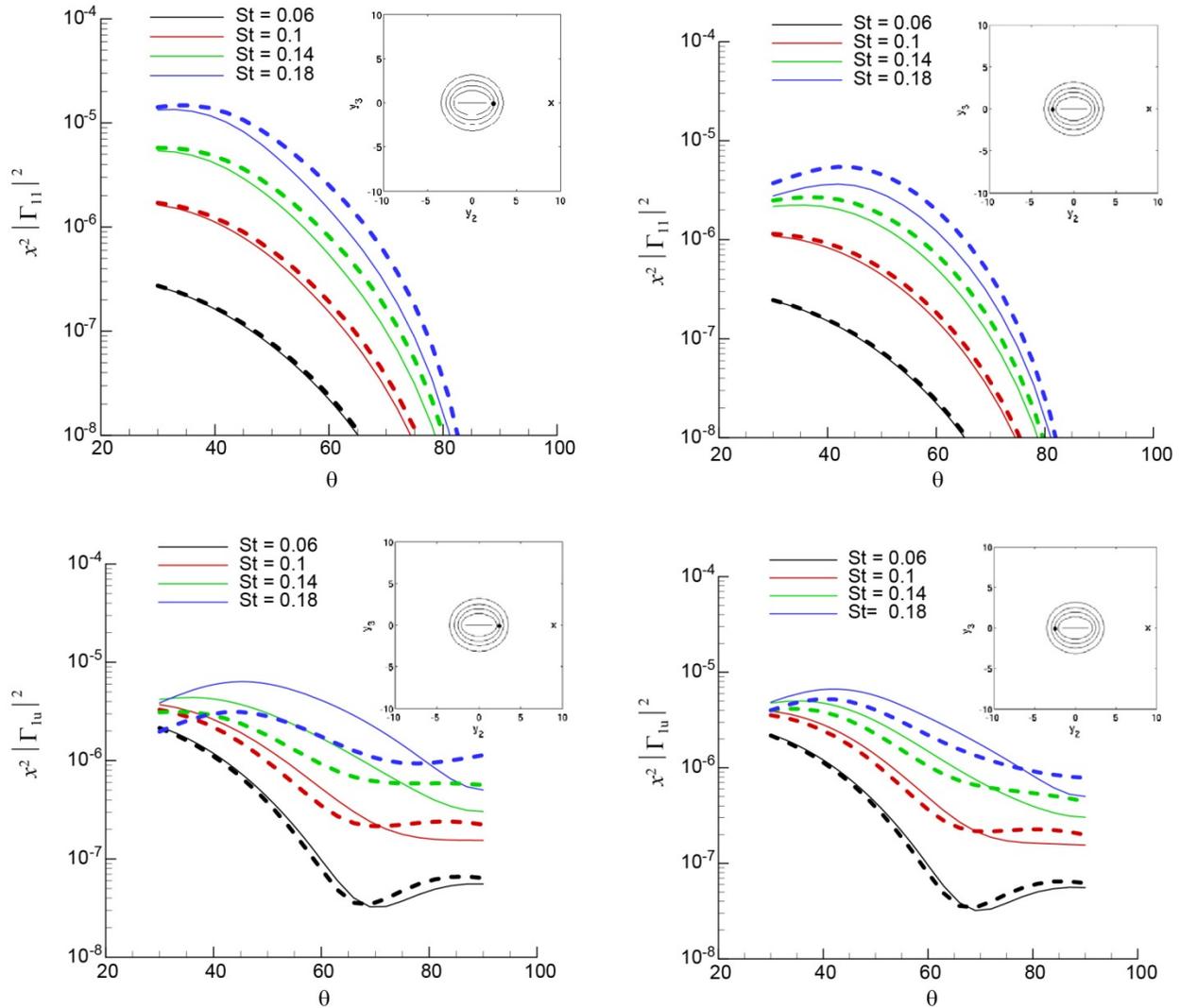


Figure 2.—Comparison of numerical solution for the Green's function (solid) with low-frequency solution (dashed) for different Strouhal numbers.

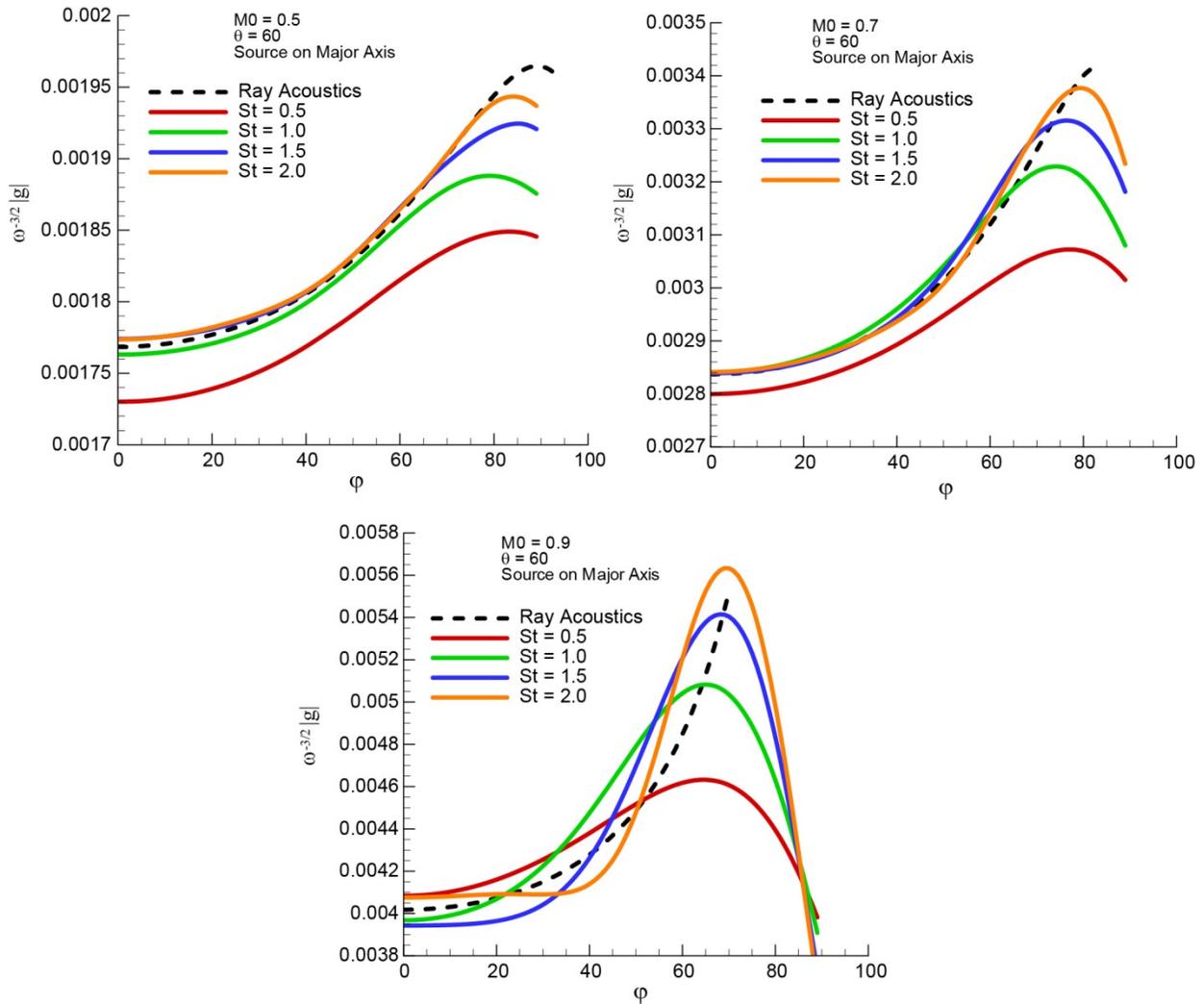


Figure 3.—Comparison of numerical solution for the Green's function (solid) with high-frequency (ray acoustics) solution (dashed) for different Strouhal numbers.

An interface was written to allow the Green's function solver to be used with mean flow profiles obtained from a RANS solution of the flow. The interface consists of a Tecplot macro to extract axial planes from a full RANS solution, plot the contours of mean streamwise velocity and export the coordinates of the contour levels to a data file. A FORTRAN code then reads these coordinates and constructs a fit of them to level curves in elliptic coordinates. The procedure was applied to a set of generic rectangular jets and to a distributed exhaust nozzle which is rectangular in overall cross section but made of up of multiple smaller nozzles. Figure 4 shows an example of the quality of the mean flow model for a 4:1 aspect ratio rectangular nozzle at various axial distances from the exit. The representation of the mean acoustic Mach number (mean streamwise velocity divided by the ambient speed of sound) by surfaces of constant u in elliptic coordinates using this method is reasonably good, except at locations very close to the nozzle exit, where the contours from the RANS solutions tend to be much flatter than the elliptical coordinates. Sufficiently far downstream the contours become circular and the mean flow resembles that of an axisymmetric jet.

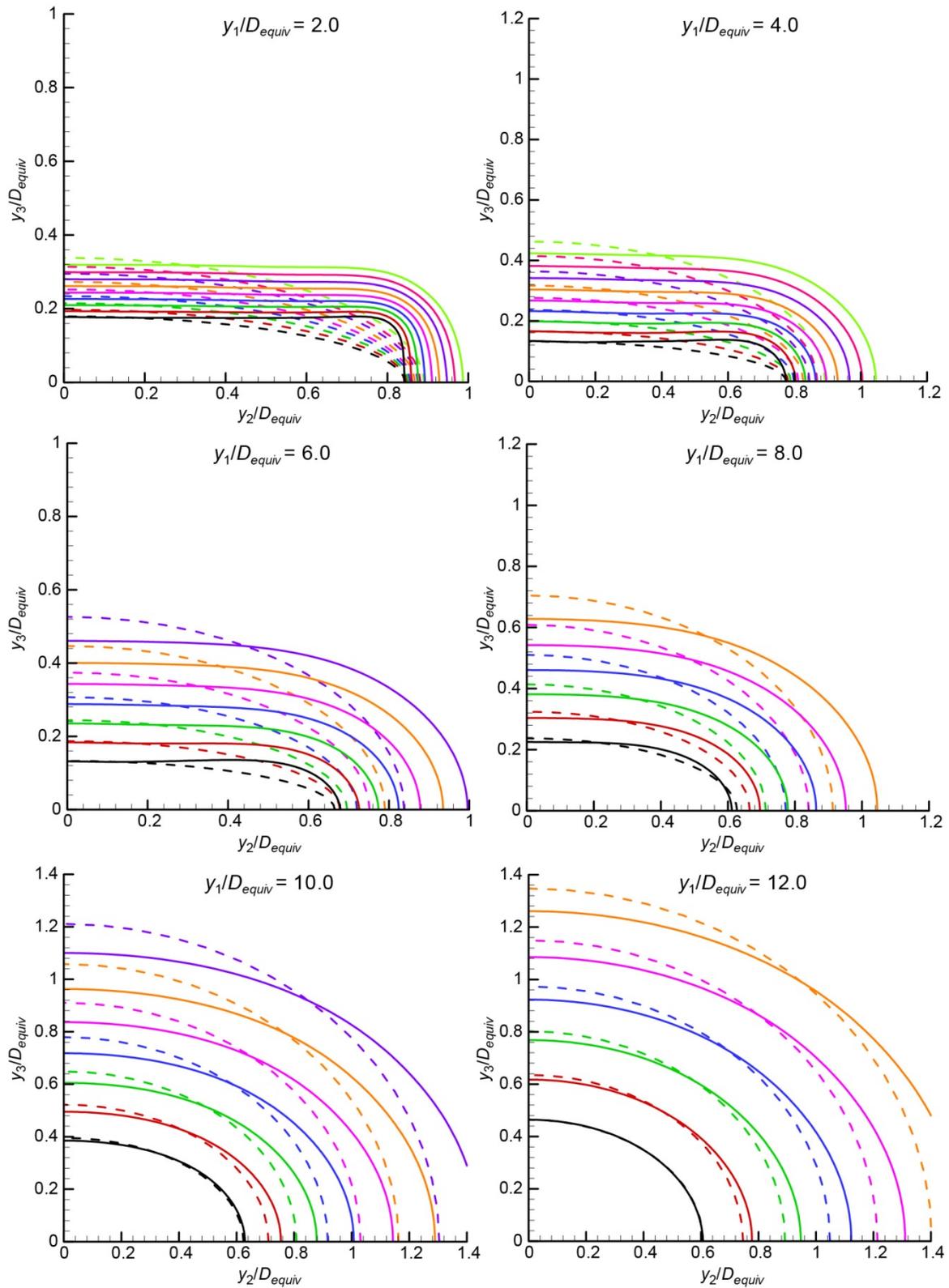


Figure 4.—Contours of mean acoustic Mach number extracted from the RANS solutions (solid) and their representation in elliptical coordinates (dashed) at a number of axial locations for an aspect ratio four rectangular jet with exit Mach number 0.9. Colors indicate contour levels

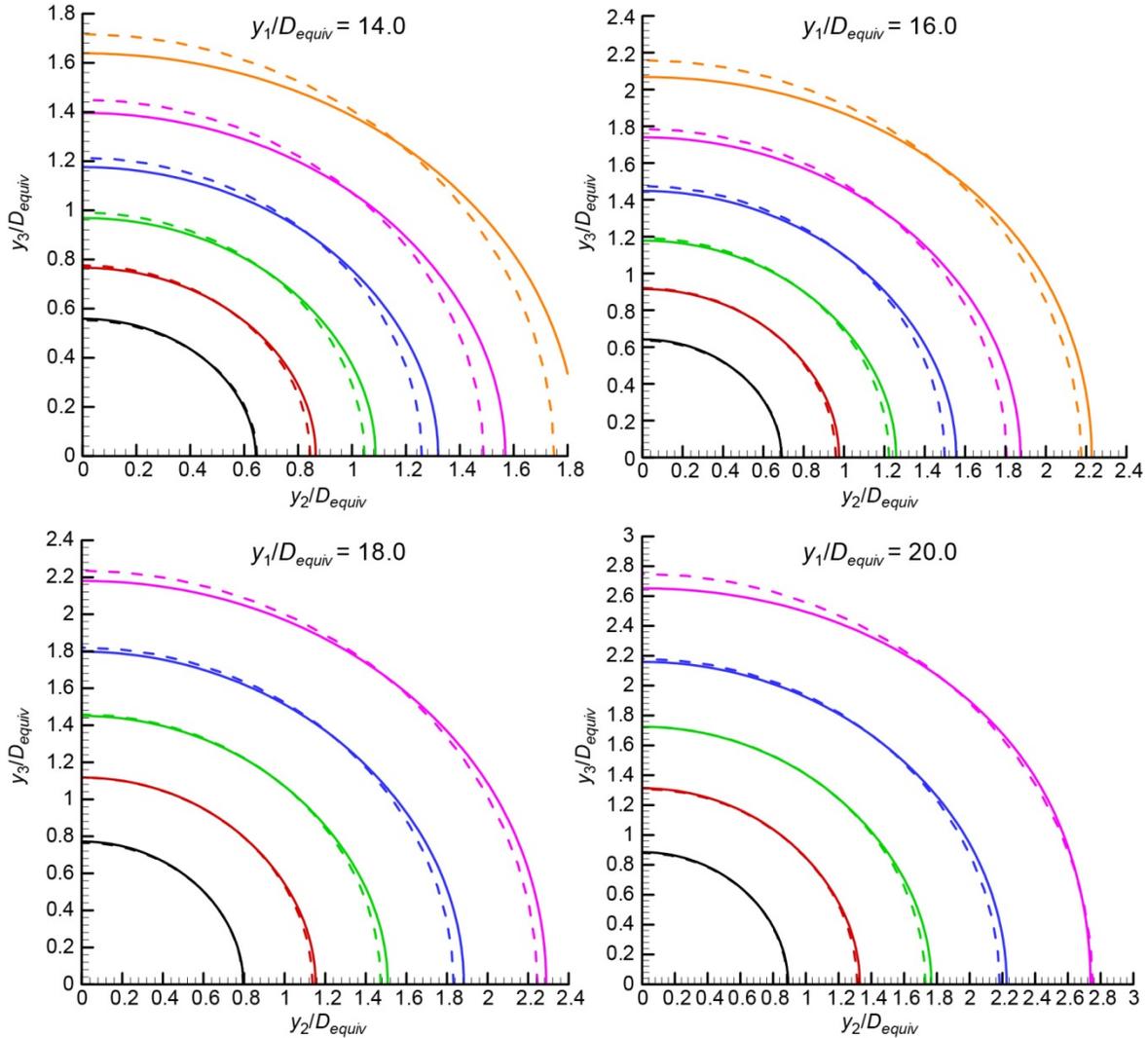


Figure 4.—Concluded.

Task 1.2—Definition of a Source Model For a Rectangular Jet

In an acoustic analogy approach to jet noise prediction, models are needed for the nominal source terms. For the acoustic analogy formulation used in this work, the source terms appearing in the formula for the acoustic spectrum are the components of the Reynolds stress auto-covariance tensor

$$R_{\lambda j \kappa l}(\mathbf{y}, \boldsymbol{\eta}, \tau) \equiv \overline{[\rho v'_{\lambda} v'_{j} - \overline{\rho v'_{\lambda} v'_{j}}](\mathbf{y}, \tau_0) [\rho v'_{\kappa} v'_{l} - \overline{\rho v'_{\kappa} v'_{l}}](\mathbf{y} + \boldsymbol{\eta}, \tau_0 + \tau)}.$$

For the initial calculations carried out under this task, a source model originally developed for round jets was used (Ref. 2). The model assumes that the turbulence is axisymmetric in the cross flow plane and takes into account the experimentally observed tails of certain components of the Reynolds stress auto-covariance. Turbulent length, time and amplitude scales were obtained from the RANS solution.

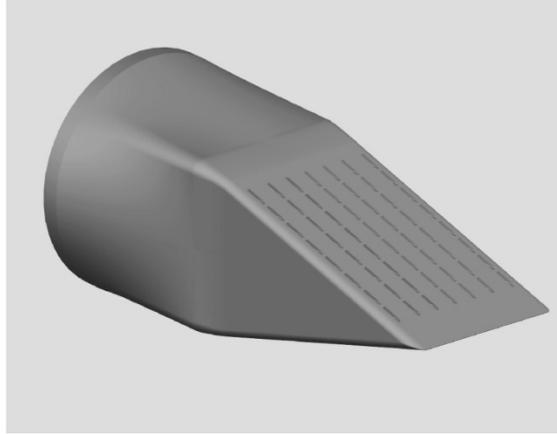


Figure 5.—SPS DEN design tested at NASA.

Task 1.3—Integration of Source and Propagation Models Into a Noise Prediction Code and Initial Comparisons With Experimental Data

The sound propagation model developed under Task 1.1 and the existing source model identified in Task 1.2 were integrated into a single noise prediction code for rectangular jets. The Slanted Pseudo Slot (SPS) Distributed Exhaust Nozzle (DEN) (Ref. 9) was used as an initial test case for this code. Acoustic tests were done on this nozzle in 2002 and a RANS solution for this flow was available for use in these calculations. Figure 5 shows a picture of this nozzle.

Initial predictions using this code for the SPS DEN nozzle were limited to frequencies below which significant jet-jet shielding effects were expected ($St \leq 1.0$). Calculations were carried out for two observer polar angles: one near the jets axis ($\theta = 30^\circ$) and one at the sideline direction ($\theta = 90^\circ$), and two azimuthal observer locations: along the nozzle major axis ($\phi = 0^\circ$) and on the minor axis ($\phi = 90^\circ$). In the first set of test runs carried out, it was assumed that the turbulence was uniform along level surfaces of the mean velocity, i.e., an elliptic ring source model was assumed. These results gave less than 0.1 dB variations in sound levels between the two azimuthal observer locations at both polar angles, contrary to experimental observations. In a second set of calculations, the turbulence levels were allowed to vary (according to the RANS solution) along the mean flow profiles. Results from these calculations at a polar observer angle of $\theta = 30^\circ$ gave a 2 to 3 dB difference between noise levels at $\phi = 0^\circ$ and $\phi = 90^\circ$, with the latter, minor axis, side being louder, in rough agreement with measured data. However, results at $\theta = 90^\circ$ still gave less than 0.1 dB azimuthal variation, unlike the data. Table 1 shows results for $\Delta SPL = \Delta SPL_{\phi=90} - \Delta SPL_{\phi=0}$ for different Strouhal numbers at polar angles $\theta = 90^\circ$ and $\theta = 30^\circ$ from the second set of predictions.

TABLE 1.— ΔSPL PREDICTIONS BETWEEN THE MINOR AND MAJOR AXES OBSERVER LOCATIONS AT VARIOUS STROUHAL NUMBERS AT TWO OBSERVER POLAR ANGLES

St	$\Delta SPL; \theta = 90^\circ$	$\Delta SPL; \theta = 30^\circ$
1.0	-0.07	2.99
0.75	-0.06	2.27
0.5	-0.05	2.8
0.25	-0.06	1.61

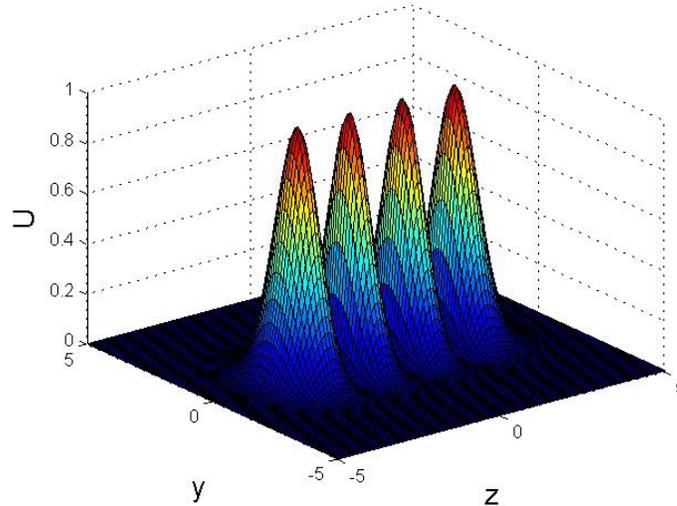


Figure 6.—Example of a flow field generated by expansion in Hermite polynomials for four individual jets.

Task 1.4—Modification of the Model to Treat Distributed Exhaust Nozzles

A model was constructed to represent the mean flow field resulting from a propulsion system involving multiple jet streams in close proximity to one another, such as the SPS DEN. In this model the mean velocity profile in axial planes is represented as a finite series of Hermite polynomials

$$U(y, z) = \frac{e^{-(y^2+z^2)}}{\sqrt{\pi}} \sum_{m=0}^{2n-2} a_{2m} \frac{1}{(2m)!(2)^{2m}} H_{2m}(z)$$

where n is the number of individual jet streams that make up the total exhaust. An example of the flow field due to four individual jet streams is shown in Figure 6.

The governing equations and boundary conditions for the Green's function in terms of Hermite polynomials were derived. This model has not developed further for numerical solution or implementation into a noise prediction code up to this time.

Task 2.0—Selection and Implementation of More Exact Numerical Methods For Sound Propagation in Non-Axisymmetric Jets

This task consisted of separate subtasks which were aimed at developing and implementing more exact numerical methods for solving the scalar Green's function problem governing sound propagation in locally parallel non-axisymmetric jets and the vector Green's function problem describing sound propagation in general, spatially evolving, mean flows.

Under the locally parallel mean flow approximation, the Green's function problem reduces to a partial differential equation for a single (scalar) Green's function. A method was developed to solve this problem by expanding both the mean flow interaction terms in the governing equation and the scalar Green's function in truncated Fourier series in the azimuthal coordinate of a circular cylindrical coordinate system. The Fourier series expansion results in a coupled system of ordinary differential equations for the Fourier modes of the Green's function. Derivatives in these equations, as well as the associated boundary conditions, are approximated by second-order central differences yielding a sparse system of linear algebraic equations that is solved using standard library routine.

The method was tested using analytical mean profiles intended to be representative of two concepts being considered by NASA for noise reduction in aircraft exhaust systems. These are the offset jet and the so-called fluid shield. The former was modeled in the initial tests by a functional form for the acoustic Mach number profile given by

$$M(r, \varphi) = M_C e^{-(1+\alpha-2\alpha \cos \varphi)r^2}$$

where (r, φ) are cylindrical coordinates and α is a constant, and the latter by

$$M(r, \varphi) = M_C \left[e^{-ar^2} + br^2 e^{-c(r-1)^4} g(\varphi) \right]$$

where a, b, c are constants and

$$g(\varphi) = \begin{cases} 0, & 0 \leq \varphi < \pi \\ -\sin \varphi, & \pi \leq \varphi < 2\pi \end{cases}$$

Contours of the acoustic Mach number profiles in a cross flow plane for these two geometries are shown in Figure 7.

The practical utility of the method depends upon being able to represent mean flows of interest in jet noise by a relatively small number of Fourier modes so that the size of the system of algebraic equations to be solved does not become excessively large. In order to assess the potential risk of encountering impractically large systems of algebraic equations for the type of jet flows of interest to NASA, tests were carried out to determine the number of Fourier modes needed to adequately represent the mean flow dependent coefficients in the equation for the Green's function for the two cases describe above.

Figure 8 shows results obtained using various numbers of Fourier modes to reconstruct the mean acoustic Mach number profile and the functions

$$\mathcal{R} = \left\{ \frac{2 \cos \theta}{[1 - M(\mathbf{y}_\perp) \cos \theta]} \frac{\partial M}{\partial y_T} + \frac{1}{\tilde{c}^2(\mathbf{y}_\perp)} \frac{\partial \tilde{c}^2(\mathbf{y}_\perp)}{\partial y_T} \right\},$$

$$\mathcal{F} = \left\{ \frac{2 \cos \theta}{[1 - M(\mathbf{y}_\perp) \cos \theta]} \frac{\partial M}{y_T \partial \varphi_0} + \frac{1}{\tilde{c}^2(\mathbf{y}_\perp)} \frac{\partial \tilde{c}^2(\mathbf{y}_\perp)}{y_T \partial \varphi_0} \right\},$$

and

$$\mathcal{N} = \left\{ \frac{[1 - M(\mathbf{y}_\perp) \cos \theta]^2}{\tilde{c}^2(\mathbf{y}_\perp)} - \frac{\cos^2 \theta}{c_\infty^2} \right\},$$

where \tilde{c}^2 is the mean of the square of the sound speed and c_∞^2 is its ambient value, which are the mean flow dependent coefficients in the equations for the Green's function, for the case of the offset jet. As the plots show, four Fourier modes are sufficient to represent these quantities for this profile.

Figure 9 shows that eight modes are required to obtain roughly the same level of accuracy for these quantities in the case of the fluid shield.

Since the numbers of modes required for these two cases was found to be quite reasonable, the method was implemented into a computer code to solve for the Green's function. The code was first tested for the case of a round jet and the results compared with those obtained from an existing (independent) dedicated round jet noise prediction code. Favorable agreement was found and the method was then applied to the two non-axisymmetric analytic test cases described above.

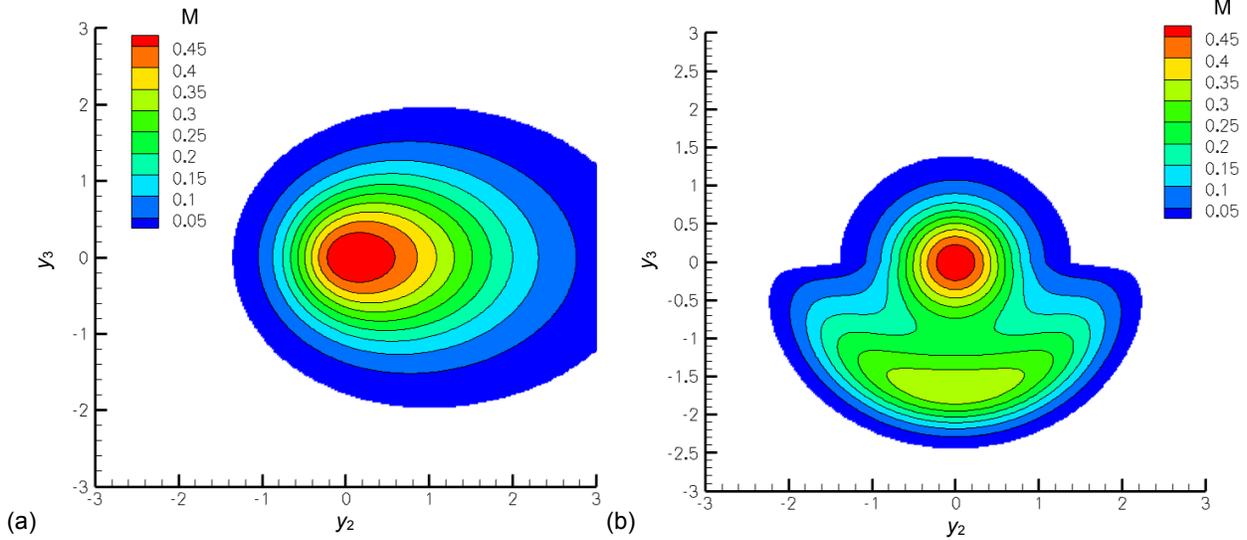


Figure 7.—Mean acoustic Mach number contours for (a) the offset jet and (b) the fluid shield

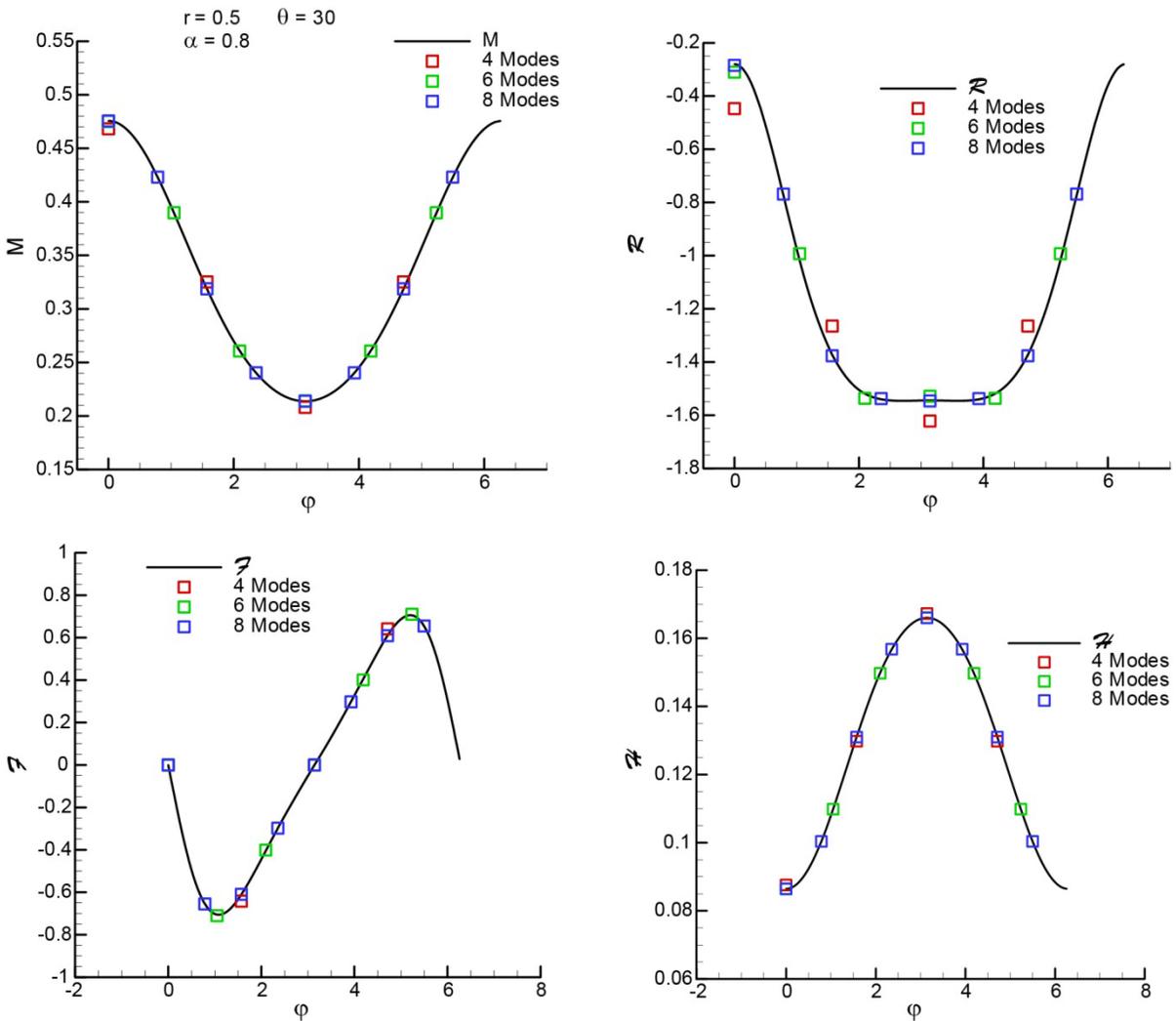


Figure 8.—Azimuthal variation of the mean-flow-dependent coefficients of the scalar Green's function equation and their reconstruction using various Fourier modes. Offset jet.

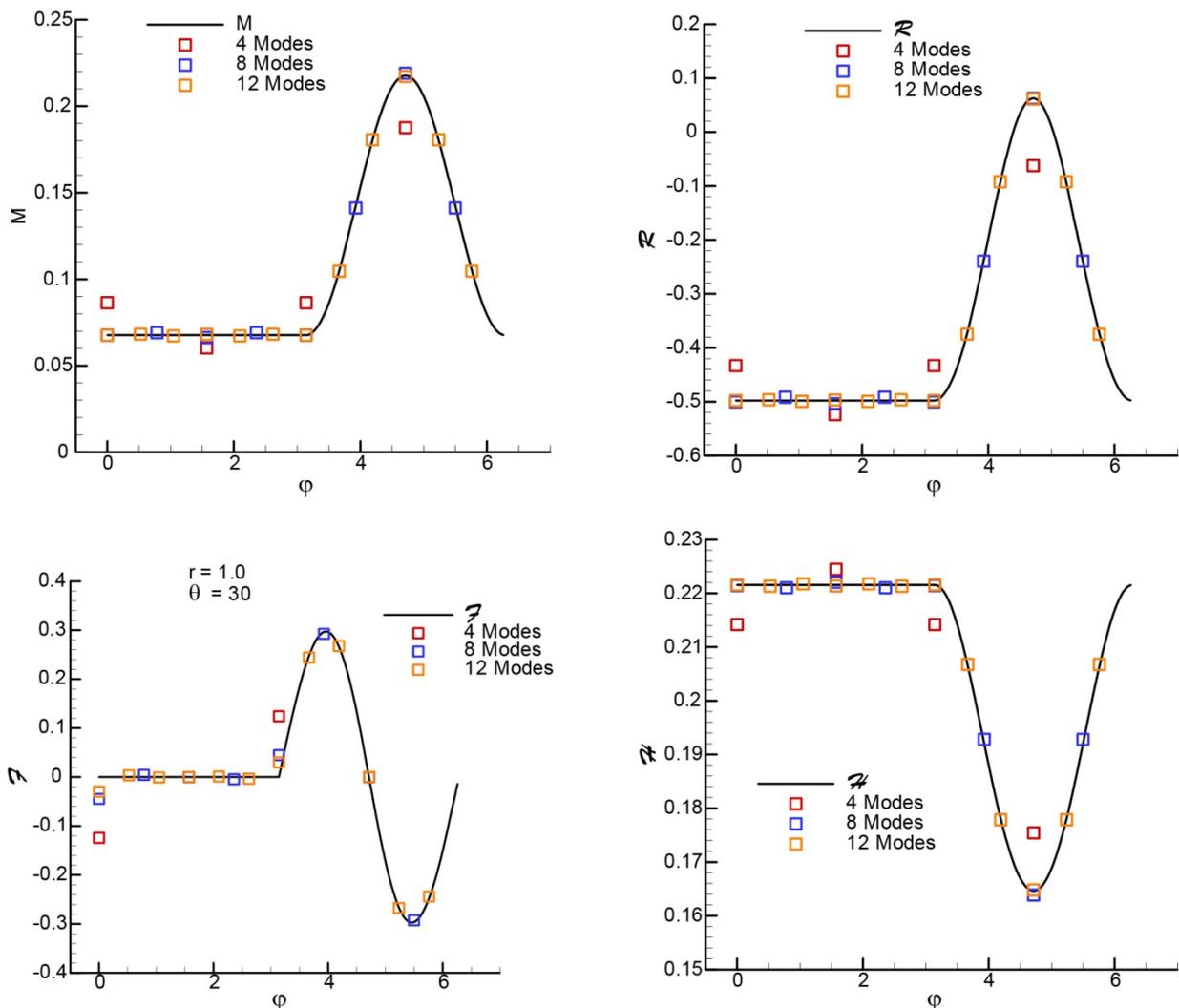


Figure 9.—Azimuthal variation of the mean-flow-dependent coefficients of the scalar Green's function equation and their reconstruction using various Fourier modes. Fluid Shield.

Figure 10 shows results for the real (a) and imaginary (b) parts of the Green's function obtained at one location in the jet ($r = 0.5$, $\varphi = 0$), at a polar angle of 30° , using the code written to implement this method with different numbers of Fourier modes to represent the mean flow dependent coefficients for the case of the offset jet. The results show that the numerical solutions for the Green's function are well converged (in terms of the required number of Fourier modes) with six Fourier modes, with no visible changes when this number is increased to eight.

Figure 11 shows corresponding results for the fluid shield. Results for this case appear to be even better converged in terms of the required number of Fourier modes, a result that was not expected given the mean flow results in Figure 9

Additional tests were carried out for these two test cases to help validate the code; for example the behavior of the appropriately scaled Green's function was demonstrated to behave in a way consistent with the high-frequency asymptotic solution as the non-dimensional frequency increased. All of the results of testing of the code for these two cases are documented in a NASA Contractor Report completed as one of the deliverables of this contract (Ref. 5).

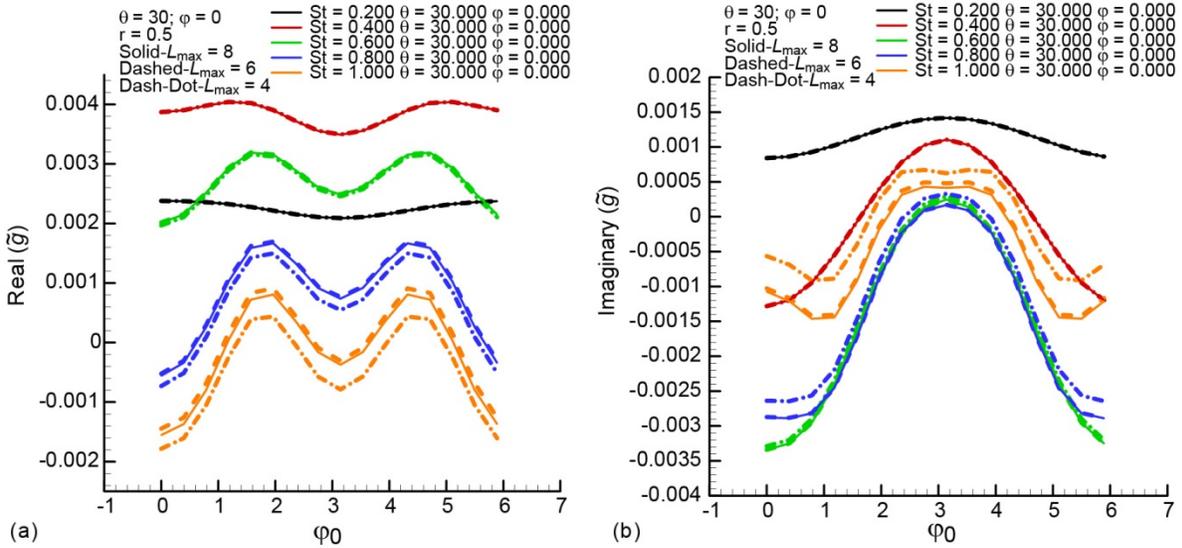


Figure 10.—Results for the (a) real part and (b) imaginary part of the Green's function versus azimuthal source location at one radial source locations using different numbers of Fourier modes for the mean flow dependent coefficients. Dashed-dot $L = 4$; dashed $L = 6$; solid $L = 8$. Offset Jet.

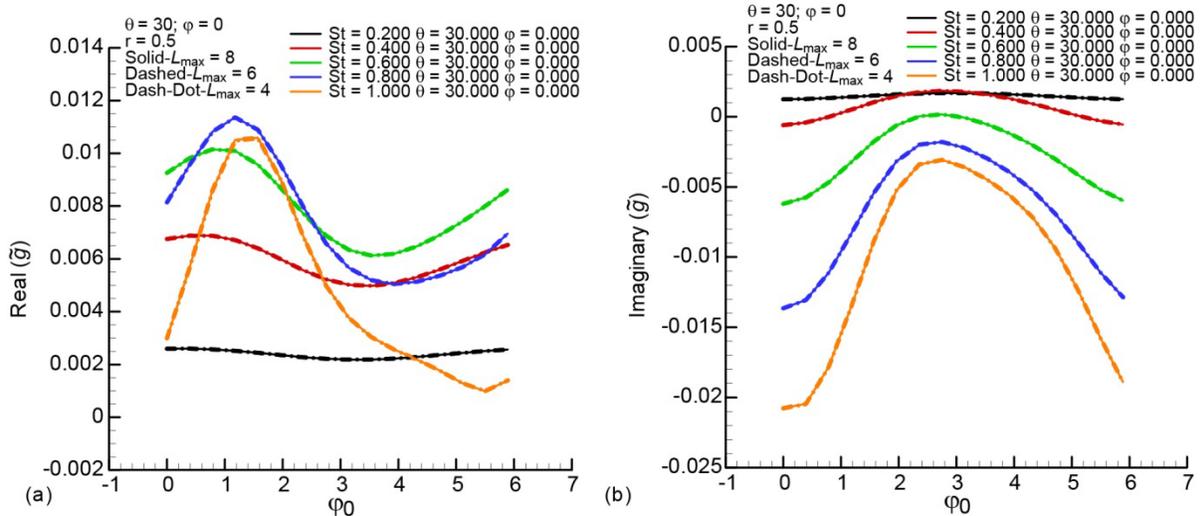


Figure 11.—Results for the (a) real part and (b) imaginary part of the Green's function vs. azimuthal source location at one radial source locations using different numbers of Fourier modes for the mean flow dependent coefficients. Dashed-dot $L = 4$; dashed $L = 6$; solid $L = 8$.

A longer-term goal of this task was to develop sound source and propagation models which can provide higher fidelity predictions of the sound field and yield new physical insights into the generation and propagation of noise from nozzles of complex geometry. To this end, working with NASA researchers, numerical methods and computational algorithms have been identified that can be used to directly solve the equations governing sound propagation in a spatially developing, non-axisymmetric jet.

The approach to developing a high-fidelity noise propagation model was to use these numerical methods on a set of progressively more complex problems to work through any numerical issues that may arise relatively early in the process and then apply any lessons learned to the problem at next level of complexity.

A code that solves the two-dimensional linearized Euler equations (which closely resemble the equations governing sound propagation in a unidirectional steady flow in the acoustic analogy formulation used in this task) using these numerical methods was made available to the contractor by NASA. As a first step in developing a general solver for sound propagation in non-axisymmetric jets, this code was validated for the special cases of zero mean flow and for a uniform mean flow in the axial direction, using analytical solutions.

Figure 12 shows comparisons of results obtained from the NASA-supplied code with the analytical solution, for a uniform mean flow, as a function of axial location at a fixed transverse location a relatively short time after the solution has settled to a time-periodic state.

Figure 13 shows comparisons at a much longer time at a point in the center of the computational domain.

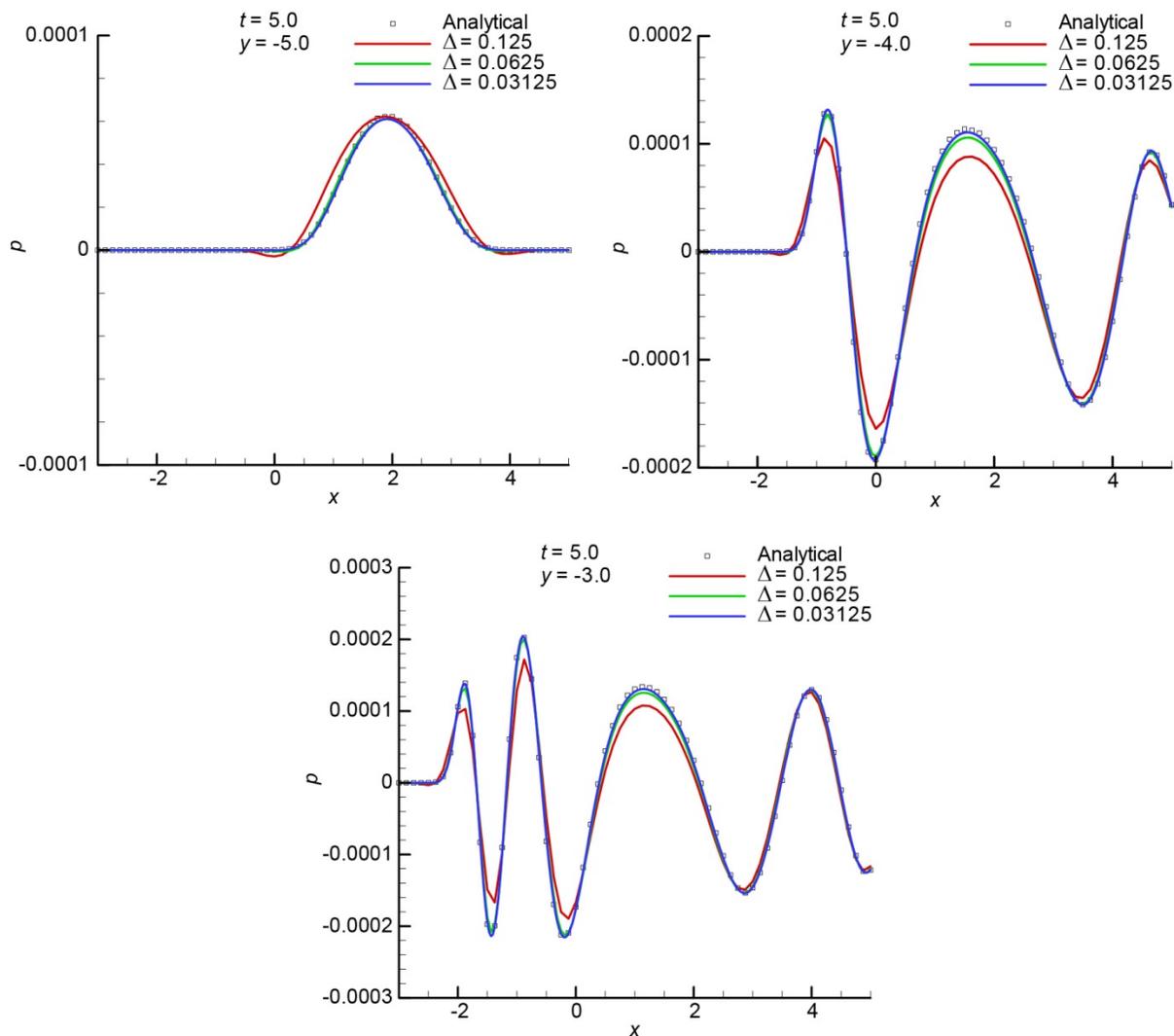


Figure 12.—Comparisons of results from the NASA-supplied code to solve the unsteady linearized Euler equations for the case of a uniform mean flow with the (numerically evaluated) analytical solution.

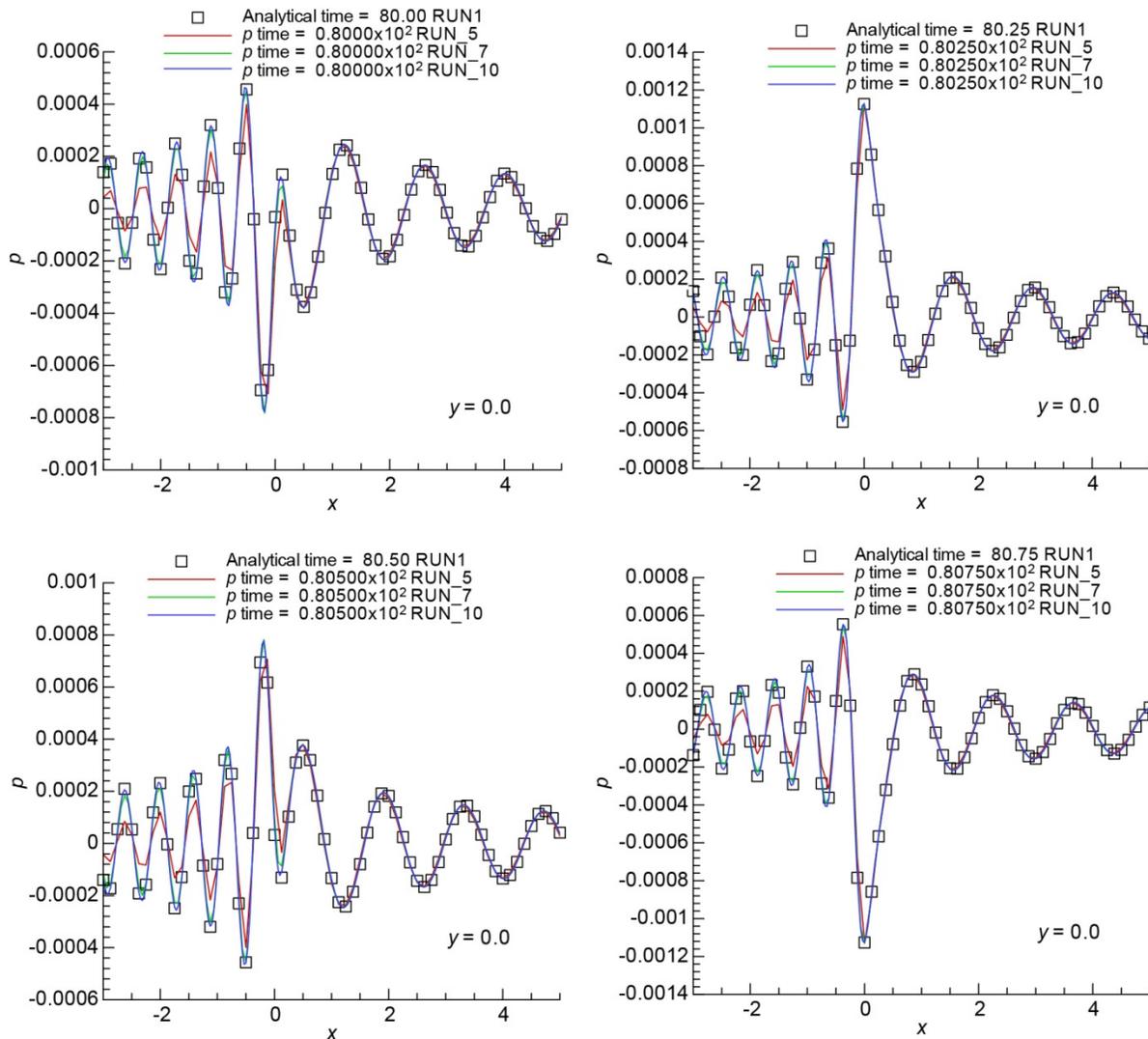


Figure 13.—Comparisons of results from the NASA-supplied code to solve the unsteady linearized Euler equations for the case of a uniform mean flow with the (numerically evaluated) analytical solution. Long-time run.

The results of the testing of the code showed that the numerical results approach the analytical solution provided the mesh size is sufficiently small and also uncovered some potential issues at the upstream boundary, where the relative errors were somewhat larger than other locations in the domain. This prompted a theoretical re-examination of the boundary conditions used in the code. A reformulated set of boundary conditions was derived (by a NASA researcher) which resulted in a much more robust solution in the vicinity of the upstream boundary.

The new boundary condition was incorporated into a new version of the code which solves the two-dimensional unsteady linearized Euler equations for a non-uniform, parallel, mean flow. This code was delivered to the contractor near the end of this task performance period and has not yet been validated against an independent numerical solution.

Significant progress was made under this sub-task toward the development of a general code to accurately predict sound propagation in jets from nozzles of complex geometry. The goals of the NASA Fundamental Aeronautics Program require tools of this type to assess potential noise-reduction technologies. Further work in this area is needed, however. A three-dimensional unsteady solver for a non-uniform mean flow is needed to solve the problems of interest in jet noise. Also, although the linearized Euler equations are very similar to the equations governing sound propagation in the acoustic

analogy formulation used in this work, they are not identical, and some modification to the code formulation (although not the basic propagation algorithm) will need to be made to solve the latter. Finally, the codes produced by NASA researchers up to now have been serial codes running on a single computational node. To make these codes more efficient for use in practical noise predictions, parallel processing computing methods will need to be incorporated.

Task 2.1—Application to Specific Nozzle Geometries—Testing and Assessment

Based on the results of the demonstrations using analytical mean profiles discussed above, the method for solving the scalar Green’s function for a locally parallel jet using expansion in orthogonal polynomials was further developed to allow use of mean profiles from a RANS solution. As a first test case, the General Electric (GE) Inverted Velocity Profile (IVP) with fluid shield nozzle was used.

A schematic of this nozzle is shown in Figure 14. The nozzle consists of core and fan streams as well as a third stream which partially surrounds the former two and is meant to ‘shield’ observer locations below it from the noise generated by their mixing.

Results from tests to determine the number of Fourier modes needed to represent the mean flow dependent coefficients in the Green’s function equation for this nozzle flow (analogous to those for the analytical tests cases in Figure 8 and Figure 9) are shown in Figure 15. Two of these terms, \mathcal{R} , \mathcal{N} , are well represented by eight Fourier modes, but the remaining term, \mathcal{Z} , which contains derivatives of the mean acoustic Mach number and sound speed with respect to the azimuthal coordinate, exhibits appreciable oscillations, as obtained from the RANS solution, which are not reproduced when this function is represented by eight modes.

The Fourier decomposition of the mean flow terms for this case were used in the Green’s function solver to obtain preliminary results for the GE-IPV nozzle flow. An example of the results obtained (analogous to results in Figure 10 and Figure 11) is shown in Figure 16, where the real (a) and imaginary (b) parts of the Green’s function are shown for different non-dimensional frequencies (Strouhal number) as computed using different numbers of Fourier modes for the mean flow dependent coefficients.

The results shown in Figure 16 seem to be relatively well converged in terms of the number of Fourier modes needed (as in the test cases in Figure 10 and Figure 11). Issues with representing mean flow dependent coefficients with significant oscillations in the azimuthal direction remain to be resolved.

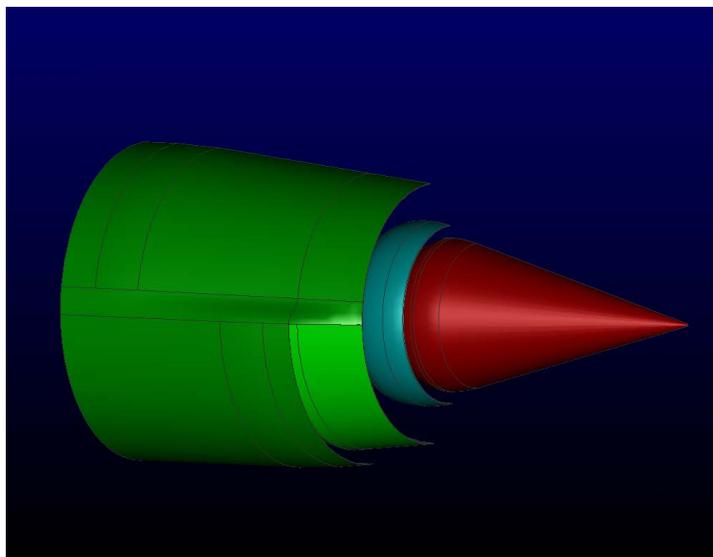


Figure 14.—GE-IPV—Dual Stream Jet with a Third Stream Shield.

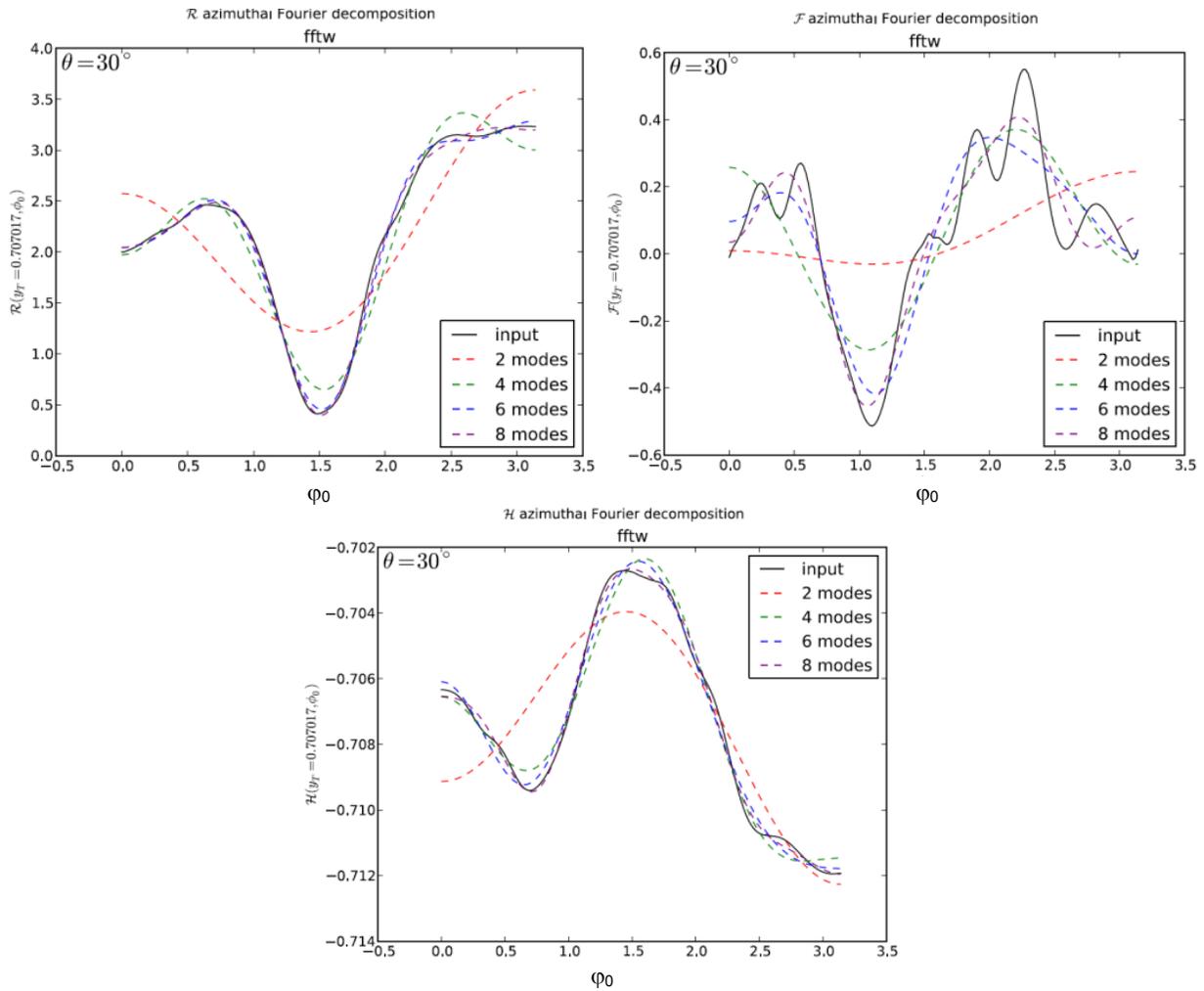


Figure 15.—Azimuthal variation of the mean-flow-dependent coefficients of the scalar Green's function equation and their reconstruction using various Fourier modes. GE IVP Nozzle with RANS mean flow.

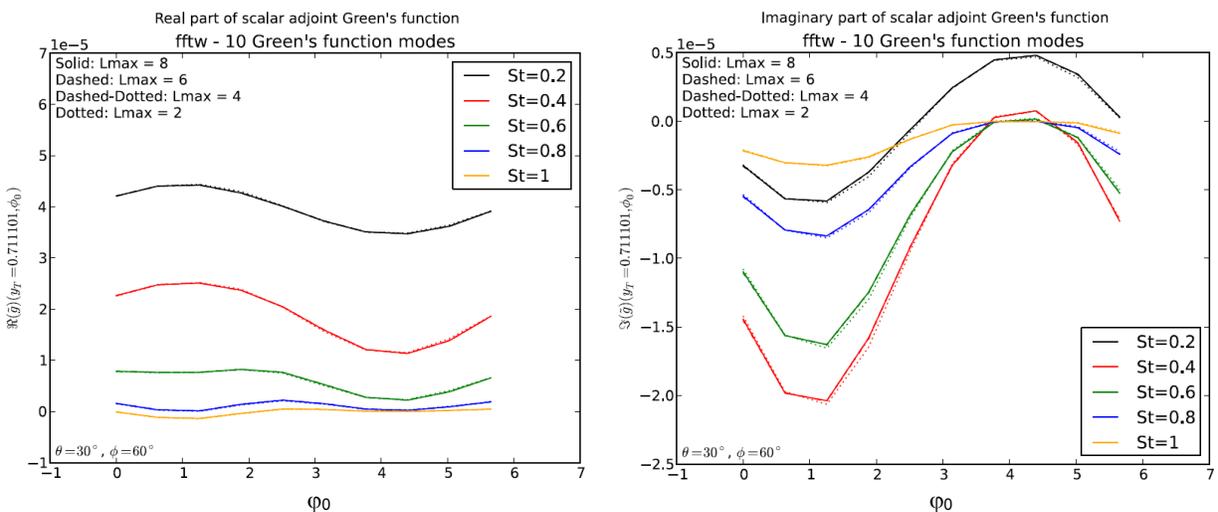


Figure 16.—Results for the (a) real part and (b) imaginary part of the Green's function vs. azimuthal source location at one radial source locations using different numbers of Fourier modes for the mean flow dependent coefficients. Dotted $L = 2$; dashed-dot $L = 4$; dashed $L = 6$; solid $L = 8$.

Task 3.0—Comparison of Numerical Results With Those of Model Developed in Year 1 and Development of Improved Model

Additional calculations using the source model originally developed for round jets, and used in Task 1.2 in the initial set of noise predictions for rectangular jets, showed that, although this model produced good spectral predictions at 90° to the jet axis and in the peak noise direction over a range of Mach numbers, less satisfactory results are obtained at intermediate angles and the problems were traced to the model used to represent the independent components of the Reynolds stress auto-covariance. To overcome this issue, a new model for the space-time Fourier transform of the fluctuating Reynolds stress auto-covariance tensor was constructed, which is more consistent with characteristics observed in experimental measurements of jet turbulence that had just become available at the time. The model is a sort of hybrid between previous frequency-domain (with frequency-dependent length scales) and time domain approaches.

The model for the space-time Fourier transform of the fluctuating Reynolds stress auto-covariance tensor, Ψ^*_{ijkl} , is

$$\Psi^*_{ijkl}(k_1, \mathbf{k}_T, \omega) = -\frac{\pi l_0 l_1 \bar{l}_T^2 (1 + \tilde{\omega}^2)^{1/2} A_{ijkl}}{U_c} \sum_{m,l=0}^{\infty} a_{m,l} (-1)^{m+l} D_{k_1}^m D_{\omega}^l \frac{1}{R} \frac{\partial}{\partial R} \frac{1}{(1+R^2)} e^{-\tilde{k}_T^2/4(1+R^2)^{1/2}}$$

where

$$\tilde{k}_T^2 \equiv (k_2^2 + k_3^2) \bar{l}_T^2 (1 + \tilde{\omega}^2)^{1/2},$$

and the frequency-dependent transverse length scale, \bar{l}_T , is given by

$$\bar{l}_T = \sqrt{2} \frac{l_{2,3}}{\left[1 + \tilde{\omega}^2 (1 + b\tilde{\omega}^2) / (1 + b)\right]^{1/4}}.$$

Further details of the model and its implementation can be found in Reference 3.

The new model was tested and calibrated for three round jets with exit velocities ranging from subsonic to moderately supersonic, over a set of observer polar angles with the goal of improving previous results at the intermediate angles. Comparisons of the predictions obtained using this model with experimental data are shown in Figure 17. Each plot shows results corresponding to a fixed far-field observer polar angle at the three Mach numbers considered. Seven polar angles are shown, covering the range of interest. The predictions are in relatively good agreement with the data. In particular, the model is able to capture the large increase in sound at supersonic speeds for angles near the jet axis. The cross-hatched regions indicate where the data is believed to be dominated by shock-associated noise, caused by slight differences in the actual exit pressure from its perfectly expanded value. The present prediction model only accounts for the turbulent mixing noise, and therefore does not include this component of supersonic jet noise.

Further details of the prediction results obtained using this model, including the breakdown of the contributions to the acoustic spectrum from individual source terms and contributions from various axial locations in the jet, as functions of observer angle and jet speed are provided in Reference 3.

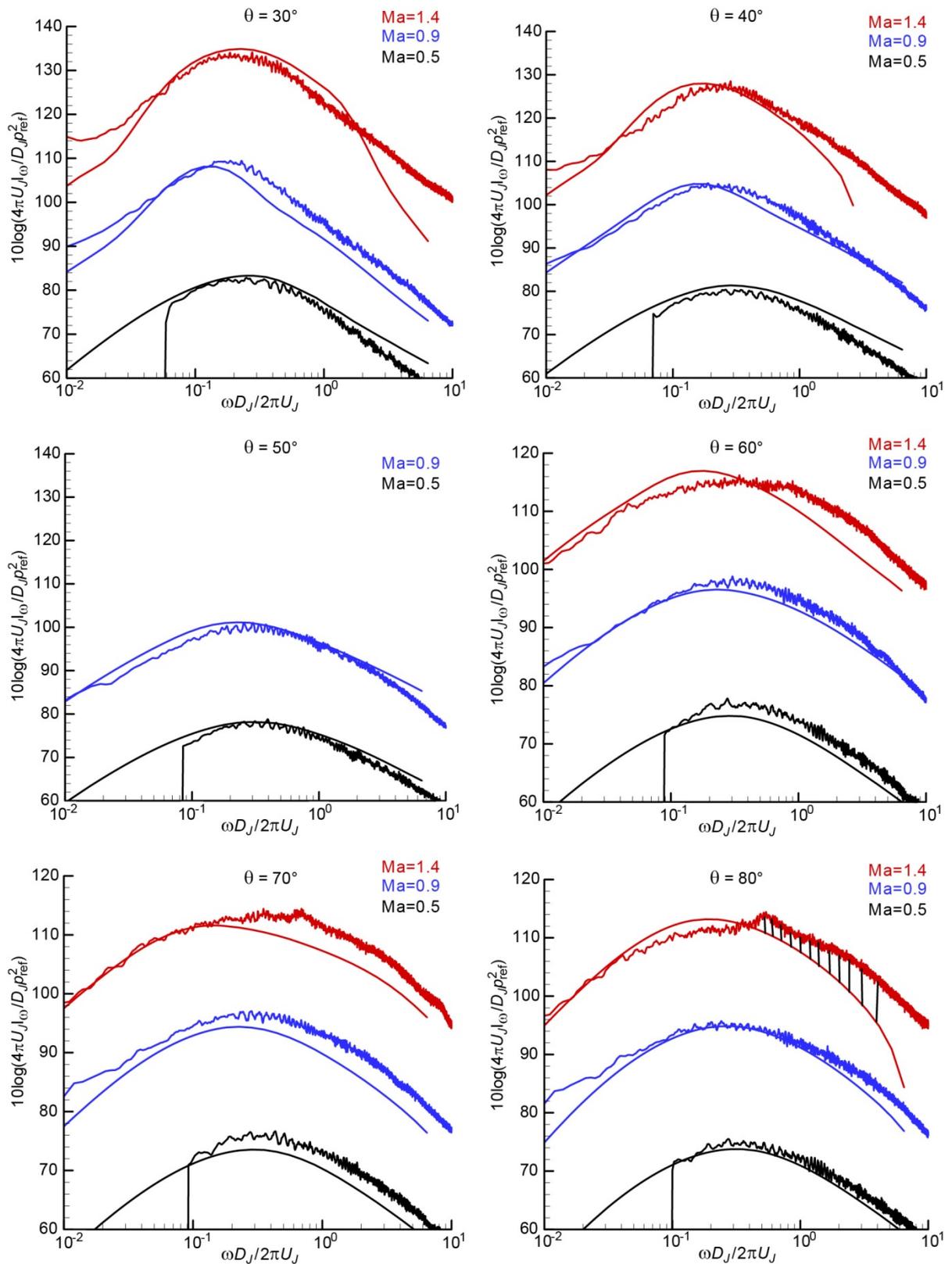


Figure 17.—Comparisons of predictions with data.

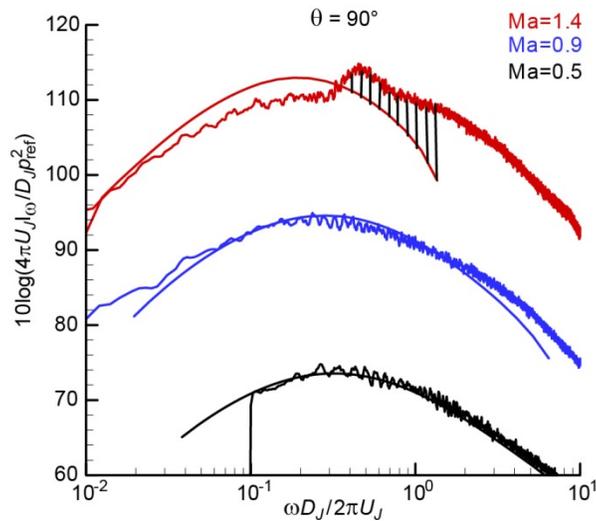


Figure 17.—Concluded.

Task 4.0—Incorporation of Improved Model in Jet Noise Prediction Scheme

The hybrid source model described under Task 3.0 was incorporated into the jet noise prediction scheme for rectangular jets described in Task 1.3. The new code was then used to make noise predictions for jets from a set of generic rectangular nozzles (the Extensible Rectangular Nozzle system (Ref. 10)) with aspect ratios two, four and eight, for jet exit Mach numbers of 0.5, 0.7 and 0.9. Results from these calculations and comparisons with NASA data (Ref. 11) taken in the Small Hot Jet Aeroacoustic Rig (SHJAR), for the aspect ratio four nozzle, are shown in Figure 18, Figure 19 and Figure 20 for $M_j = 0.5$, 0.7, 0.9, respectively. Note that the polar angles indicated in these figures are relative to the nozzle inlet. Results for the other two nozzles (aspect ratios two and eight), as well as breakdowns of the contributions to the acoustic spectrum from individual source terms, are presented in Reference 4.

The results show that the code is capable of predicting the experimentally observed azimuthal variation in the sound field.

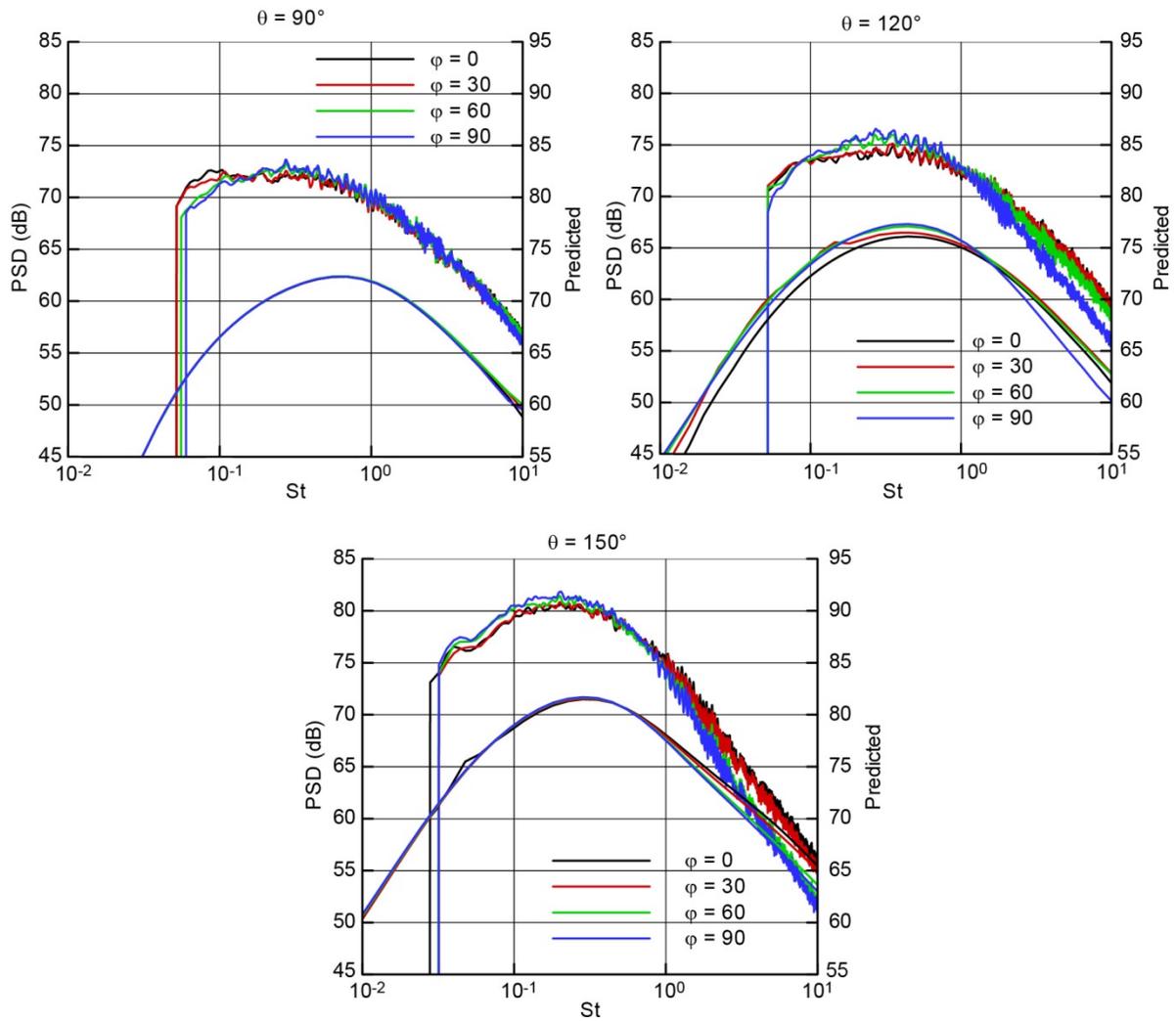


Figure 18.—Comparisons of noise prediction with SHJAR data – AR4, $M_J = 0.5$.

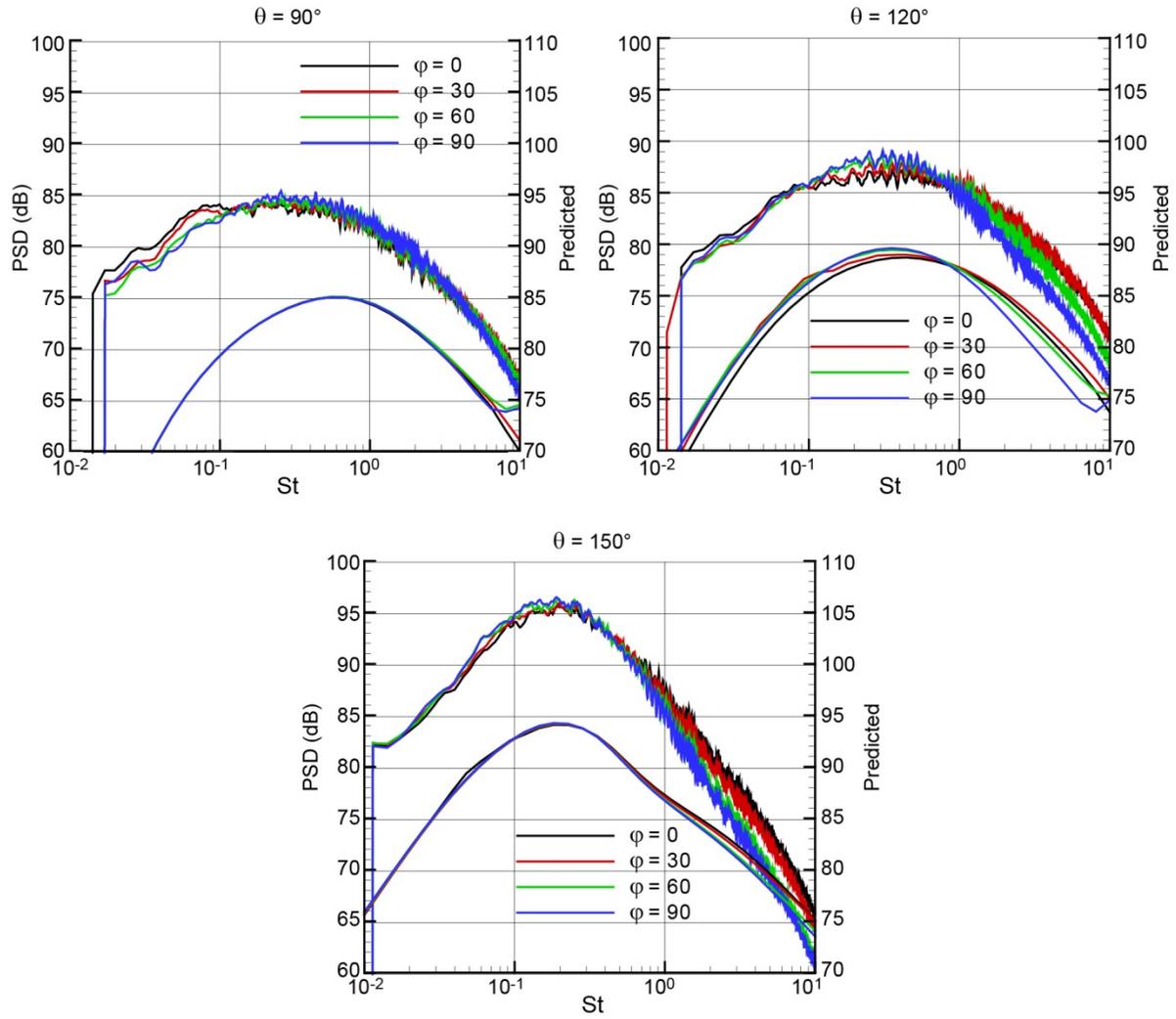


Figure 19.—Comparisons of noise prediction with SHJAR data – AR4, $M_J = 0.7$.

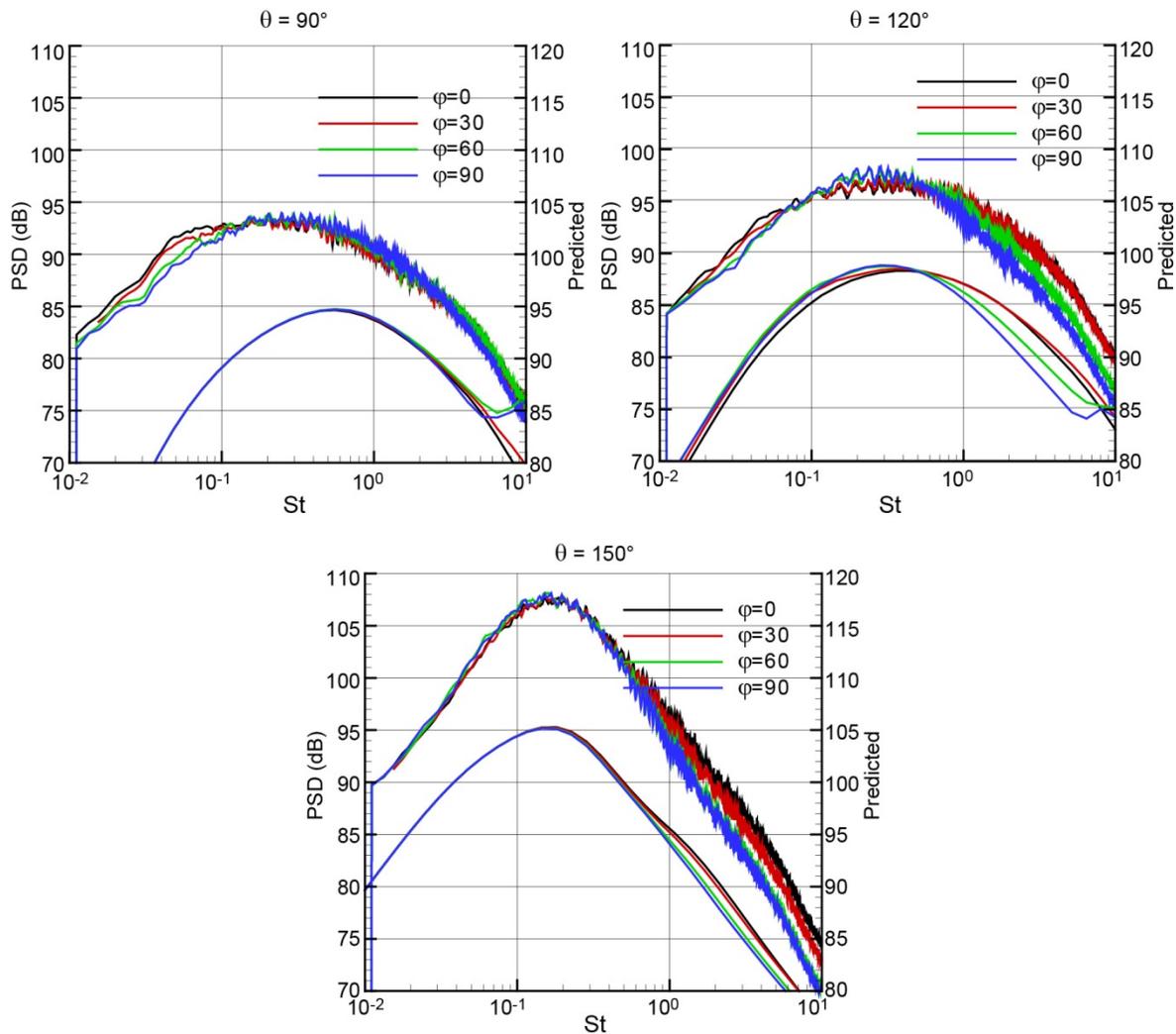


Figure 20.—Comparisons of noise prediction with SHJAR data – AR4, $M_j = 0.9$.

Task 5.0—Further Development of Prediction Method—Application to Other Geometries and Flow Conditions

Work under this task consisted of further developing the source and propagation models constructed under this project to provide more robust noise prediction tools for use in evaluating concepts under NASA’s Fundamental Aeronautics Program, and extending the range of geometries and flow conditions for which reduced-order model noise predictions can be made.

Of particular interest in jet noise is the ability to predict, by modeling the correct physical mechanism, the very different shapes of the small-angle acoustic spectrum relative to that at 90° , and the sharp rise in sound level that has been experimentally found to occur when the jet exit acoustic Mach number becomes supersonic.

Most acoustic analogy analyses introduce some sort of source compactness assumption that has the effect of artificially coupling all the circumferential acoustic source modes to each circumferential acoustic propagator mode. This results in source models that lump all the circumferential modes together in an overall source term which is likely to be dominated by the contributions from the higher order circumferential modes that probably contain most of the turbulent energy. However, careful analysis shows that each circumferential source mode only couples to the corresponding propagator mode—so that each circumferential mode of the acoustic spectrum is given by the product of the corresponding acoustic

propagator and source modes. This can potentially have a large effect on the acoustic spectrum since the spectral shapes of the lower order source modes can be quite different from those of the energy-containing modes.

A proper modeling of the sound sources then would be one based on the individual Fourier modes of the fluctuating Reynolds stress auto-covariance tensor. Models for these terms were formulated and incorporated into an existing code for predicting noise from round jets. Preliminary testing and calibration of this model for round jets, using newly available experimental data, was started in the final months of this contract, and promising results were obtained. Further testing, and application to non-circular jets, will require additional work outside the scope of this task order.

The ability to predict noise generated and propagated by jet flows in the vicinity of solid surfaces is expected to become more important as proposed designs that meet the noise-reduction goals of NASA's Fundamental Aeronautics Program for next generation aircraft suggest that integration of the propulsion system with the airframe will be needed. One important source of noise arising from the interaction of a turbulent jet flow with solid surfaces is the edge noise generated by the interaction of a turbulent jet with the edge of a wing.

Work under this task supported an effort to develop a RANS-based prediction code for edge noise based on an extension of the classical Rapid Distortion Theory (RDT) to transversely sheared base flows (Refs. 6 and 7)

The generalized RDT formulation was applied to the problem of a two-dimensional turbulent jet interacting with the downstream edge of a semi-infinite flat plate. This simple geometry can be used to represent the experimental configuration of a subsonic jet emanating from a large-aspect-ratio rectangular nozzle in the vicinity of a flat plate, recently considered at NASA Glenn (Ref. 12). A Computer Aided Design rendering of the experimental set up is shown in Figure 21.

Figure 22 shows results from the RDT-based predictions for a jet exit acoustic Mach number, Ma , of 0.7, four polar angles measured from the downstream jet axis $\theta = 90^\circ, 75^\circ, 60^\circ, 45^\circ$, and two azimuthal angles, corresponding to observer locations above and below the plate ($\psi = \pm 90^\circ$). Results for two additional jet exit acoustic Mach numbers are presented in References 7 and 8.

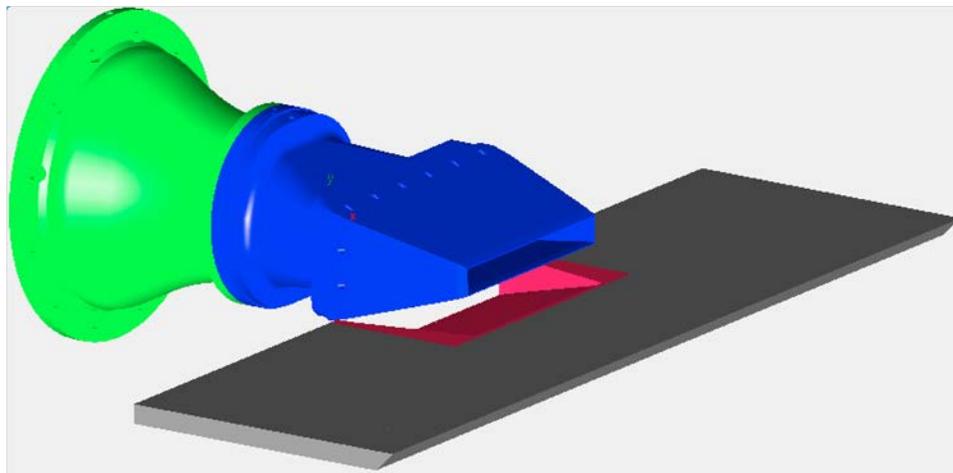


Figure 21.—Experimental configuration for jet surface interaction.
Courtesy Dr. James Bridges NASA Glenn.

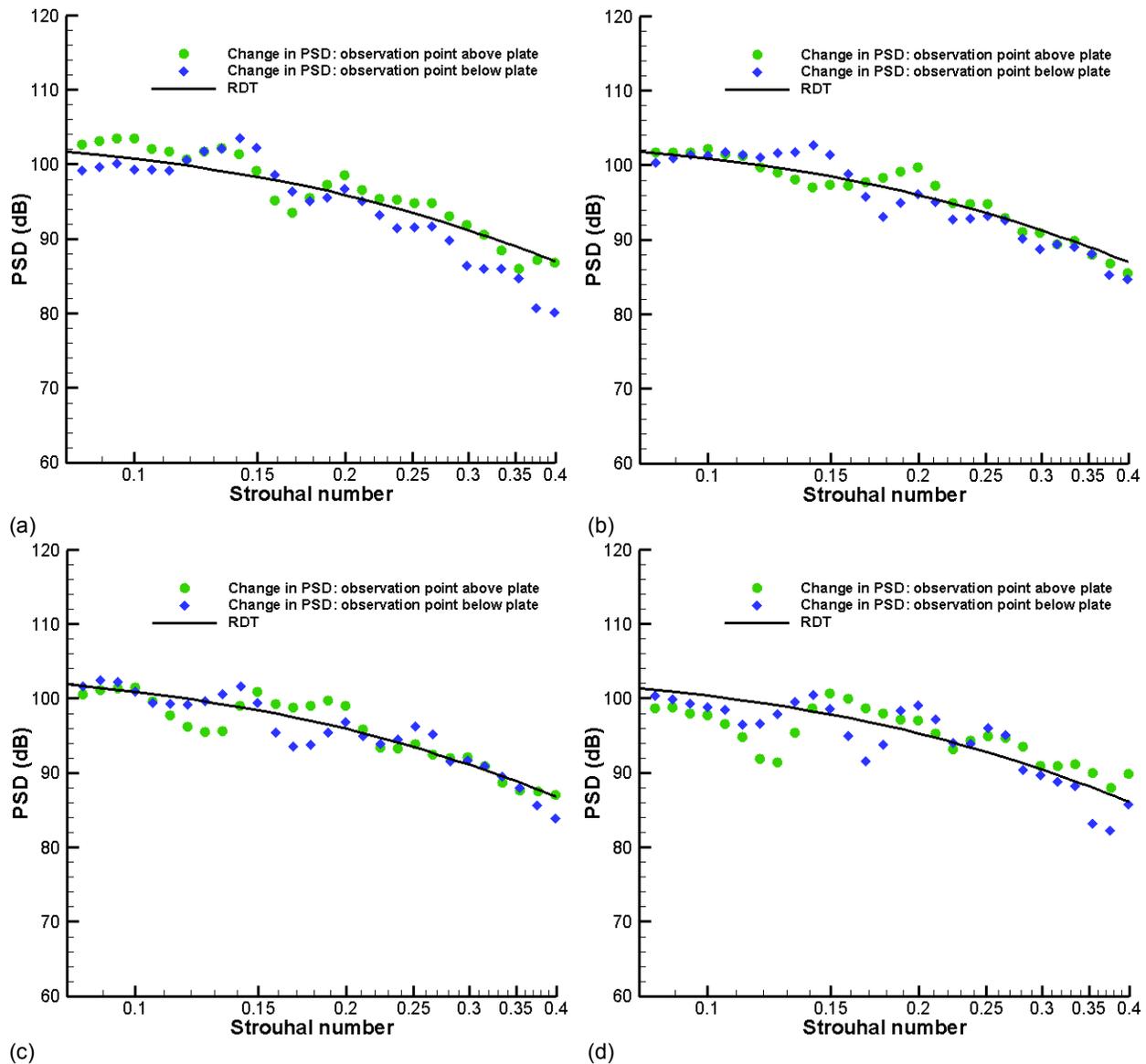


Figure 22.—Predicted (curves) and measured (green/blue symbols above/below the plate) Power Spectral Density of the far-field pressure fluctuations at 100 equivalent diameters from the nozzle exit as a function of Strouhal number. (a) $\theta = 90^\circ$; (b) $\theta = 75^\circ$; (c) $\theta = 60^\circ$; and (d) $\theta = 45^\circ$.

The results show that the RDT-based predictions are in good quantitative agreement with the data. The theory indicates that the sound radiated above the plate is the same as that below (i.e., dipole-like directivity) and this is also observed (to within experimental uncertainty) in the data. The agreement is best at 90° (parts (a) of the figures), where the low Mach number edge-generated noise is predicted to be maximum.

The encouraging initial results obtained from this effort are expected to lead to further work on this problem as well as other configurations involving jet surface interactions.

Appendix A.—State of the Art Review

Modeling and Prediction of the Noise From Non-Axisymmetric Jets

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A.1 Introduction

Current jet noise prediction methods are generally based on an acoustic analogy formulation, wherein the governing Navier-Stokes equations are rearranged to obtain a linear inhomogeneous equation (or system of equations) where all nonlinear terms are taken as a nominal acoustic source. Lighthill (1952, 1954) pioneered this approach when he showed that an analogy can be made between the density fluctuations in a real flow with those produced by a quadrupole source distribution in a uniform medium at rest. Improvements to this approach for jet noise prediction were subsequently proposed which sought to explicitly account for acoustic/mean flow interaction effects, thereby eliminating the need for source models to represent these effects. The NASA Glenn jet noise prediction code JeNo (Khavaran, Bridges and Georgiadis, 2005) is based on a form of the acoustic analogy of Lilley (1974). The effects of sound refraction through a non-uniform mean velocity and temperature field are obtained by computing the Green's function for the Lilley equation for an axisymmetric jet. Reynolds-averaged Navier-Stokes solutions are used as input for the mean flow and to obtain the length, time, and turbulence kinetic energy scales for source models. The JeNo code has been tested on a variety of jet flows and has been shown to be capable of providing accurate predictions for the spectral and directivity characteristics of round subsonic jets, as well as the acoustic spectra of supersonic jets at polar angles near 90° to the jet axis. Recent work has identified the physical effects that need to be included in the propagation and source models in order to predict the peak sound levels at small angles to the jet axis for supersonic jets (Goldstein and Leib, 2007). Development is also currently under way to extend the flow and source models to deal with heated jets (Khavaran and Kenzakowski, 2007 a,b). These efforts have continued to focus on circular jets.

New technological developments for aircraft noise suppression will involve complex geometrical configurations, and there is a need to extend the currently available noise prediction schemes beyond axisymmetric jets. The main impediment to the accurate prediction of noise from non-axisymmetric jets is the lack of an adequate model for the sound propagation through general transversely sheared mean flows that can be used for design and screening purposes to guide future nozzle technology development for civil aviation noise reduction.

The objective of this task is to develop source and directivity models that can be used to predict noise from non-axisymmetric turbulent jets. In this report, we present a brief review of some recent work in non-axisymmetric jet noise prediction. We highlight some recent advances in numerical methods that are particularly relevant to this problem, and discuss the use of high-frequency geometric acoustics methods.

For some applications, the introduction of appropriate physical modeling approximations, based on geometry or mean flow characteristics, into the general acoustic analogy formulation can provide a way of substantially simplifying the problem to be solved and reducing the computational effort required to make predictions. Models based on these approximations may be capable of providing the leading-order effects of a non-axisymmetric mean flow on the spectral and directivity characteristics of the sound field and its variation with respect to the relevant parameters, such as aspect ratio. Such physical models may also be amenable to further, mathematical, approximation, such as high- or low-frequency asymptotics, leading to additional simplification. We discuss one approximation in particular which could be useful for certain classes of jets.

In addition to the propagation effects, a noise prediction method for non-axisymmetric jets must adequately model the acoustic source terms as specified by the analogy formulation. These take the form

of two-point, time-delayed, density-weighted, fourth-order correlations of the fluctuating velocity and enthalpy. Significant simplifications are needed to reduce the general form of these quantities to a manageable level, and extensive databases are available which have guided source-modeling efforts for round jets. The source structure can be expected to be significantly different in non-circular jets, with the potential for greater sensitivity to the geometrical details. Much less data is available for the turbulence correlations in non-circular jets. This will need to be an area of continued experimental and theoretical research.

Any new jet noise prediction models and computational codes which are developed must be validated against experimental data. We review the available noise data from jets exiting simple rectangular and elliptic nozzles which could be used for this purpose, and summarize the results of the experimental observations. As an example of the type of revolutionary noise-reduction concepts currently being studied, and for which more advanced noise-prediction tools are required, we review the results of some recent tests on the use of distributed exhaust nozzles for jet noise reduction. Finally, we provide some recommendations on how to proceed to reach the objective of this task.

A.2 Modeling and Numerical Methods for Noise From Non-Axisymmetric Jets

Various forms of acoustic analogy which explicitly account for mean flow refraction effects have been proposed for jet noise prediction. These may vary in the choice of independent variables, and/or the ultimate form of the ‘source’ terms which must be modeled. When the flow field through which the sound is supposed to propagate is approximated as a unidirectional, transversely sheared mean flow, the governing equations for all these approaches reduce to the third-order, moving-medium wave equation

$$\mathcal{L} p' = \Gamma \quad (1.1)$$

Where p' is a quantity associated with the fluctuating pressure, Γ is a source term and

$$\mathcal{L} = \frac{D}{Dt} \left[\frac{D^2}{Dt^2} - \nabla c^2 \nabla \right] + 2c^2 \nabla U \cdot \nabla \frac{\partial}{\partial x_1} \quad (1.2)$$

with $U = U(x_2, x_3)$ and $c = c(x_2, x_3)$ being the mean streamwise (in the x_1 direction) velocity and mean sound speed, respectively, both of which are functions of the cross stream coordinates, x_2, x_3 . This problem is usually solved by introducing a Green’s function which must, in general, be computed numerically.

As stated above, most current jet noise prediction methods have concentrated on circular jets with locally parallel, axisymmetric mean flows. For this case, (1.2) can be reduced to an ordinary differential operator by Fourier transformation in x_1 and t , and Fourier expansion in the azimuthal coordinate. There is a significant increase in the computational resources required to make noise predictions in non-axisymmetric jets compared with axisymmetric ones, and the vast majority of the computational effort lies in computing the propagation through the spatially varying base flow, i.e., the Green’s function. Methods for obtaining the Green’s function (or modeling the mean flow refraction effects) in non-axisymmetric flows can be broadly grouped into those using more or less fully numerical methods, methods based on asymptotic (generally high-frequency) expansions and those using models based on physical properties of the flow. In the following subsections we highlight a few methods within each of these categories.

A.2.1 Numerical Methods

Tam and Pastouchenko (2002) developed a semi-empirical prediction method for the noise from non-axisymmetric turbulent jets, building upon the work of Tam and Auriault (1999) for axisymmetric jets. Their approach, like those based more formally on an acoustic analogy, accounts for mean flow refraction

effects using a Green's function and constructs models for the statistics of the source terms. Results from a Reynolds-averaged Navier-Stokes solution are used for the mean flow in the Green's function calculation and to obtain the length, time, and turbulence kinetic energy scales in the source models. Solutions for the adjoint Green's function of the linearized Euler equations with a locally parallel mean flow approximation (which can be reduced to a problem of the form (1.1)) is obtained by transforming the problem into a sound scattering problem which is solved numerically. The numerical method is based on the Dispersion Relation Preserving (DRP) scheme (Tam and Webb, 1993) and requires careful treatment of the boundary conditions, to avoid reflections back into the computational domain. Tam and Pastouchenko (2002) surround their computational domain with a perfectly matched layer, and include artificial selective damping to avoid contaminating the solution with spurious short wavelength disturbances. Comparisons of the results using this method with experimental data were, for the most part, confined to polar angles near 90° to the jet axis, where the refraction effects, and therefore the azimuthal asymmetry of the sound field, are relatively small.

There is a wide variety numerical methods that could potentially be applied to compute the Green's function for a non-axisymmetric jet. It is important, however, that any numerical method used to compute sound propagation through a non-axisymmetric mean flow be able to accurately and efficiently represent the propagation of waves over a range of frequencies and wavelengths, and avoid introducing errors due to imposing boundary conditions on a finite computational domain. Two schemes which seem to hold promise for this application are briefly described.

The Green's Function Discretization (GFD) method was designed to obtain accurate solutions for convected wave equations of the type arising in aerodynamic noise applications. This method was originally developed to solve the Helmholtz equation (Caruthers, French and Raviprakash 1995), and has been applied to the convected Helmholtz equation for both uniform and non-uniform mean flows (Francescantonio and Casalino 1999). The idea behind this approach is to make use of the expected local wave character of the solution to minimize the number of grid points required to accurately represent the solution. An interpolation formula is derived using an appropriate Green's function, and this formula is then used to discretize the governing equations and boundary conditions. In its original form, the method was only applicable to differential equations for which the free-space Green's function was known.

The GFD method has recently been adapted to the linearized Lilley equation for use in various aeroacoustic problems (Casalino and Bodony, 2006; Genito, Casalino and Botte, 2007; Casalino and Genito, 2007). Discretization of the Lilley equation and boundary conditions is affected by employing local shape functions which are derived using the Green's function of the convected Helmholtz equation. Perfectly matched layers are added to the computational domain to model non-reflecting boundary conditions. Spectral-like accuracy is claimed for this method, with accuracy preserved with as few as three or four points per wavelength. Applications of this method to jet noise calculations have, to this point, been limited to test cases involving two-dimensional and axisymmetric parallel mean flows with analytically prescribed mean velocity and density profiles (Casalino and Bodony, 2006; Casalino and Genito, 2007).

A class of highly accurate finite difference algorithms for computational aeroacoustics applications has been developed by Goodrich (1996, 1997). This method is based on the use of a local propagator, which directly incorporates the wave dynamics into the solution, along with high-order interpolation polynomials. The method has been tested for the linearized Euler equations with uniform mean flow (Goodrich, 1999) and was found to be more efficient than lower-order methods when high accuracy over long times is required. Implementation of appropriate boundary conditions associated with these higher order schemes has been discussed by Goodrich and Hagstrom (1999). These algorithms are currently being tested on a wider variety of systems of partial differential equations. They seem to have features which would make them well suited for computing sound propagation in non-axisymmetric mean flows.

A.2.2 High-Frequency Asymptotic Methods

High-frequency asymptotic methods have often been used to reduce the computational resources required to make practical predictions of aerodynamically generated noise. Geometrical acoustics theory

(ray theory) provides a framework for the high-frequency asymptotic approximation of sound propagation through general (non-axisymmetric, non-parallel) mean flows.

A high-frequency solution for the Green's function of (1.2) for an arbitrary uni-directional, transversely sheared mean flow was derived by Goldstein (1982). This solution was extended to treat sound sources located internal to a nozzle by Goldstein and Leib (2000). The main difficulty with geometrical acoustics is the appearance of caustics and associated shadow zones into which rays cannot penetrate, and where the theory predicts no sound field. In the parallel mean flow approximation, the ray acoustics solution breaks down at polar angles in the far-field close to the jet axis—the so-called the 'zone of silence'. The ray acoustics solution can be extended into the zone of silence by generalizing the approach to include complex rays. The latter are defined as trajectories obtained by solution of the ray equations for which the initial conditions and 'distance' along the rays take on complex values. In problems for which a solution can be formally obtained in terms of Fourier transform, such as a round jet (Wundrow and Khavaran, 2004), this solution can be used to guide the complex generalization of the ray acoustics solution. More general procedures for problems which are not amenable to Fourier transform methods, such as a non-axisymmetric jet, have been proposed by Chapman et al. (1999), who provide numerous specific examples.

As a practical matter, obtaining the Green's function for the Lilley equation using ray acoustics amounts to solving a nonlinear system of ordinary differential equations to trace the trajectories to the far field. The quantities appearing in the formula for the Green's function (Goldstein 1982): the phase function (eiconal) and the ray-spreading factor (which gives the amplitude of the sound field) can be computed simultaneously with the ray trajectories. A fairly large number of rays need to be computed, but all the rays are independent of one another, and so this method is well suited for parallel computing techniques employing a large number of relatively inexpensive processors. A computer code implementing the ray acoustics solution (for real rays) on a cluster of personal computers was developed by Leib and Goldstein (2001).

In ray acoustics calculations based on the direct form of the Green's function, such as those discussed above, rays are traced into the acoustic far-field beginning at specified source locations and initial directions within the jet. The far-field polar and/or azimuthal location reached by these rays is determined by the ray trajectory. For noise predictions, it is required to compute the sound field at specified locations in the far-field, to, for example, compare with experimental data where microphones are located at fixed locations. In order to determine the initial direction of rays which reach the desired locations in the far-field an iterative procedure is generally adopted. This step can add considerably to the overall computational times if good initial guesses for the initial conditions are not available. The best way to overcome this problem would be to consider the adjoint problem, and trace rays from the desired locations in the far field back into the jet.

Practical implementation of complex ray calculations involves additional considerations beyond those of the usual ray tracing. Among the issues arising in complex ray calculations, is the fact that the functions defining the mean field through which the rays travel (the mean velocity and temperature in the case of jet noise) are considered to be complex-valued functions of complex variables. In jet noise predictions, the mean velocity and temperature fields are obtained from a RANS solution, and so are only known numerically at discrete points in the flow domain. In order to compute complex rays, it is required to analytically continue the functions defined by these solutions to complex variables. Cubic spline fitting is used in the current version of the JeNo code (for real variables) to interpolate the mean velocity and temperature from the RANS solution for the computation of the Green's function for an axisymmetric flow. Amodei et al (2006) found that fitting based cubic splines, or other polynomial fits, is not appropriate for continuation into the complex plane, because of the appearance of discontinuities and other numerical difficulties. Their method applied a two-dimensional discrete cosine transform to the discrete data for their variable index of refraction, and used the resulting expansion as the fitting function. The coefficients in these expansions can be obtained by Fast-Fourier Transform.

Another issue in complex ray tracing is that of finding which (and all) among the infinite set of the initial rays which emanate into the complex space from a given source location intersect the real physical

space at the observation points of interest. Most current complex ray algorithms (see, for example, Egorchenko and Kravtsov, 2001) combine a Newton iteration procedure (similar to that used by Leib and Goldstein, 2001 for the direct Green's function) with an ordinary differential equation solver (like Runge-Kutta) to find the complex initial conditions for rays which arrive at a given point in the physical space. Various methods have been devised to avoid large global searches involving all possible rays by limiting the search to certain expected ranges. These methods are, to a large extent, dependent upon the particular problem being considered. It does not appear that this iterative process can be avoided by using an adjoint formulation since, if all the possible rays which arrive at a desired observation point are traced back into the flow (with complex 'initial' conditions), it would still be necessary to determine which of these rays originated from real source locations within the jet.

An alternative to dealing with the additional complications associated with complex rays would be to include non-parallel mean flow effects in the ray acoustics formulation. Durbin (1983) has shown that the inclusion of non-parallel mean flow effects eliminates the strict zone of silence in the ray acoustic solution, and that a continuous result can be obtained as the jet axis is approached. The fully non-parallel case involves three-dimensional ray tracing. Implementations of the three dimensional ray tracing solution have been carried out by Khavaran and Krejsa (1993) and Khavaran (1996).

Three-dimensional ray tracing requires a 3-D surface fit of the mean flow field from the CFD solution, which can limit the robustness of a prediction scheme based on this method. Noise predictions using this approach were made by Khavaran (1996) for a 2:1 aspect ratio supersonic elliptic jet. The results showed qualitative agreement with the experimentally observed azimuthal variation of the sound field. The method was found to be quite time consuming, and it was necessary to limit the source volume to the most energetic region of the jet. It should be noted, however, that this work was done before the wide spread availability of the relatively large compute clusters and associated software that are now commonly used. As noted earlier, ray acoustic methods are very well suited to parallel computing techniques and it should be possible to significantly reduce the time required to obtain these solutions if the ray trajectories are computed simultaneously.

A.2.3 Physical Modeling

A full numerical solution of the Green's function for the linearized Lilley equation (1.1) for a non-axisymmetric mean flow would be prohibitively time consuming to be useful in a nozzle design environment. Although less computationally intensive, geometric acoustic methods involve a relatively high level of overhead, in terms of sorting and interpreting the relevance of the many possible rays, and are best used on advanced clusters of computers. These methods are currently useful as research tools, and continued improvements in computational algorithms and hardware may make them feasible for inclusion in engineering design process in the future. In order to develop a prediction method which can provide faster, is somewhat more approximate, results, simplifying assumptions can be introduced.

If it is assumed (in the context of a locally parallel mean flow approximation) that the levels curves of the mean velocity and temperature fields (on which these quantities are constant) in a given axial slice of the jet coincide and are concentric, they can be considered as forming a one coordinate in a generalized cylindrical system, and the mean flow will vary only in the direction normal to these surfaces. For example, it may be reasonable to assume that the level curves of the mean flow are concentric with the nozzle exit geometry in some cases. The solution will then be periodic along these level curves and the problem can be reduced to a set of ordinary differential equations by expanding in a Fourier series. This reduces the differential operator to a form which closely resembles that for the axisymmetric case—except that in the non-axisymmetric case there is coupling among the equations governing the Fourier modes. Not all geometries of interest can be conveniently mapped in this way, and some may require a large number of modes to represent the solution. However, certain generic shapes are well suited to this approach and it can be used to study the azimuthal variation of the noise field radiated by jets exiting from nozzles of these shapes without recourse to large scale computations. A more detailed description of this approach, along with a specific example, is given in Section 4.

The discussion above has touched on a few approaches for modeling sound propagation in non-axisymmetric mean flows in the context of an acoustic analogy formulation. The different approaches have varying levels of fidelity and computational expense. It should be pointed out that, in addition to adequately modeling the sound propagation in a non-axisymmetric jet, accounting for the effects of asymmetry in the source model is equally important to the overall success of a noise prediction method based on an acoustic analogy.

A.3 Experimental Data and Observations

A.3.1 Simple Rectangular and Elliptic Jets

Acoustic spectra from subsonic jets emanating from non-circular nozzles have been measured at NASA Glenn (Zaman and Tam 1999; Tam and Zaman 2000). Rectangular nozzles with aspect ratios of 3:1 and 8:1, and elliptic 3:1 aspect ratio nozzles were studied. The results show relatively modest azimuthal variation in the sound spectra for the nozzles and flow conditions considered. For the rectangular nozzles, the largest differences were found at small angles to the jet axis ($\theta = 25^\circ$), where the high-frequency (Strouhal numbers greater than about 0.45) portion of the spectrum in the minor axis plane exceeds that on the major axis.

Flow data was also taken for these jets (in a different facility) to provide additional information to help understand the azimuthal dependence of the noise. Mach number contours in the cross-flow plane at various streamwise locations for the 8:1 rectangular nozzle show that the mean flow contours are nearly rectangular in the region very close to the nozzle exit, spread to become more elliptical further downstream, and eventually become circular sufficiently far from the nozzle exit. It was also found that the jets from larger aspect ratio nozzles had shorter potential core lengths.

An extensive set of flow and acoustic data for rectangular jets of various aspect ratios, and for a variety of operating conditions, has been obtained at Georgia Tech (Massey, Ahuja and Messersmith, 2002; Massey, Ahuja and Gaeta, 2004). Their acoustic measurements indicate that the rectangular jets they tested were generally quieter (by up to 2 to 3 dB) than an equivalent round jet on an Overall Sound Pressure Level (OASPL) basis. Their narrow-band spectral data showed that the higher aspect ratio rectangular jets were louder at high frequency, but quieter at low frequencies than their baseline circular jets. They also found significant differences in the spectra measured in different azimuthal planes, with the largest differences appearing at polar angles of 40° to the jet axis. These differences are, as would be expected, greater for nozzles of higher aspect ratio.

Flow measurements taken in a separate facility indicate that the rectangular jets in their experiments mixed faster than an equivalent round jet, with the potential core length shortening with increasing aspect ratio. Mach number profiles taken at various axial locations downstream of the nozzle exit suggest that the jets tended to maintain their asymmetry for a significant distance from the nozzle exit.

Narrow-band far-field spectral data from non-circular supersonic jets is available from the NASA Langley Jet Noise Lab (Norum, 1994). Data is available for jets from Mach 1.54 aspect ratio 2 elliptic, Mach 1.98 aspect ratio 3 elliptic and Mach 2.00 aspect ratio 7.6 rectangular nozzles. This data was analyzed by Tam (1998) in the context of his proposed two universal spectral shapes, and by Tam and Pastouchenko (2002) for comparison with their noise predictions in non-axisymmetric jets.

A.3.2 Distributed Exhaust Nozzles

In order to meet the increasingly stringent national and international noise standards being imposed on both current and future aircraft, it will be necessary to develop technologies which are capable of delivering dramatic reductions in jet noise levels relative to the current state of the art. Revolutionary new concepts will be needed to achieve this goal. One such concept which is currently being investigated is the idea of an integrated exhaust/airframe system, where the propulsion system is an integral part of the airframe.

An important element of this concept is the use distributed exhaust nozzles (DEN) integrated into the airframe. The DEN replaces the usual single-jet exhaust plume from a conventional nozzle by an array of much smaller jets. The smaller-sized jets result in a shift of the peak noise levels to higher frequency, where atmospheric attenuation is more effective. With the right amount of spacing of the small jets, their ultimate coalescence into a single jet can be made to occur at relatively low velocity, providing a lower level of low-frequency noise than from a single jet plume.

The design of such nozzles requires careful consideration of the size, shape and spacing of the openings through which the flow will exit, as well as the monitoring of any associated aerodynamic penalties. There appears to be significant potential for the application of DEN, but further research is needed. In particular, there is presently no available way of predicting how changes in the many relevant parameters will affect the radiated far-field noise.

Kinzie et al. (2001) carried out steady flow computations and far-field noise measurements on a distributed exhaust model consisting of a series of horizontal slots. The nozzle tested in this work resulted in minimal thrust loss, but also very little noise reduction, because the individual jets merged quickly to form a single plume. This work highlighted the importance of having the individual jets which emerge from the distributed exhaust nozzle maintain their separate identities for a sufficiently long distance downstream in order to obtain any significant noise reduction.

Tests on distributed nozzles designed with the help of CFD calculations (using a Northrop Grumman code) were made by Gaeta et al. (2002) and Kinzie et al. (2005). These nozzles showed significant differences in terms of spectral content and far-field directivities from baseline round nozzles, and demonstrated the potential for substantial noise reduction using distributed nozzles.

Gaeta et al. (2002) found that their DEN operated at subsonic exhaust velocities resulted in lower noise levels at relatively low frequencies and that the peak spectral noise levels are shifted to higher frequencies relative to an ‘equivalent’ round nozzle. At supersonic exit conditions, they found that the DEN does not exhibit the screech tones found in the round jet, and that the broadband shock-associated noise is significantly reduced. Gaeta et al. (2002) document the polar and azimuthal directivity of the DEN and compare the results with the round nozzle.

Kinzie et al. (2005) tested two DENs, one very similar to the one test by Gaeta et al. (2002) and one where the exit holes were tear-dropped shaped (DROPS nozzle), both designed with the aid of the Northrop Grumman CFD code. Large sound level reductions, based on overall sound pressure level (OASPL) were found, particularly for the DROPS nozzles, and some significant azimuthal variations were found relative to their baseline round nozzle.

For all of the nozzles tested by Gaeta et al. (2002) and Kinzie et al. (2005), a low-frequency ‘hump’ was found in the sideline (polar angle of 90°) spectra in the loudest of the azimuthal planes measured. Gaeta et al. (2002) carried out a series of tests varying the nozzle exit velocity and conjectured that this hump could be a manifestation of the trailing edge noise generated at the apex of the DEN nozzle. Subsequent tests by Gaeta et al. (2004) with modified trailing edge conditions, however, showed no effect on the frequency or level of this hump. The possibility that this spectral hump was caused by sources internal, or very close to, the nozzle exit was investigated by Gaeta et al. (2004) by varying the height of the internal center body which serves to split the incoming stream to feed the upper and lower surfaces of the nozzle. A lower peak frequency was indeed found for the hump with a thicker center body, but a scaling of the expected peak Strouhal number based on this height did not quite match the observed peak frequencies. Gaeta et al. (2004) point out that the low-frequency hump in DEN noise spectra shares some characteristics with that with produced by a jet exhausting over a portion of a wing. The true source of this spectral peak is not known at this time. This illustrates the additional complexities which may be involved in predicting the noise from distributed exhaust nozzles systems relative to round or simple non-circular nozzles. Additional noise generation mechanisms, beyond those typically considered in jet noise, may need to be included to predict all the features found in the noise signatures of these nozzles.

Gaeta et al. (2004) also studied the effects of simulated forward flight on the noise from a DEN. They found that the trends observed under static conditions, namely a shift of the peak noise to higher frequency and a reduction of low-frequency noise relative to a round nozzle, were also present with

forward flight. Non-zero forward flight velocity generally reduces the noise from jet exhaust plumes, and Gaeta et al. (2004) found, as expected, that increasing the flight velocity reduced the noise from DEN over most of the frequency range. The low-frequency ‘hump’ found in the static tests was still present with forward flight, but its peak was shifted to higher frequency and it was found to persist over a more limited frequency range as the flight velocity was increased.

A.4 Conclusions and Recommendations

To aid in the development of aircraft noise reduction technologies, prediction tools are needed. The ultimate goal is to have a prediction code which can handle all effects which can be potentially important in noise generation and reduction: complex geometries, sound interaction with airframe surfaces, mechanisms associated with shock-associated noise, etc. Significant development work will be required to achieve this goal. One step toward this goal is to extend the capability of current jet noise prediction schemes (such as the JeNo code) to non-circular geometries.

It is desirable to have an accurate, efficient and robust numerical method for computation of the propagation of sound through spatially evolving, non-axisymmetric mean flows. Advances in numerical algorithms and boundary conditions show promise for jet noise prediction. Their development and implementation in the context of a jet noise prediction scheme should be pursued. In order to be able to obtain results in the more near term, the best choice would be to begin by considering relatively simple, generic nozzle exit geometries and to introduce appropriate modeling assumptions to minimize the overall computational effort. Study of the azimuthal variation of sound radiation from simple non-circular geometries, such as rectangular or elliptical ones, can provide valuable information on the effects of nozzle geometry, aspect ratio, etc., on far-field sound directivity. The modeling assumptions and computer codes can be validated with existing experimental data.

A quasi-parallel mean flow approximation seems to be appropriate, and the problem should be formulated in terms of the adjoint Green’s function for the Lilley equation. Fourier transforms can be taken in the streamwise direction as well as in time to obtain the equation and boundary conditions for the reduced Green’s function. Mean flow solutions for the particular geometries to be considered will be needed, and it is expected that these can be obtained from the WIND code with the support of personnel in the Inlets and Nozzles Branch who are currently involved with the continued development of this code. Functional fits and/or models for the mean velocity and temperature profiles in the cross-flow plane at each axial slice in the jet will need to be constructed from the CFD solutions.

A model which results in the reduction of the partial differential operator of the reduced Green’s function problem for a general transversely sheared mean flow to that of an ordinary differential equation is highly desirable, since it provides considerable simplification of the final boundary value problem that must be solved numerically.

If the mean flow is modeled as having concentric level surfaces, these surfaces can be considered as coordinate surfaces in a general cylindrical coordinate system. The mean flow will then vary only in the direction normal to these surfaces and a transformation can be made such that the solution is periodic in one direction. The partial differential equation can then be reduced to a system of coupled ordinary differential equations by expanding in Fourier modes. A generic case to consider is that where the mean flow level surfaces, $U = U(u)$ and $\tilde{c}^2 = \tilde{c}^2(u)$, form a concentric series of ellipses. The conformal mapping

$$z = x_2 + ix_3 = A \cosh(u + iv) \quad (1.3)$$

maps the domain $0 \leq u < \infty$, $-\pi \leq v < \pi$ to elliptical cylindrical coordinates, with lines of constant u and v forming confocal ellipses and hyperbolas, respectively.

Introduction of this mean flow model, along with the transformation (1.3) and a Fourier series expansion in v , into the reduced Lilley equation, results in relatively minimal coupling among the

resulting equations for the Fourier modes. Discretization of the ordinary differential operator in u using second-order finite-differences yields a potentially large—but highly banded—system of simultaneous equations. It may be feasible to solve this system of equations directly using a sparse system solver. If core memory becomes a problem (for example, if the number of grid points required becomes too large) an iterative method along with Fast Fourier transforms could be used.

An important practical application for this work is to predict the noise from rectangular jets. A rectangular nozzle can also be mapped into a periodic semi-infinite domain, but the transformation involves more complicated, multi-valued, special functions (Nehari, 1952). Experimental measurements and numerical computations (Zaman and Tam, 1999 ; Tam and Pastouchenko, 2002) of jets flows from rectangular nozzles show that the mean velocity contours in the cross flow plane become generally elliptical beyond a certain distance from the nozzle exit, and that they become nearly circular sufficiently far downstream. A simple approach to predicting the sound field from a rectangular jet, would be to represent the mean cross-stream velocity (and sound speed) distribution in each axial slice by concentric ellipses, with the latter becoming more circular with increasing downstream distance from the nozzle exit. The ability of this simple model to provide sufficiently accurate predictions would depend upon the particular application considered. It could be expected, for example, that the high-frequency portion of the spectrum, which is generated in the region relatively close to the nozzle exit, where the profiles would differ most from the elliptic shape, would be less well predicted than that generated further downstream.

A better representation of the mean flow in a rectangular jet is given by a concentric family of super ellipses. A conformal mapping between Cartesian coordinates and the level curves of a super ellipse does not seem to exist. It has been found, however, that a slight modification of the elliptical coordinate transformation, results in a system which looks very much like a super ellipse. The modified transformation is given by

$$\begin{aligned} x_2 &= A[\cosh(u)\cos(v) + \alpha \tanh(u)\cos(3v)] \\ x_3 &= A[\sinh(u)\sin(v) + \beta \tanh(u)\sin(3v)]. \end{aligned} \tag{1.4}$$

Figure 1 shows an example of this transformation with $\alpha = 0.0$ and $\beta = 0.2$.

Comparisons of the surfaces of constant u in figure 1 with experimentally measured (Zaman and Tam, 1999) and numerically computed (Tam and Pastouchenko, 2002) mean velocity distributions in rectangular jets suggest that these surfaces would provide a better fit to the mean profiles than the elliptic coordinates. The (u, v) coordinate system obtained from the transformation (1.4) is, however, non-orthogonal. This results in a more complicated differential operator for the governing equation, as well as additional coupling among the equations for the Fourier modes beyond the case of an ellipse.

The values of the coefficients α and β used in constructing the coordinate system shown in figure 1, however, are quite small so that this transformation could, in some sense, be considered a small perturbation about the elliptical transformation. Figure 2 shows one of the level surfaces of u from the elliptical transformation (1.3) and the modified form (1.4). This suggests that it could be possible to set up a perturbation type procedure where modeling the level surfaces of the mean flow from a rectangular nozzle as concentric ellipses could be considered the lowest-order approximation, with the improved fitting of the mean flow associated with the modified transformation (1.4) treated as a small correction.

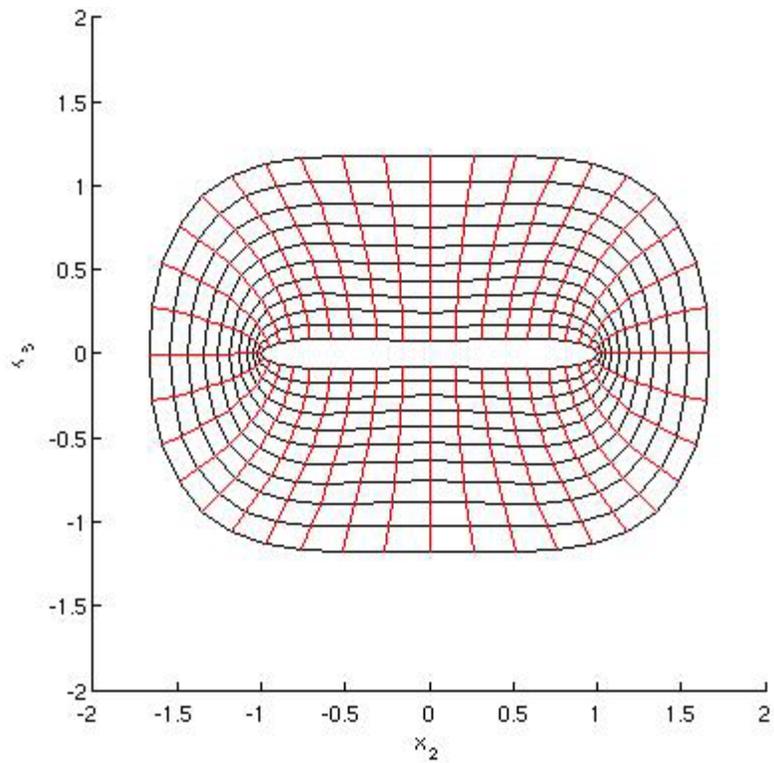


Figure 1.—Coordinate system generated by the transformation (1.4).
Lines of constant u (black), lines of constant v (red).

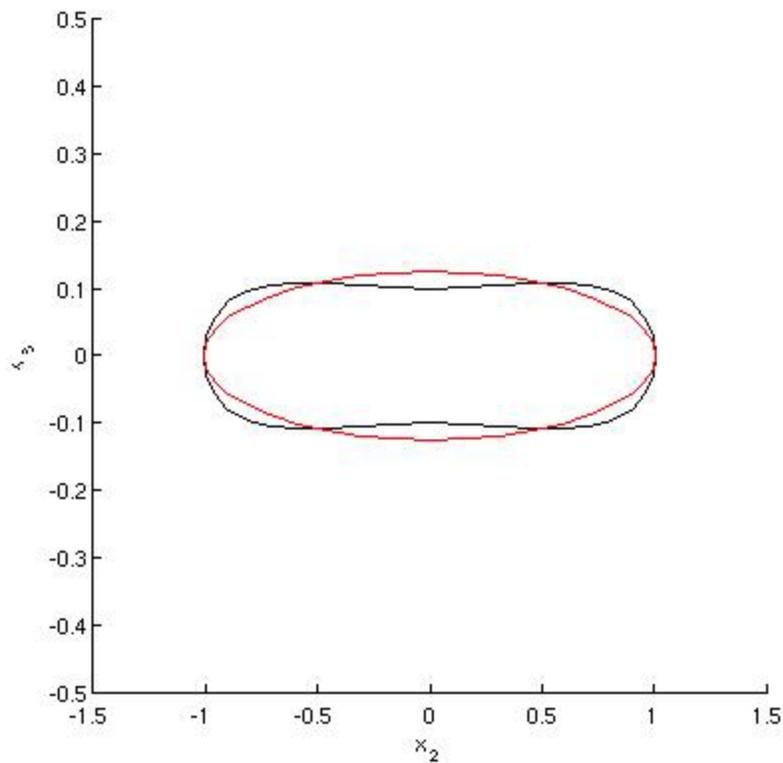


Figure 2.—Comparison of the level surfaces $u = 0.125$ for an ellipse (red) and from transformation (1.4) (black).

It would seem appropriate then to begin with the elliptical coordinate transformation (1.3) and treat the mean flow as a function of u alone in these variables. Numerical results could be obtained for this case using either analytically prescribed functions for the mean velocity and temperature, or, by fitting the results obtained from a RANS solution to set of ellipses. Corrections due to the modified transformation (1.4) could then be implemented and tests carried out to assess the relative importance of any differences obtained in a variety of cases. This approach would result in a computational code which could be used to model sound propagation through a rectangular jet in a relatively simple manner without requiring excessively long computational times. The code could be made to run on cluster of computers to, for example, simultaneously obtain solutions for different frequencies and/or far-field polar angles.

As a longer term goal, an efficient numerical method for solution of the sound propagation through general transversely sheared mean flows should be developed. A full numerical solution would have the advantage of not being restricted to particular geometries and would be better suited to studying the detailed effects of more complex mean flow fields on the far-field sound. The disadvantage is, of course, that it will be considerable more complicated and much more time consuming to obtain results. The propagation algorithms and boundary condition treatment developed by Goodrich (1999) and Goodrich, Hagstrom and Lorenz (2006) would seem to be good candidates for solution of this problem. These methods have been demonstrated to provide highly accurate results in reasonable amounts of time when applied to model problems.

In addition to the need to compute the propagation of sound through the jet, models for the acoustic sources are needed to make jet noise predictions using an acoustic analogy approach. The current version of the JeNo code simplifies the description of the source terms by approximating the statistics of the turbulence as isotropic and quasi-normal. The relevant, second-order, space-time correlations are expressed as separable, exponential functions of space and time. This source model has been demonstrated to produce results in good agreement with experimental data for the acoustic spectrum for round subsonic jets and supersonic jets near 90° to the jet axis.

For more general applications, such as high speed jets near the peak noise direction and more complicated geometric configurations, it is desirable to have available a more general description of the source terms. A very general class of models for the source terms in the formula for the acoustic spectrum was proposed by Goldstein and Leib (2007). The model allows for non-normal and axisymmetric turbulence and allows for the exact inclusion of effects, such as source non-compactness, which are often neglected or treated approximately in other models. Development of these models is ongoing, and it is intended to make use of the latest results of this work to model the acoustic sources in non-circular jets.

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- Progress on Green's Function Calculations for Non-Axisymmetric Jets, NASA Acoustics Technical Working Group, April 23-24, 2013, Cleveland, Ohio.

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- Leib, S.J. and Goldstein, M.E. "Hybrid Source Model for Predicting High-Speed Jet Noise," *AIAA Journal*, Vol. 49, No. 7, 2011.
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14. ABSTRACT The objective of this task was the development of source and directivity models to predict noise from non-axisymmetric jets. The technical approach was based on an acoustic analogy for which sound propagation is described by a Green's function for an appropriate linear operator, and models are constructed for the statistics of the nominal sound sources. A literature survey was carried out to assess the (then) current state of the art in non-axisymmetric jet noise prediction. Reduced-order propagation models were developed that include three-dimensional effects associated with sound propagation from non-circular nozzles using conformal mapping, expansion in series' of orthogonal functions and numerical methods. Source models were constructed that incorporated features of turbulent jet flow statistics found in recent experiments. These source models were integrated with the relevant Green's function solvers to construct noise prediction codes. Results from these noise prediction codes were compared with acoustic data provided by NASA. Work under this task also supported an effort to develop a code for numerical solution of the full acoustic analogy equations, for use in validating the reduced-order models and performing high-fidelity calculations for cases of special interest. Theoretical formulations and models for the noise generated by the interaction of a jet with the trailing edge of a flat plate were developed and implemented into a prediction code to extend to the geometrical configurations for which reduced-order predictions can be made.					
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