

BUCKLING OF STIFFENED, FLAT, PLYWOOD PLATES IN COMPRESSION

A SINGLE STIFFENER PERPENDICULAR TO STRESS

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BUCKLING OF STIFFENED, FLAT PLYWOOD PLATES IN COMPRESSION--

A SINGLE STIFFENER PERPENDICULAR TO STRESS¹

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Summary and Conclusions

A mathematical analysis of the critical stress of a plywood plate in edge compression stiffened by means of a stiffener glued to the plate is presented in this report. The analysis yields an approximate formula for the minimum stiffness of the stiffener required to cause the plate to buckle into 2 half-waves rather than into 1. The data from 371 tests of 60 plates suggest a modification of the formula by an empirical factor, probably made necessary by the original lack of flatness of the plates.

Introduction

The critical buckling stress of a flat plywood plate with simply supported edges in edgewise compression can be increased appreciably by the addition of a small stiffener at the center of the plate and parallel to its loaded edge. A sufficiently large stiffener will restrain the plate so that it will buckle in two half-waves with a node at the stiffener. If the depth of the stiffener is slightly reduced and the plate again loaded, the node between the 2 half-waves will move away from the stiffener, and the buckle pattern will show 2 unequal half-waves. Such buckle patterns are consistent with the mathematical analysis given herein. Further reduction of the depth of the stiffener will allow the plate to buckle in a single half-wave somewhat flattened in the vicinity of the stiffener. This flat spot will decrease in length as the depth of the stiffener is decreased until, when the stiffener is entirely removed, the buckle trace will be nearly a perfect sine curve. This evolution of the buckle pattern as viewed along the center line perpendicular to the stiffener is shown by traces of the various patterns in figure 1.

¹This is one of a series of progress reports prepared by the Forest Products Laboratory relating to the use of wood in aircraft, issued in cooperation with the Army-Navy-Civil Committee on Aircraft Design Criteria. Original report published in 1946.
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If the stiffener is sufficiently stiff, the plate may be considered as two plates each having a length-to-width ratio of half that of the entire plate. The critical stress is therefore greater than that of the unstiffened plate. As the stiffener is reduced in depth, the critical stress remains constant at this higher value until the plate buckles into a single half-wave. From this point the critical stress steadily decreases with the reduction of the stiffener down to the critical stress of the unstiffened plate. Many of the plates having stiffeners of nearly the critical size act peculiarly in their buckling behavior. There is a tendency for the critical stress to increase slightly as the decreasing stiffener size passes the critical point.

This report presents a mathematical analysis of the critical stress of such plates in which an approximate formula is derived for the minimum stiffness of the stiffener necessary to cause the plate to act as two plates. The data from 371 tests of 60 plates suggest an empirical modification of the formula for use in design.

Mathematical Analysis

Consider a plywood panel (fig. 2) with a central horizontal stiffener, and let the dimensions of the panel be such that without the stiffener it would buckle into a single half-wave when subjected to a sufficiently large compressive load. All edges are assumed to be simply supported. It is also assumed that the torsional rigidity of the stiffener can be neglected and that the effect of the stiffener is concentrated along the center line of the panel. An approximate method will be developed to determine the least stiffness of the stiffener that will cause the panel to buckle into 2 half-waves instead of 1 when sufficient compressive load is applied. A method of derivation will be adopted that will furnish a means of interpreting the buckling patterns that are experimentally observed. It will be assumed that the panel is initially not perfectly flat. The initial small departure from perfect flatness can be represented by the following double Fourier's series:

$$w_0 = \sum \sum a_{ij} \sin i \alpha x \sin j \beta y \quad (1)$$

where $\alpha = \frac{\pi}{a}$, $\beta = \frac{\pi}{b}$, and i and j are integers that may take values from 1 to ∞ . Later it will be assumed for purposes of approximation that only 2 of the coefficients a_{ij} are different from 0. The early stages of the calculations will be carried out for the general form.

Since the panel is not initially perfectly flat, it will deflect laterally under the action of even a small compressive load P per inch of edge. The deflected surface at any load will be represented in the following form:

$$w = \sum \sum b_{ij} \sin i \alpha x \sin j \beta y \quad (2)$$

In accordance with (1), since the stiffener is attached to the panel the initial departure of the stiffener from perfect straightness is given by

$$w_{0S} = \sum \sum a_{ij} \sin i \alpha x \sin j \frac{\pi}{2} = \sum A_i \sin i \alpha x \quad (3)$$

where

$$A_i = \sum_{j \text{ odd}} (-1)^{\frac{j-1}{2}} a_{ij} \quad (4)$$

In accordance with (2), the deflection of the stiffener under compressive load, as measured from a straight line, is given by

$$w_s = \sum B_i \sin i \alpha x \quad (5)$$

$$B_i = \sum_{j \text{ odd}} (-1)^{\frac{j-1}{2}} b_{ij} \quad (6)$$

The energy method will be used to determine the values of the coefficients b_{ij} in terms of the coefficients a_{ij} for a given value P of the load per inch of loaded edge. These coefficients determine the shape of the panel in equilibrium under a given load. For this purpose it is necessary to calculate the flexural energies of the plate and stiffener and the loss in potential energy of the load due to the vertical shortening of the distance between the loaded edges associated with the bending of the plate.

The flexural energy of the plate is given by the expression

$$W_1 = \frac{h^3}{24\lambda} \int_0^a \int_0^b \left\{ E_1 \left[\frac{\delta^2(w - w_0)}{\delta x^2} \right]^2 + E_2 \left[\frac{\delta^2(w - w_0)}{\delta y^2} \right]^2 + 2E_L \sigma_{TL} \frac{\delta^2(w - w_0)}{\delta x^2} \frac{\delta^2(w - w_0)}{\delta y^2} + 4\lambda \mu_{TL} \left[\frac{\delta^2(w - w_0)}{\delta x \delta y} \right]^2 \right\} dy dx \quad (7)$$

where coefficients have the meanings given them in Forest Products Laboratory Reports Nos. 1312 and 1316 and in the table of notation.

After substituting (1) and (2) in (7) and performing the integrations, it is found that

$$W_1 = \frac{h^3 ab}{96\lambda} \left\{ E_1 \alpha^4 \sum \sum i^4 (b_{1j} - a_{1j})^2 + E_2 \beta^4 \sum \sum j^4 (b_{1j} - a_{1j})^2 + \right. \\ \left. 2A\alpha^2\beta^2 \sum \sum i^2 j^2 (b_{1j} - a_{1j})^2 \right\} \quad (8)$$

where

$$A = E_L \sigma_{TL} + 2\lambda \mu_{LT} \quad (9)$$

The flexural energy of the stiffener is given by the expression

$$W_2 = \frac{(EI)_s}{2} \int_0^a \left[\frac{\delta^2 (w_s - w_{0s})}{\delta x^2} \right]^2 dx \quad (10)$$

where $(EI)_s$ denotes the stiffness of the stiffener as calculated by the method described in Forest Products Laboratory Report No. 1557.

After using equations (3) and (5) and performing the integrations, it is found that

$$W_2 = \frac{(EI)_s \alpha^4 a}{4} \sum i^4 (B_1 - A_1)^2 \quad (11)$$

The loss in potential energy of the load is equal to the work done by the load during the shortening of the panel due to bending. Hence,

$$W_3 = \frac{P}{2} \int_0^a \int_0^b \left[\left(\frac{\delta w}{\delta y} \right)^2 - \left(\frac{\delta w_0}{\delta y} \right)^2 \right] dy dx \quad (12)$$

where P is the applied load per inch of edge of the panel.

After using equations (1) and (2) and performing the integrations, it is found that

$$W_3 = \frac{P\beta^2 ab}{8} \sum \sum j^2 (b_{1j}^2 - a_{1j}^2) \quad (13)$$

For equilibrium the expression

$$W = W_1 + W_2 - W_3 \quad (14)$$

is a minimum.

In accordance with (8), (11), and (13),

$$W = D_1 \frac{ab}{8} \left\{ \sum \sum \lambda_{1j} (b_{1j} - a_{1j})^2 + \frac{2\gamma a}{b} \alpha^4 \sum i^4 (B_i - A_i)^2 - \frac{P\beta^2}{D_1} \sum \sum j^2 (b_{1j}^2 - a_{1j}^2) \right\} \quad (15)$$

where

$$\lambda_{1j} = \alpha^4 i^4 + \delta \beta^4 j^4 + 2\epsilon \alpha^2 \beta^2 i^2 j^2 \quad (16)$$

$$\delta = \frac{D_2}{D_1} \quad (17a)$$

$$\epsilon = \frac{K}{D_1} \quad (17b)$$

$$\gamma = \frac{(EI)_s}{D_1 a} \quad (17c)$$

$$D_1 = \frac{E_1 h^3}{12\lambda} \quad (18a)$$

$$D_2 = \frac{E_2 h^3}{12\lambda} \quad (18b)$$

$$K = \frac{Ah^3}{12\lambda} \quad (18c)$$

The conditions for equilibrium are then

$$\frac{\delta W}{\delta b_{1j}} = 0, \quad \begin{array}{l} i = 1, 2, 3, \dots \\ j = 1, 2, 3, \dots \end{array} \quad (19)$$

It is possible to complete the analysis based on this infinite system of equations. Approximate results, however, can be obtained by supposing that all coefficients b_{1j} and a_{1j} are 0 with the exception of b_{11} , b_{12} , a_{11} , and a_{12} . This means that only 2 "modes" of deflection are considered, the mode (1,1) in which the deflection is proportional to $\sin \alpha x \sin \beta y$ and the mode (1,2) in which the deflection is proportional to $\sin \alpha x \sin 2\beta y$. The mode (1,1) has no nodes except at the edges of the panel, while the mode (1,2) has a node at the stiffener also. The consideration of these two modes is sufficient to provide an explanation of the observed buckling patterns. Further, the consideration of 2 modes only leads in a simple way to an approximate expression for the stiffness of the stiffener necessary to make the plate buckle into 2 half-waves.

Equation (19), after omitting all coefficients except those mentioned previously, yields two equations, which are:

$$\frac{\delta W}{\delta b_{11}} = \frac{D_1 a b}{8} \left\{ 2\lambda_{11}(b_{11} - a_{11}) + \frac{4\gamma \alpha^4 a}{b}(b_{11} - a_{11}) - \frac{P\beta^2 2b_{11}}{D_1} \right\} = 0 \quad (20)$$

$$\frac{\delta W}{\delta b_{12}} = \frac{D_1 a b}{8} \left\{ 2\lambda_{12}(b_{12} - a_{12}) - \frac{P\beta^2}{D_1} 8b_{12} \right\} = 0$$

From these equations the following values for $\underline{b_{11}}$ and $\underline{b_{12}}$ are readily found:

$$b_{11} = \frac{a_{11}}{1 - \frac{P}{P_{11}}} \quad (21)$$

$$b_{12} = \frac{a_{12}}{1 - \frac{P}{P_{12}}} \quad (22)$$

$$P_{11} = \frac{D_1 \left(\lambda_{11} + \frac{2\gamma \alpha^4 a}{b} \right)^4}{\beta^2} \quad (23)$$

$$P_{12} = \frac{D_1 \lambda_{12}^4}{4\beta^2} \quad (24)$$

After introducing the expressions for λ_{11} and λ_{12} and making the substitutions indicated by equations (17) and (18), the following values are obtained for $\underline{P_{11}}$ and $\underline{P_{12}}$:

$$P_{11} = \frac{\pi^2}{12\lambda} \left[E_1 \frac{b^2}{a^2} + 2A + E_2 \frac{a^2}{b^2} \right] \frac{h^3}{a^2} + 2\pi^2 \frac{b}{a^4} (EI)_s \quad (25)$$

$$P_{12} = \frac{\pi^2}{12\lambda} \left[E_1 \frac{b^2}{4a^2} + 2A + E_2 \frac{4a^2}{b^2} \right] \frac{h^3}{a^2} \quad (26)$$

It may be seen from equations (21) and (22) that $\underline{P_{11}}$ is the critical load at which the stiffened panel buckles in a single half-wave and that $\underline{P_{12}}$ is the critical load at which the stiffened panel will buckle in two half-waves. If the panel buckled in two waves, there would be a node at the stiffener of an initially flat panel. If, however, the panel is not perfectly flat initially, the mode (1,1) may be expected to be present in the Fourier representation of its surface. The amplitude of this mode will be increased to a greater or lesser extent, depending upon the stiffness of the stiffener, as the load approaches the critical load $\underline{P_{12}}$. The resulting combination of the modes (1,1) and (1,2) will lead to a buckling pattern consisting of two unequal half-waves, and the stiffener

will be bent. If the stiffener is very stiff, the development of mode (1,1) to any great extent is prevented, and the panel buckles into two nearly equal half-waves. An illustration will be given later of the relative amplification of the two modes for different stiffnesses of stiffener. Equation (26) agrees with equation (1) of Forest Products Laboratory Report No. 1316 for the case of a buckle form of two half-waves, and equation (25) agrees with the same equation for the case of a buckle form of a single half-wave, except for the added term due to the presence of the stiffener. It should be noted that equations (25) and (26) are written for critical load per inch of edge, while equation (1) of Report No. 1316 is written for critical stress.

Let $(EI)_s$ be increased through a series of values. A study of equations (25) and (26) together with (21) and (22) shows that the panel will buckle in mode (1,1) with some amplification of the mode (1,2) until $(EI)_s$ is so large that

$$P_{11} = P_{12}$$

that is, until

$$\frac{\pi^2}{12\lambda} \left[E_1 \frac{b^2}{a^2} + 2A + E_2 \frac{a^2}{b^2} \right] \frac{h^3}{a^2} + 2\pi \frac{2b}{a^4} (EI)_s = \frac{\pi^2}{12\lambda} \left[E_1 \frac{b^2}{4a^2} + 2A + E_2 \frac{4a^2}{b^2} \right] \frac{h^3}{a^2}$$

From this equation it follows that the critical value of $(EI)_s$ at which the plate will buckle into two half-waves with a node at the stiffener is

$$(EI)_{scr} = \frac{h^3 b}{8\lambda} \left[E_2 \frac{a^4}{b^4} - \frac{1}{4} E_1 \right] \quad (27)$$

A square panel of isotropic material with a central horizontal stiffener will be considered for the purpose of illustrating the relative rates of increase of the amplitudes b_{11} and b_{12} of the modes (1,1) and (1,2) from their initial values a_{11} and a_{12} for different stiffnesses of the stiffener as the load approaches a critical value. Since the panel is isotropic

$$E_1 = E_2 = A = E$$

By using these relations and the fact that the panel is square, equations (25) and (26) are reduced to the forms

$$P_{11} = \frac{Eh^3}{12\lambda} \frac{\pi^2}{a^2} \left[4 + 2 \frac{12\lambda}{h^3 a} (EI)_s \right] = D\alpha^2 (4 + 2\gamma) \quad (28)$$

$$P_{12} = \frac{25}{4} \frac{Eh^3}{12\lambda} \frac{\pi^2}{a^2} = \frac{25}{4} D\alpha^2 \quad (29)$$

It will be convenient to write equations (21) and (22) in the forms

$$\frac{b_{11}}{a_{11}} = \frac{1}{1 - \frac{P}{P_{11}}} \quad (30)$$

$$\frac{b_{12}}{a_{12}} = \frac{1}{1 - \frac{P}{P_{12}}} = \frac{1}{1 - \frac{P_{11}}{P_{12}} \frac{P}{P_{11}}} \quad (31)$$

For given values of γ it is then possible to calculate $\frac{b_{11}}{a_{11}}$ and $\frac{b_{12}}{a_{12}}$ for various values of P . For example, if $\gamma = 0.5$, $P_{11} = 5D\alpha^2$ and $P_{12} = 6.25D\alpha^2$. The following results are then readily obtained:

if $P = 0.5P_{11}$	$b_{11} = 2a_{11}$	$b_{12} = 1.67a_{12}$
$P = 0.7P_{11}$	$b_{11} = 4a_{11}$	$b_{12} = 2.5a_{12}$
$P = 0.9 P_{11}$	$b_{11} = 10a_{11}$	$b_{12} = 3.57a_{12}$

These results and those for other values of γ are plotted in figure 3.

The least value of γ for which the panel will buckle into 2 half-waves is found to be 1.125 by equating P_{11} and P_{12} as given by equations (28) and (29). The curves show that for values of γ in the neighborhood of this critical value there is nearly equal amplification of the amplitudes of the modes (1,1) and (1,2). The vertical asymptotes at the right in figure 3 correspond to the critical load P_{11} when $\gamma < 1.125$ and to P_{12} when $\gamma > 1.125$.

From this example it is clear that the initial lack of flatness of the panel will complicate the observed form of the deflected surface of the panel as the load is gradually increased to the buckling load. The initial departure from perfect flatness must be small if the preceding discussion is to apply. If it is not, membrane stress will be developed, and the situation will be more complicated. This may account for some of the scatter of the experimental points.

For the purpose of obtaining approximate results in relatively simple form, only two modes (1,1) and (1,2) of the deflection pattern were considered. More exact results can be obtained by carrying the analysis through for the general form of equation (15). A general analysis has

been carried out by Lundquist² from a somewhat different standpoint for the case of several horizontal stiffeners. Timoshenko³ has treated the case of an isotropic panel with horizontal stiffeners. Values of γ for isotropic plates, obtained by substituting in equation (17c) the values of $(EI)_s$ given by equation (27) based on only the 2 modes considered, are found to average about 6 percent less than those given by Timoshenko.³

Description of Stiffened Plates

The plates were cut from yellow birch plywood of aircraft grade. Three different thicknesses and constructions of plywood were selected: (1) 3 plies of 1/16-inch veneer, the direction of the grain of the core perpendicular to that of the faces; (2) 4 plies of 1/20-inch veneer, the direction of the grain of the 2 inner plies parallel to each other and perpendicular to that of the outer plies; and (3) 5 plies of 1/20-inch veneer, the direction of the grain of each ply perpendicular to that of adjacent plies. The plates varied in width from 8 to 12 inches and in length from 6 to 20 inches. In the tests the load was applied to edges of the plates having the 8- to 12-inch dimensions. The direction of the grain of the face plies of each plate was parallel or perpendicular to the direction of the load.

The stiffeners were made of sitka spruce and varied in width from 1/8 to 3/8 inch and in depth from 0 to 1 inch. Each stiffener was cut 1-1/4 inches shorter than the width of the plate to which it was glued, as shown in figure 2.

Before assembling, both the plates and the stiffeners were conditioned in an atmosphere of 65 percent relative humidity and a temperature of 70° F. After assembly, they were stored under the same conditions until the time of test.

Method of Test

The following conditions are assumed in the theory and were approximated in the tests: The plywood is initially nearly flat; all four edges of the panel remain in a plane and are simply supported; and the loaded edges are displaced with respect to each other uniformly across the width of the specimen.

²Lundquist, E., Journal of Aeronautical Sciences 6: 269, 1939.

³Timoshenko, S., Theory of Elastic Stability, p. 378 and table 41, p. 379, New York, 1936.

The restraining frame used (fig. 4) is described in Forest Products Laboratory Reports Nos. 1316-D, E, and G except that the conditions at the loaded edges of the test specimen were slightly modified. The loaded edges of the specimen were fitted into segmented cylindrical steel rods instead of the continuous rods previously used, which were in turn held in position by the loading head shown in figure 5. The top and bottom loads were applied to the heads through two sets of knife edges, each set free to rotate about a single axis perpendicular to the plane of the specimen. The unloaded edges of the test specimen were positioned as shown in figure 6.

The segmented rod was used to increase accuracy in the determination of critical loads of low values. In the present investigation it was necessary to choose the dimensions of the plates such that the critical loads obtained with a heavy stiffener were well below the proportional limit of the material and also to determine the critical loads after the stiffeners were removed. The critical loads when the stiffeners were removed were so low that they were unduly influenced by the restraint imposed by the solid rod previously used.

The testing procedure was as follows:

The dimensions of the specimen were measured, the thickness to 0.001 inch and the length and width to 0.01 inch. The width and depth of the stiffener were measured to 0.001 inch.

The edges of the specimen to be tested were snugly fitted into rectangular slots in the segmented cylindrical steel loading rods. The segments were spaced upon the specimen so as to allow approximately 1/16 inch between them. Tension springs whose free length was approximately 9/10 of the length of the unloaded edge of the specimen were attached to screws fastened to the loading rods 3/16 inch from the unloaded edges of the specimen. Clamps were applied to the stiffener near its ends to prevent its separation from the plywood plate. A specimen prepared for test is shown in figure 7.

The restraining frame was placed on the bed of the testing machine and centered beneath the upper set of knife edges. The 2 guide rails for 1 face of the specimen were adjusted by means of the adjusting screws to be vertical and in a plane. The lower loading head was placed on the lower set of knife edges and centered in the restraining frame. The specimen was set in the frame; the upper loading head was placed on the specimen and the movable posts of the restraining frame adjusted to be vertical and just to clear the unloaded edges of the specimen. Then the adjustable rails were moved toward the specimen until a 0.002-inch clearance was obtained between the tension springs and rails. The edge conditions at time of test are shown in figures 5 and 6. A stiffened plate ready for test is shown in figure 8.

Data for load-strain curves were observed, and the load increments were so chosen as to define clearly the slope of the curve. The strain was measured by means of metaelectric strain gages mounted on each face of the specimen at the center of the width and just below the stiffener. The strain was read to the nearest 0.00001 inch. These gages were mounted directly opposite each other and were connected in series so that they recorded the average compressive strain. Loading of the specimen was continued until the strain began to decrease.

Each plate was tested first with a stiffener of 1-inch depth (perpendicular to plate). Then the depth was reduced and the specimen tested again. This was continued until the stiffener was reduced to nothing and the plate alone had been tested.

Coupons were cut from the specimen after test, from which the elastic properties were obtained. They include 1 for a static bending test, the face grain parallel to the span; 1 for a static bending test, the face grain perpendicular to the span; 2 for compression tests, the face grain parallel to the direction of applied stress; 2 for compression tests, the face grain perpendicular to the direction of applied stress; and 1 for a plate shear test.

The static bending tests were made to determine the effective modulus of elasticity in bending parallel and perpendicular to the face grain. The specimens were tested as simply supported beams centrally loaded. The coupons were 2 inches wide, and the spans used were 48 or 24 times the thickness of the coupon when the face grain was parallel or perpendicular to the span, respectively.

The coupons that were tested in compression were 4 inches long by 1 inch wide by the thickness of the plywood and were tested in a device that provided lateral support to keep the specimen in the plane of the load. The effective moduli of elasticity in compression were thus obtained in the two different grain directions.

Plate shear tests were made to determine the shear modulus of the plywood. The coupons were cut into squares in which the dimensions of the sides were 30 to 40 times the thickness. The test is described in Forest Products Laboratory Report No. 1301.

The stiffeners, prior to attachment to the plywood, were tested as simply supported beams centrally loaded on a span of 14 inches to determine the modulus of elasticity.

Determination of the Critical Load

The critical stress of flat rectangular plywood plates subjected to a uniformly distributed compressive stress applied either parallel or perpendicular to the direction of the face grain of the plywood and buckling in (n) half-waves has been defined by formula (1) in Forest Products Laboratory Report No. 1316 as follows:

$$P_{cr} = \frac{\pi^2}{12\lambda} \left[E_1 \frac{b^2}{n^2 a^2} + 2A + E_2 \frac{n^2 a^2}{b^2} \right] \frac{h^2}{a^2} \quad (32)$$

This formula was substantiated experimentally in Forest Products Laboratory Report No. 1316-D. In this substantiation the original lack of flatness of the panel was compensated for by slightly bending the edges of the plate. It was then possible to determine accurately the critical load from the load deflection curve obtained in the test. In the present investigation it was not practical to use this method because of the presence of the stiffener, and a different method was adopted and checked against equation (32).

The critical stress can be defined as the stress at which the compressive strain in the center of the plate is relieved. The relieving action, however, does not take place instantly due to lack of original perfect flatness of the plate and variations in its elastic properties. The stress at which the average compressive strain at the center of the plate stops increasing and starts decreasing with an increasing load is taken as the critical stress (fig. 9). In other words, the critical stress is the stress at which the slope of the load-strain curve becomes parallel to the load axis.

Computed critical stresses for each plate without a stiffener ($n = 1$) and for each plate having a stiffener of sufficient stiffness to hold the buckle in two half-waves ($n = 2$) are compared with observed values in figure 10. The computed and observed values of stress are each divided by the proportional limit stress of the material and are plotted as abscissas and ordinates respectively. The inclined straight line in the figure passes through points that indicate perfect agreement between the theory and test results. Although the individual points scatter considerably, the average of the experimental critical stresses was within 2 percent of the average of the computed critical stresses. The method used in the present investigation for the determination of the critical stress is therefore, on the average, an adequate one, but the observed critical stress of an individual panel may be in error.

Explanation of Tables

The pertinent data obtained from the tests are presented in tables 1 and 2. The first seven columns in table 1 contain a description and dimensions of the stiffened plates. In column 8 are listed the critical stresses of the plates with the stiffeners removed as obtained from equation (32) with $n = 1$. Column 9 contains the critical stresses obtained when the stiffener is very stiff, which were computed by use of the same formula with $n = 2$. Columns 10 and 11 contain the corresponding values obtained from test. These values were obtained by dividing the observed critical loads by the area of the cross section of the plate. Ratios of critical stresses to the proportional limit stress of the plywood are presented in the next four columns. The values in columns 12, 13, 14, and 15 are those in columns 8, 9, 10, and 11, respectively, each divided by the values in column 22. The next seven columns contain data from minor tests of the plywood and the stiffeners. The effective moduli of elasticity of the plywood in bending measured parallel and perpendicular to the side of length a are presented in columns 16 and 17. These values were obtained from the static bending tests using the formula

$$E_1 \text{ or } E_2 = \frac{P'L^3}{4bd^3y}$$

where P' is load in pounds, L is span in inches, b is breadth in inches, d is depth (thickness of plywood) in inches, and y is deflection at center of span in inches.

The effective moduli of elasticity of the plywood in compression measured parallel and perpendicular to the side of length a are presented in columns 18 and 19 and were obtained from the laterally supported column compression tests by the formula

$$E_a \text{ or } E_b = \frac{P'L}{Ay}$$

where P' is load in pounds, L is gage length in inches (2 inches), A is cross sectional area in square inches, and y is deformation in gage length in inches.

Column 20 contains the moduli of elasticity of the sitka spruce stiffeners as determined from the static bending test using the formula

$$E_s = \frac{P'L^3}{4bd^3y}$$

where P' is load in pounds, L is span in inches, b is breadth in inches, d is depth in inches, and y is deflection at center of span in inches.

Column 21 contains the shear moduli in the LT plane obtained by using the data from the plate shear test and the formula

$$\mu_{LT} = \frac{3}{2} \frac{Pu^2}{h^3w}$$

where P is load in pounds, u is gage length in inches, h is thickness in inches, and w is deflection in inches.

The proportional limit stress of the plywood material was observed during the laterally supported column compression tests and computed by the formula

$$P_{PL} = \frac{P'}{A}$$

where P' is load at proportional limit in pounds and A is cross sectional area in square inches.

Column 23 contains some pertinent remarks concerning the behavior of the stiffened plate during test.

Table 2 contains the observed critical stress and the two ratios plotted in figure 11 for each depth of stiffener. The observed critical stress was obtained by dividing the observed critical load by the cross sectional area of the stiffened plate. The ratios presented are ratios of the increase in critical stress due to a stiffener of less than the critical stiffness to the increase due to a very heavy stiffener. The denominator in each case is obtained by subtracting the observed critical stress of the plate without a stiffener from that of the plate with a very heavy stiffener. The numerator of the observed ratio is obtained by subtracting the observed critical stress of the plate without a stiffener from that of the plate with a stiffener of that particular depth. The numerator of the computed ratio is obtained by using equation (34).

Analysis of Experimental Results

The mathematical analysis suggests that the ratio obtained from experiment of the increase in the critical stress due to a stiffener of less than the critical stiffness to the increase due to a heavy stiffener be plotted against the similar ratio obtained by theory. The critical load per inch of width of a plate with a stiffener of less than the critical stiffness is given by equation (25), and the critical stress is

$$P_{cr} = \frac{\pi^2}{2\lambda} \left[E_1 \frac{b^2}{a^2} + 2A + E_2 \frac{a^2}{b^2} \right] \frac{h^2}{a^2} + 2\pi^2 \frac{b}{ha^4} (EI)_s \quad (33)$$

A comparison of this equation with equation (32) when $n = 1$ shows that the first part of the right hand member is equal to the critical stress P_{crp} of the plate with the stiffener removed, provided the plate has such dimensions that it will buckle into a single half-wave. Equation (33) can therefore be written

$$P_{cr} - P_{crp} = 2\pi^2 \frac{b}{ha^4} (EI)_s \quad (34)$$

The right hand member is the increase in the critical stress due to the presence of the stiffener.

The maximum critical stress that the stiffened plate can attain is obtained when the plate buckles into two half-waves. The critical load per inch of width of the plate is given by equation (26), and the

critical stress $P_{cr_{cm}}$ is given by equation (32) when $n = 2$. Using equation (32), when $n = 2$ and when $n = 1$ the following equation can be obtained:

$$P_{cr_{cm}} - P_{crp} = \frac{\pi^2 h^2 b^2}{4 \lambda a^4} \left[E_2 \frac{a^4}{b^4} - \frac{1}{4} E_1 \right] \quad (35)$$

This equation can be combined with (34) and (27) with the following result:

$$\frac{P_{cr} - P_{crp}}{P_{cr_{cm}} - P_{crp}} = \frac{(EI)_s}{(EI)_{scr}} \quad (36)$$

If values of the left hand member of this equation are plotted as ordinates and values of the right hand member as abscissas, a straight line connecting the origin with the point 1,1 is obtained. At this point

$$P_{cr} = P_{cr_{cm}}$$

and, therefore, greater values of the ordinate cannot be obtained. The stiffness of the stiffener, however, is not limited, and therefore the values of the abscissa are not limited. This fact is represented by a line starting at the point 1,1 and drawn parallel to the X-axis. These two lines are drawn in figure 11.

The experimental data are also plotted in figure 11 as points. The ordinates of these points were determined by substituting experimental values in the left hand member of equation (36). The critical stress of each panel with the heaviest stiffener with which it was tested was substituted for $P_{cr_{cm}}$. These values were taken from column 11 of table 1. The values for P_{crp} are those obtained from tests of plates from which the stiffeners were removed and were taken from column 10 of the same table. The experimental values of P_{cr} were taken from table 2.

The abscissas of the experimental points were calculated by use of the expression

$$\frac{2\pi^2 \frac{b}{ha^4} (EI)_s}{P_{cr_{cm}} - P_{crp}} \quad (37)$$

which is mathematically equivalent to the right hand member of equation (36). In this expression, experimental values were substituted in the denominator and theoretical values in the numerator. The stiffness of the stiffener $(EI)_s$ is computed about a neutral axis, the position of

which is found by the following formula obtained from Forest Products Laboratory Report No. 1557, equation (68):

$$z_n = \frac{1}{2} \frac{d + h}{\frac{2ahE_a}{\pi\alpha\left(\frac{E_a}{E_b}\right)^{\frac{1}{4}} t d E_s} + 1 + \frac{E_a h}{E_s d}}$$

where

$$\alpha = \sqrt{k + \sqrt{k^2 - 1}}$$

$$k = \frac{\sqrt{E_a E_b}}{2\mu_{LT}} - \frac{\sigma_{TL} E_L}{\sqrt{E_a E_b}}$$

The plotted points in figure 11 scatter considerably but nevertheless group themselves around the two straight lines OAC. They more approximately follow the curved broken line from O to B and then along BC. The scatter can be attributed to several causes: There are small errors due to the approximate method used in the theory; there are also some approximations made in the derivation of the formula for $(EI)_s$; and, probably most important, the plates tested were not perfectly flat. As is pointed out in the mathematical analysis, even a very small departure from perfect flatness complicates the situation considerably. Also, this irregularity affects the accuracy of the observations of the critical load, as is illustrated by the scatter of the points in figure 10. Although the plywood was carefully selected for initial flatness, the cutting of the specimen and the gluing of the stiffeners tend to warp the plate.

All the plates reach unit ordinate between abscissas of 0.7 and 1.7. Therefore, for design purposes, stiffeners having a computed ratio of

$\frac{(EI)_s}{(EI)_{scr}}$ (abscissa of fig. 11) of 1.8 or more are sufficiently stiff to produce the maximum critical load of the plate. If the computed ratio is equated to 1.8 and the equation solved for $(EI)_s$, a formula for the critical size stiffener is obtained, thus:

$$(EI)_{scr} = \frac{h^3 b}{4.6\lambda} \left[E_2 \frac{a^4}{b^4} - \frac{1}{4} E_1 \right] \quad (37)$$

The critical stresses of plates for which values of the ratio $\frac{(EI)_s}{(EI)_{scr}}$ lay between the limits of 1.2 to 1.7 were sometimes higher than that indicated by the theory; that is, higher than p_{perm} . Also, in this range the stiffeners seemed likely to fail in bending when the plate buckled. It is suggested that this range be avoided in the design of stiffened plates.

Notation

The following symbols are the major ones used in this report. All quantities are in inch and pound units.

a = the length of the loaded sides of a plywood plate for compressive loads.

b = the length of the unloaded sides of a plywood plate for compressive loads.

E_a = effective modulus of elasticity of plywood in compression measured parallel to the side of length a of plywood plates.

E_b = effective modulus of elasticity of plywood in compression measured perpendicular to the side of length a of plywood plates.

E_1 = effective modulus of elasticity of plywood in bending measured parallel to the side of length a of plywood plates.

E_2 = effective modulus of elasticity of plywood in bending measured perpendicular to the side of length a of plywood plates.

E_L = modulus of elasticity of wood in the direction parallel to the grain (assumed to be $\frac{20}{21}(E_1 + E_2)$).

E_s = modulus of elasticity of sitka spruce stiffener in the direction parallel to the grain, as determined from a static bending test.

d = depth of stiffener (perpendicular to the plane of the plywood plate).

t = width of stiffener (parallel to the plane of the plywood plate).

σ_{cr} = critical compressive stress for the buckling of a stiffened plywood plate. Computed values are obtained by use of equation (25).

σ_{crp} = critical compressive stress for the buckling of a plywood plate having no stiffener.

σ_{crm} = critical compressive stress for the buckling of a stiffened plate having a stiffener large enough to cause a two half-wave buckle.

σ_{pL} = proportional limit stress of the plywood material as obtained by compression minor tests.

$(EI)_s$ = a measure of the stiffness of the stiffener. Product of modulus of elasticity and moment of inertia.

$(EI)_{scr}$ = the minimum stiffness of the stiffener required to cause a plywood plate to buckle into two half-waves.

σ_{TL} = Poisson's ratio of extension in the longitudinal direction to compression in the tangential direction due to a compressive stress in the tangential direction (assumed to be 0.02 for yellow birch).

μ_{LT} = shear modulus in the LT plane as determined by the plate shear test.

z_n = distance from the center of the plywood to the neutral axis which has been shifted by attaching a stiffener to the plywood.

Table 1.—Computed and observed critical stresses for stiffened and unstiffened plywood panels in compression, with data from minor tests

Description	Dimensions				Critical stresses				Ratios				Information from minor tests						Remarks		
	Thickness of plywood (in.)	Length of unplywood loaded (in.)	Width of unplywood loaded (in.)	Edge angle (deg.)	Computed	Observed	Computed	Observed	Computed	Observed	Computed	Observed	$\frac{F_c}{F_{cr}}$	$\frac{F_c}{F_{cr}}$	$\frac{F_c}{F_{cr}}$	$\frac{F_c}{F_{cr}}$	$\frac{F_c}{F_{cr}}$	$\frac{F_c}{F_{cr}}$		$\frac{F_c}{F_{cr}}$	$\frac{F_c}{F_{cr}}$
(1) (2) (3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	
	Inch	Inches	Inches	inches	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	
1 1:1:1	0.180	10.00	15.00	0.375	548	995	611	11,031	10,107	0.195	0.120	0.203	160	12,280	599	11,609	3,530	117	5,100	0.312 inch stiffener failed at 1,990 lb. per sq. in.	
2	0.179	10.02	15.02	0.390	513	969	584	973	112	0.204	0.110	0.205	153	12,250	490	11,570	1,615	149	4,750	0.310 inch stiffener failed at 1,930 lb. per sq. in.	
3	0.178	10.00	16.01	0.293	570	11,027	674	936	1,108	0.195	0.128	0.189	185	12,450	715	11,786	1,940	141	5,270	0.307 inch stiffener failed at 1,750 lb. per sq. in.	
4	0.179	10.01	16.03	0.185	580	11,056	634	963	1,150	0.273	0.158	0.254	170	12,470	766	11,376	1,700	158	3,872		
5 1:1:1	0.182	10.00	14.00	0.375	738	11,299	833	11,365	1,209	0.168	0.236	0.168	365	11,608	1,233	946	1,645	158	3,525		
6	0.183	10.02	14.02	0.272	792	11,465	879	11,461	1,236	0.137	0.262	0.135	363	11,840	1,061	1,068	1,540	164	3,395		
7	0.182	10.00	14.00	0.168	792	11,427	846	11,311	1,227	0.108	0.242	0.108	380	11,782	1,174	1,018	1,607	159	3,495		
8	0.183	10.01	14.02	0.185	787	11,376	738	11,273	1,301	0.266	0.222	0.287	365	11,660	1,136	1,016	1,660	184	2,615		
9 1:1:1	0.180	8.00	25.01	0.128	963	11,646	995	11,451	1,235	0.102	0.233	0.102	166	12,535	812	11,517	1,433	172	4,095		
10	0.182	8.00	25.02	0.130	905	11,505	1,040	11,334	1,188	0.133	0.213	0.133	163	12,266	702	11,508	1,495	132	4,825		
11	0.175	8.01	14.02	0.126	944	11,635	827	11,598	1,202	0.190	0.217	0.190	214	12,333	696	11,745	1,683	150	4,665	0.312 inch stiffener failed at 1,990 lb. per sq. in.	
12	0.176	8.01	13.01	0.126	907	11,781	869	11,648	1,200	0.193	0.132	0.193	180	12,300	750	11,567	1,818	154	4,528	0.310 inch stiffener failed at 1,930 lb. per sq. in.	
13	0.179	8.00	12.92	0.125	989	11,984	869	11,682	1,247	0.195	0.217	0.195	170	12,430	854	11,564	1,710	144	4,008	0.307 inch stiffener failed at 1,750 lb. per sq. in.	
14	0.179	8.01	12.01	0.127	936	12,092	856	11,859	1,212	0.174	0.194	0.174	171	12,380	602	11,515	1,320	150	4,410		
15	0.180	8.00	13.00	0.148	922	11,886	868	11,628	1,201	0.110	0.189	0.110	170	12,379	685	11,638	1,588	148	4,595		
16	0.177	8.02	12.01	0.148	951	12,091	951	11,890	1,220	0.145	0.220	0.145	184	12,388	480	11,612	1,760	156	4,312		
17 1:1:1	0.186	8.01	12.02	0.123	11,499	12,420	11,394	12,146	1,401	0.148	0.173	0.148	465	12,225	11,261	1,233	1,708	209	3,735	0.468 inch stiffener failed at 2,260 lb. per sq. in.	
18	0.186	8.01	12.03	0.153	11,462	12,547	11,411	12,413	1,388	0.166	0.166	0.166	429	12,416	11,244	1,408	1,495	202	3,822		
19	0.188	8.01	6.00	0.130	11,491	12,153	11,062	11,688	1,369	0.133	0.263	0.133	418	12,118	473	11,320	1,226	200	4,038	0.362 inch stiffener failed at 1,830 lb. per sq. in.	
20	0.193	8.01	6.01	0.128	11,581	12,206	11,106	11,946	1,402	0.162	0.282	0.162	495	12,190	453	11,340	1,296	200	3,928	1.000 inch stiffener failed at 2,040 lb. per sq. in.	
21	0.189	8.00	12.02	0.183	11,424	12,395	11,411	12,413	1,387	0.137	0.137	0.137	406	12,120	11,087	1,363	1,570	210	3,681	Plates apparently damaged on fire	
22	0.186	8.01	12.01	0.187	11,368	12,194	11,345	12,130	1,415	0.163	0.163	0.163	414	11,973	11,295	1,279	1,703	204	3,295	0.192 inch stiffener failed at 1,080 lb. per sq. in.	
23 1:1:1	0.180	10.00	6.00	0.130	589	786	422	701	1,323	0.131	0.231	0.131	171	11,500	511	1,652	1,652	160	1,768		
24	0.180	10.00	6.01	0.119	566	722	539	738	1,316	0.104	0.301	0.104	151	11,400	593	1,332	1,332	160	1,768		
25 1:1:1	0.185	10.01	7.01	0.299	911	11,342	829	11,382	1,272	0.140	0.247	0.140	413	12,393	417	11,247	1,115	1,580	205	3,350	
26	0.185	10.02	7.00	0.299	865	11,400	917	11,510	1,288	0.146	0.305	0.146	508	12,062	451	11,400	1,257	1,670	200	3,005	
27	0.189	10.02	8.00	0.166	11,037	11,865	904	11,115	1,287	0.150	0.256	0.150	399	12,470	495	11,458	1,222	1,568	198	3,612	
28	0.188	10.01	8.01	0.187	994	11,091	886	11,156	1,364	0.158	0.323	0.158	421	12,466	365	11,338	1,214	1,538	203	2,744	
29 1:1:1:1:1	0.231	10.00	14.03	0.257	11,591	12,642	11,698	12,454	1,312	0.136	0.326	0.136	481	12,348	1,042	1,801	1,680	186	5,100	0.244 inch stiffener failed at 2,340 lb. per sq. in.	
30	0.234	10.02	15.02	0.258	11,612	12,516	11,749	12,580	1,296	0.146	0.310	0.146	662	12,370	1,048	1,756	1,951	184	5,640		

Table 1.—Computed and observed critical stresses for stiffened and unstiffened plywood panels in compression, with data from minor tests (continued)

Plating No.	Description	Dimensions		Critical stresses		Stresses		Information from minor tests										Remarks					
		Width of plywood (h)	Length of loaded edge (b)	Computed P _{cr}	Observed P _{cr}	Computed P _{cr}	Observed P _{cr}	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈	P ₉	P ₁₀		P ₁₁	P ₁₂			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	
			Inch	Inches	Inch	Inches	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.
31	Parallel	0.235	10.00	34.02	10.25	11,990	12,560	11,726	12,702	10,226	10,368	10,245	10,364	647	12,159	11,032	11,690	11,454	11,100	11,000	11,000	11,000	0.362 inch stiffener failed at 2,980 lb. per sq. in.
32	Parallel	0.231	10.02	13.98	3.50	11,618	12,716	11,685	12,684	11,350	11,594	11,444	11,594	681	12,362	11,054	11,562	11,775	11,775	11,775	11,775	11,775	0.503 inch stiffener failed at 2,370 lb. per sq. in.
33	Parallel	0.232	10.02	34.02	12.61	11,521	12,426	11,577	12,667	11,356	11,536	11,348	11,536	646	12,051	11,096	11,632	11,604	11,604	11,604	11,604	11,604	0.358 inch stiffener failed at 2,680 lb. per sq. in.
34	Parallel	0.231	10.02	34.02	2.52	11,547	12,656	11,685	12,715	11,348	11,536	11,348	11,536	635	12,349	11,038	11,782	11,692	11,692	11,692	11,692	11,692	0.369 inch stiffener failed at 2,370 lb. per sq. in.
35	Parallel	0.231	10.03	16.01	2.52	11,631	12,120	11,795	12,331	11,358	11,426	11,352	11,426	665	12,140	11,024	11,660	11,570	11,570	11,570	11,570	11,570	0.362 inch stiffener failed at 2,980 lb. per sq. in.
36	Parallel	0.192	10.00	18.01	1.85	11,855	11,957	11,957	12,097	11,299	11,333	11,336	11,336	417	11,625	11,100	11,090	11,700	11,700	11,700	11,700	11,700	0.503 inch stiffener failed at 2,370 lb. per sq. in.
37	Parallel	0.193	10.00	16.01	1.85	11,791	12,121	12,121	12,266	11,292	11,327	11,327	11,327	393	11,666	11,146	11,222	11,678	11,678	11,678	11,678	11,678	0.503 inch stiffener failed at 2,370 lb. per sq. in.
38	Parallel	0.186	10.00	13.02	1.20	11,947	12,171	12,171	12,171	11,980	11,983	11,983	11,983	459	12,393	11,208	11,450	11,578	11,578	11,578	11,578	11,578	0.503 inch stiffener failed at 2,370 lb. per sq. in.
39	Perpendicular	0.185	10.01	6.00	1.87	11,796	11,339	11,647	11,647	11,647	11,647	11,647	11,647	216	12,268	11,134	11,168	11,836	11,836	11,836	11,836	11,836	0.503 inch stiffener failed at 2,370 lb. per sq. in.
40	Perpendicular	0.176	10.00	5.00	1.23	11,733	11,647	11,647	11,647	11,647	11,647	11,647	11,647	238	12,268	11,107	11,707	11,820	11,820	11,820	11,820	11,820	0.503 inch stiffener failed at 2,370 lb. per sq. in.
41	Perpendicular	0.185	10.02	6.00	1.87	11,798	11,434	11,434	11,507	11,212	11,321	11,321	11,321	401	12,122	11,168	11,168	11,446	11,446	11,446	11,446	11,446	0.503 inch stiffener failed at 2,370 lb. per sq. in.
42	Perpendicular	0.234	10.00	17.02	3.79	11,753	12,082	11,660	12,083	11,367	11,435	11,435	11,435	347	12,244	11,109	11,870	11,766	11,766	11,766	11,766	11,766	0.503 inch stiffener failed at 2,370 lb. per sq. in.
43	Perpendicular	0.232	10.00	15.50	3.80	11,726	12,210	11,580	12,076	11,363	11,465	11,465	11,465	333	12,268	11,130	11,696	11,754	11,754	11,754	11,754	11,754	0.503 inch stiffener failed at 2,370 lb. per sq. in.
44	Perpendicular	0.224	10.00	8.00	2.52	11,415	12,316	11,339	11,903	11,609	11,609	11,609	11,609	576	12,176	11,193	11,749	11,848	11,848	11,848	11,848	11,848	0.503 inch stiffener failed at 2,370 lb. per sq. in.
45	Perpendicular	0.175	10.00	6.50	1.90	11,693	11,628	11,543	11,543	11,543	11,543	11,543	11,543	220	12,651	11,213	11,872	11,715	11,715	11,715	11,715	11,715	0.503 inch stiffener failed at 2,370 lb. per sq. in.
46	Perpendicular	0.233	12.02	9.02	3.83	11,180	12,157	11,649	11,649	11,337	11,616	11,616	11,616	482	12,176	11,193	11,749	11,848	11,848	11,848	11,848	11,848	0.503 inch stiffener failed at 2,370 lb. per sq. in.
47	Perpendicular	0.235	12.00	9.82	3.76	11,261	12,071	11,645	11,645	11,417	11,685	11,685	11,685	313	12,297	11,271	11,799	11,799	11,799	11,799	11,799	11,799	0.503 inch stiffener failed at 2,370 lb. per sq. in.
48	Perpendicular	0.233	12.01	11.00	3.80	11,235	11,543	11,543	11,543	11,388	11,483	11,483	11,483	302	12,292	11,315	11,823	11,704	11,704	11,704	11,704	11,704	0.503 inch stiffener failed at 2,370 lb. per sq. in.
49	Perpendicular	0.231	12.01	17.04	4.01	11,195	11,667	11,031	11,662	11,237	11,371	11,371	11,371	295	12,330	11,092	11,794	11,794	11,794	11,794	11,794	11,794	0.503 inch stiffener failed at 2,370 lb. per sq. in.
50	Perpendicular	0.233	12.00	19.02	3.80	11,283	11,630	11,283	11,538	11,295	11,324	11,324	11,324	251	12,342	11,128	11,820	11,768	11,768	11,768	11,768	11,768	0.503 inch stiffener failed at 2,370 lb. per sq. in.
51	Perpendicular	0.232	12.00	18.03	3.81	11,184	11,656	11,103	11,657	11,246	11,344	11,344	11,344	229	12,213	11,248	11,723	11,723	11,723	11,723	11,723	11,723	0.503 inch stiffener failed at 2,370 lb. per sq. in.
52	Perpendicular	0.233	12.01	16.00	3.02	11,145	11,966	11,036	11,790	11,226	11,349	11,349	11,349	205	12,200	11,226	11,648	11,648	11,648	11,648	11,648	11,648	0.503 inch stiffener failed at 2,370 lb. per sq. in.
53	Perpendicular	0.235	12.00	15.00	3.02	11,157	12,226	11,188	12,012	11,243	11,468	11,468	11,468	143	12,293	11,062	11,665	11,712	11,712	11,712	11,712	11,712	0.503 inch stiffener failed at 2,370 lb. per sq. in.
54	Perpendicular	0.189	12.00	20.00	3.80	11,638	11,974	11,019	11,919	11,181	11,276	11,276	11,276	215	12,429	11,134	11,327	11,327	11,327	11,327	11,327	11,327	0.503 inch stiffener failed at 2,370 lb. per sq. in.
55	Perpendicular	0.189	11.98	15.00	3.77	11,621	11,074	11,074	11,164	11,284	11,284	11,284	11,284	181	12,294	11,162	11,328	11,328	11,328	11,328	11,328	11,328	0.503 inch stiffener failed at 2,370 lb. per sq. in.
56	Perpendicular	0.190	12.02	16.00	3.79	11,618	11,801	11,801	11,801	11,177	11,345	11,345	11,345	206	12,032	11,105	11,976	11,330	11,330	11,330	11,330	11,330	0.503 inch stiffener failed at 2,370 lb. per sq. in.
57	Perpendicular	0.182	12.00	18.00	3.75	11,356	11,744	11,660	11,744	11,284	11,444	11,444	11,444	151	12,155	11,125	11,263	11,605	11,605	11,605	11,605	11,605	0.503 inch stiffener failed at 2,370 lb. per sq. in.
58	Perpendicular	0.181	11.98	16.00	3.75	11,390	11,947	11,947	11,947	11,112	11,271	11,271	11,271	132	12,196	11,148	11,244	11,568	11,568	11,568	11,568	11,568	0.503 inch stiffener failed at 2,370 lb. per sq. in.
59	Perpendicular	0.175	11.98	14.00	3.77	11,404	11,125	11,125	11,125	11,112	11,312	11,312	11,312	148	12,321	11,112	11,244	11,568	11,568	11,568	11,568	11,568	0.503 inch stiffener failed at 2,370 lb. per sq. in.
60	Perpendicular	0.178	12.00	6.00	3.77	11,331	11,596	11,596	11,596	11,150	11,270	11,270	11,270	142	12,859	11,148	11,387	11,609	11,609	11,609	11,609	11,609	0.503 inch stiffener failed at 2,370 lb. per sq. in.

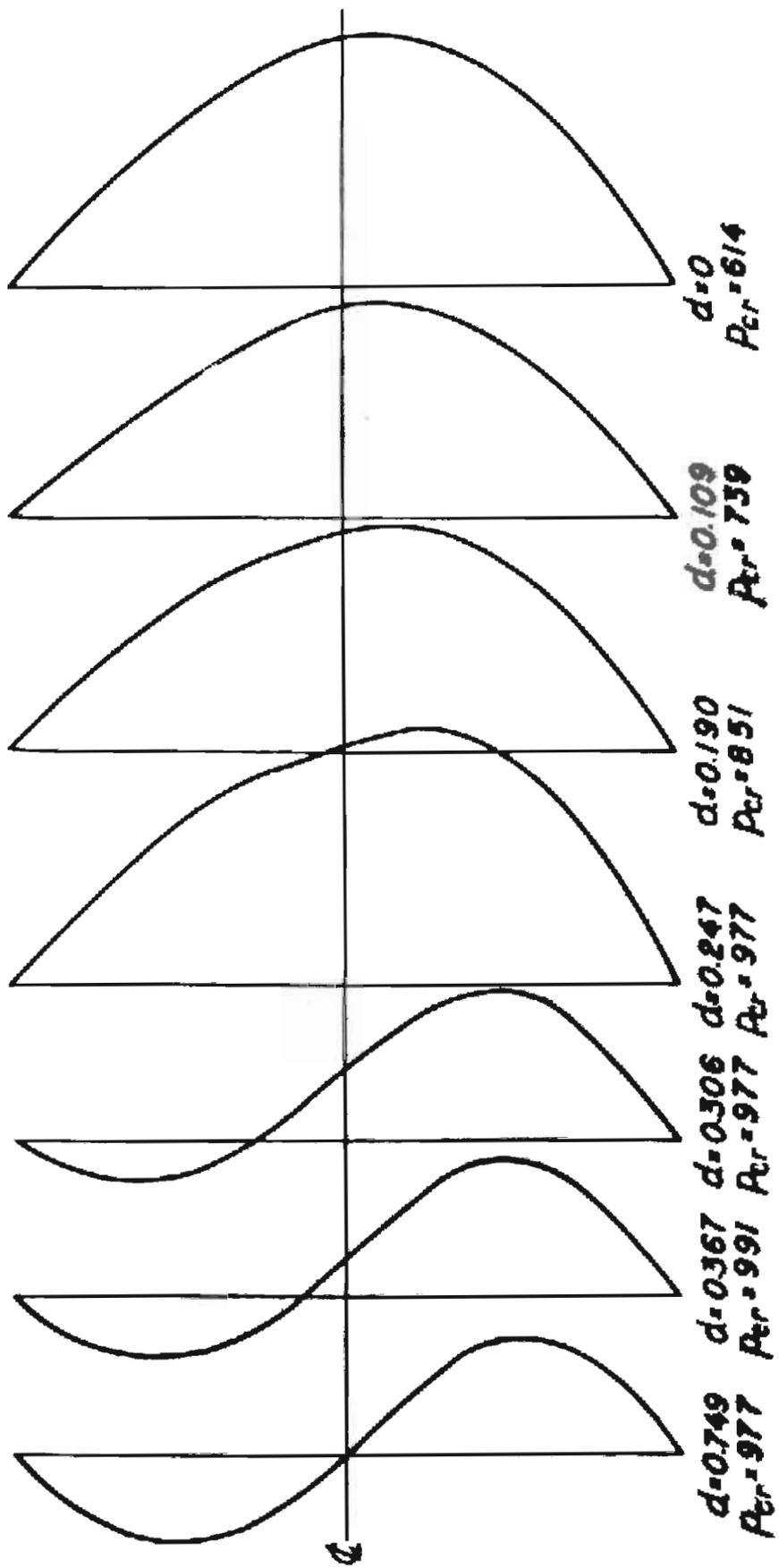


Figure 1.--Typical buckle patterns as viewed along the centerline perpendicular to the stiffener of a stiffened plate obtained by making a trace on the vertical centerline of the plate while it is buckled in the testing machine. (Plate No. 4)

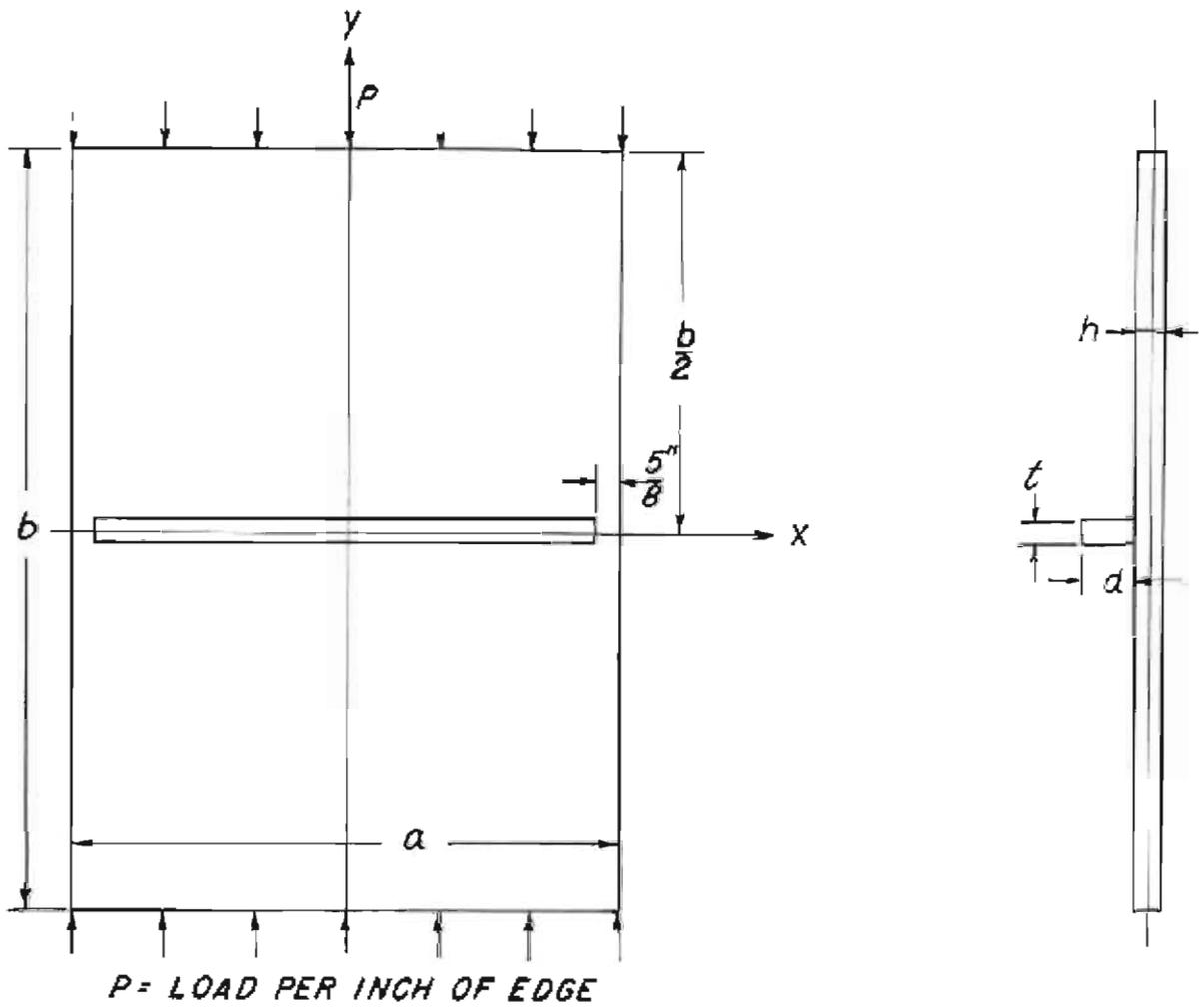


Figure 2.--A stiffened plate, showing dimensions, position of stiffener, and direction of stress.

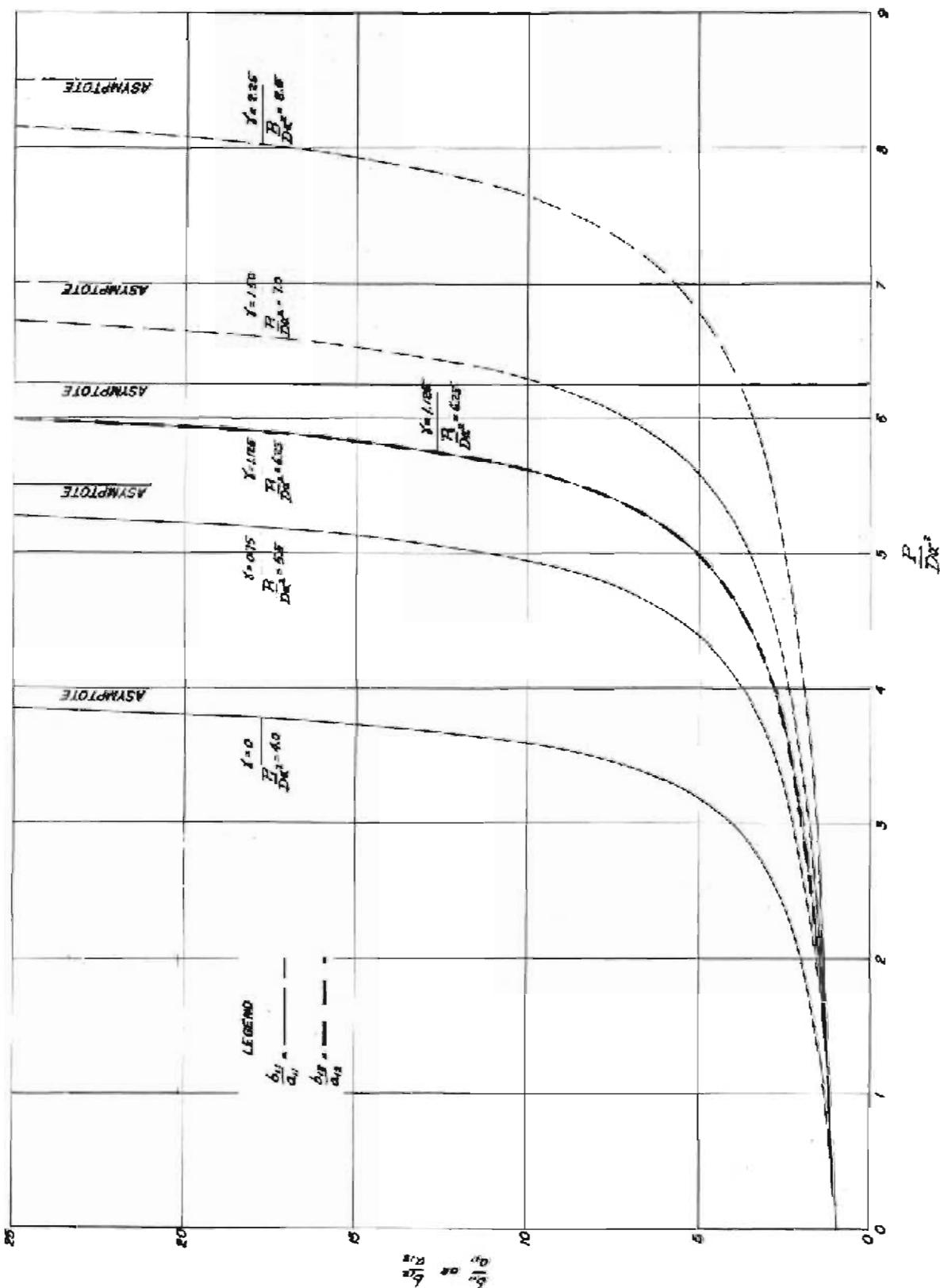


Figure 3.--The amplification of the first and second modes for an isotropic plate with one horizontal stiffener for stiffeners with different stiffness ratios $\gamma = \frac{(EI)_s}{D \text{ plate } x^3}$

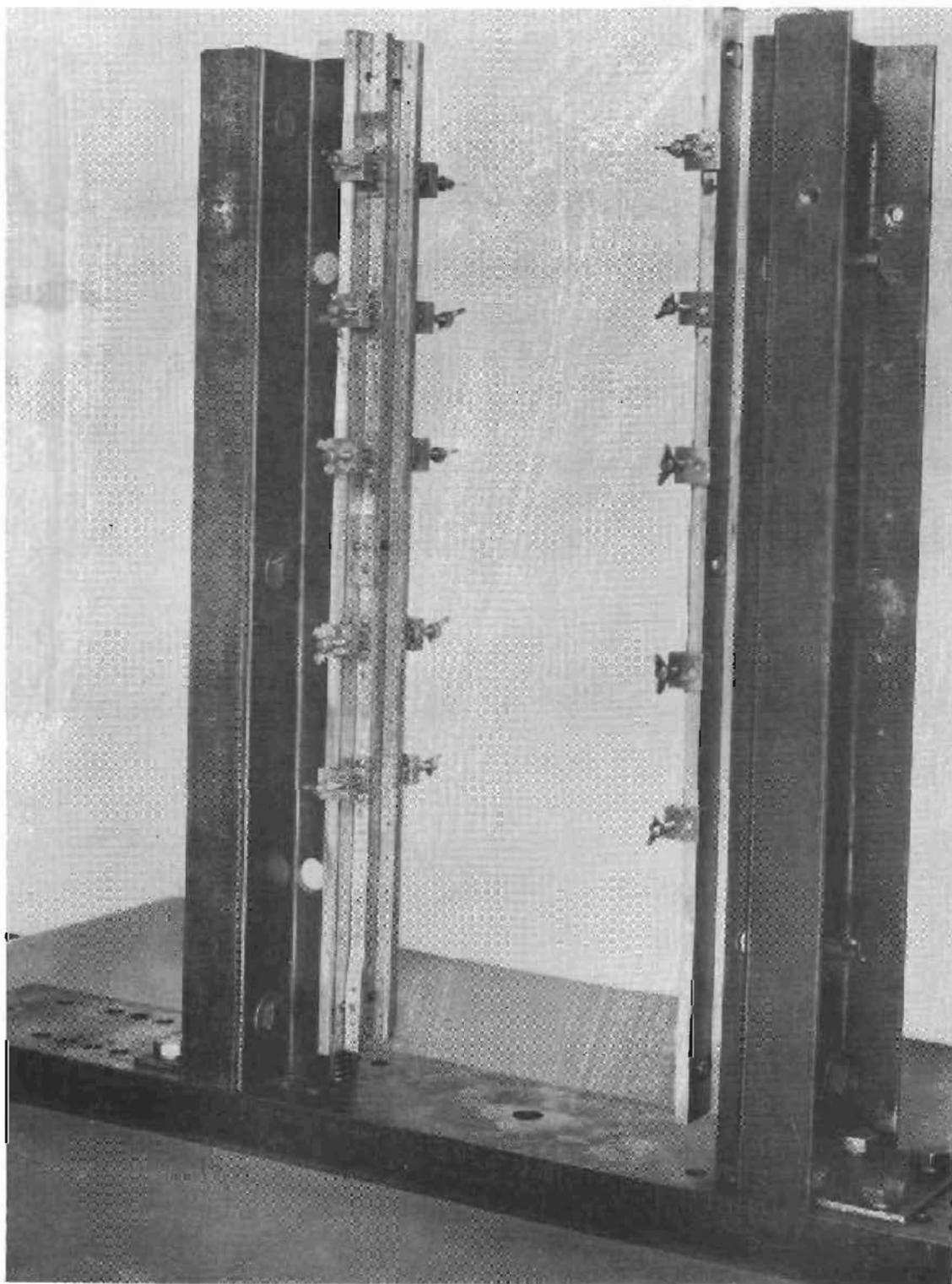


Figure 4.--Side view of panel buckling apparatus showing movable post in place.

Z M 68856 F

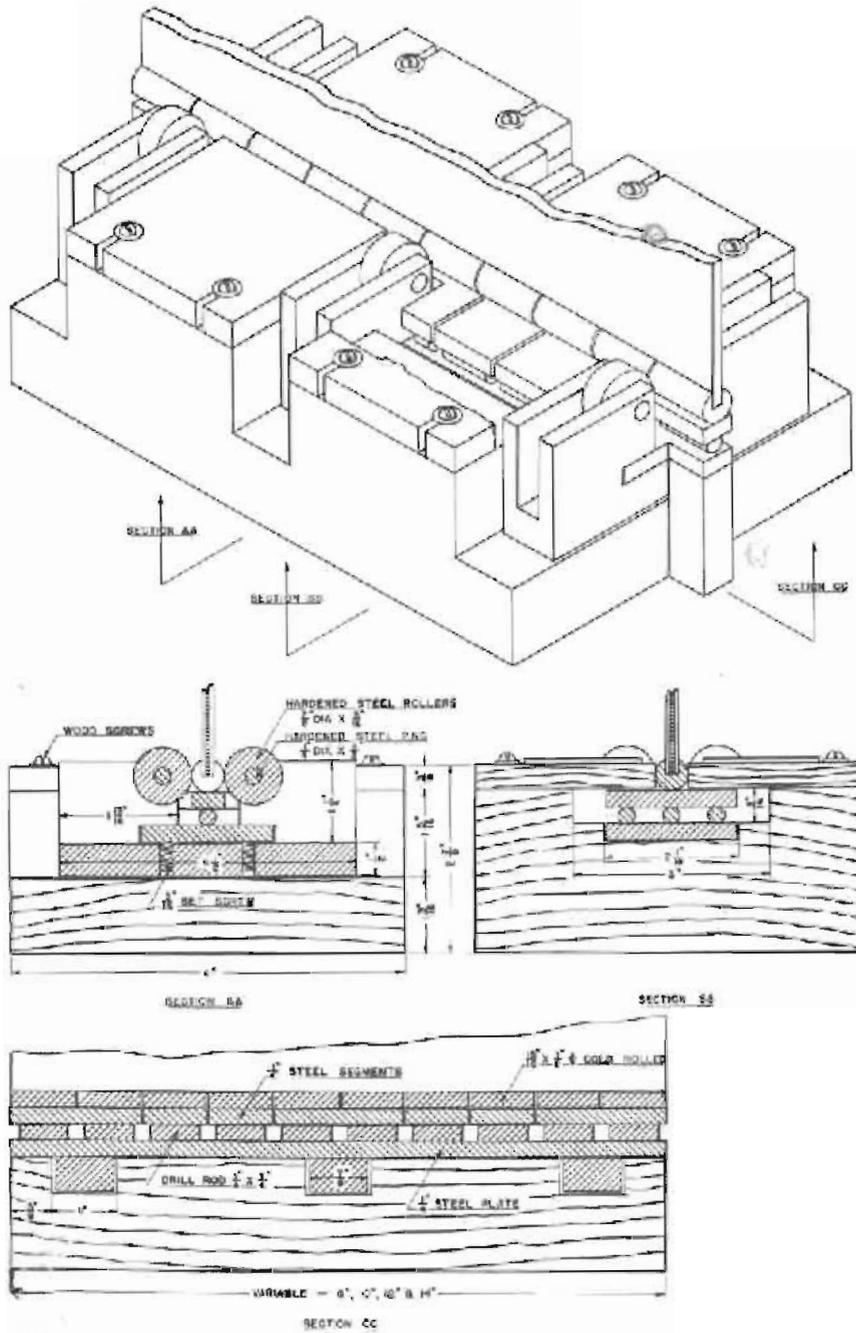


Figure 5.--Design details of loading head used in tests of stiffened flat plywood plates.

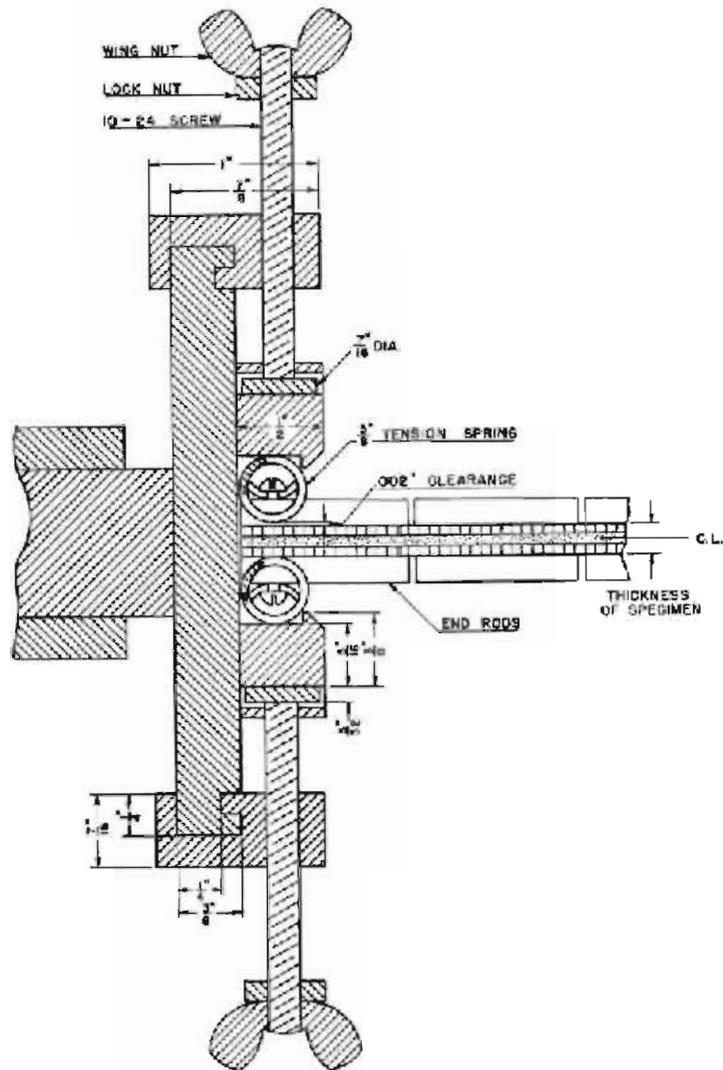


Figure 6.--Detail of edge support for panel buckling test of stiffened flat plywood plates.

Z N 57773 Y

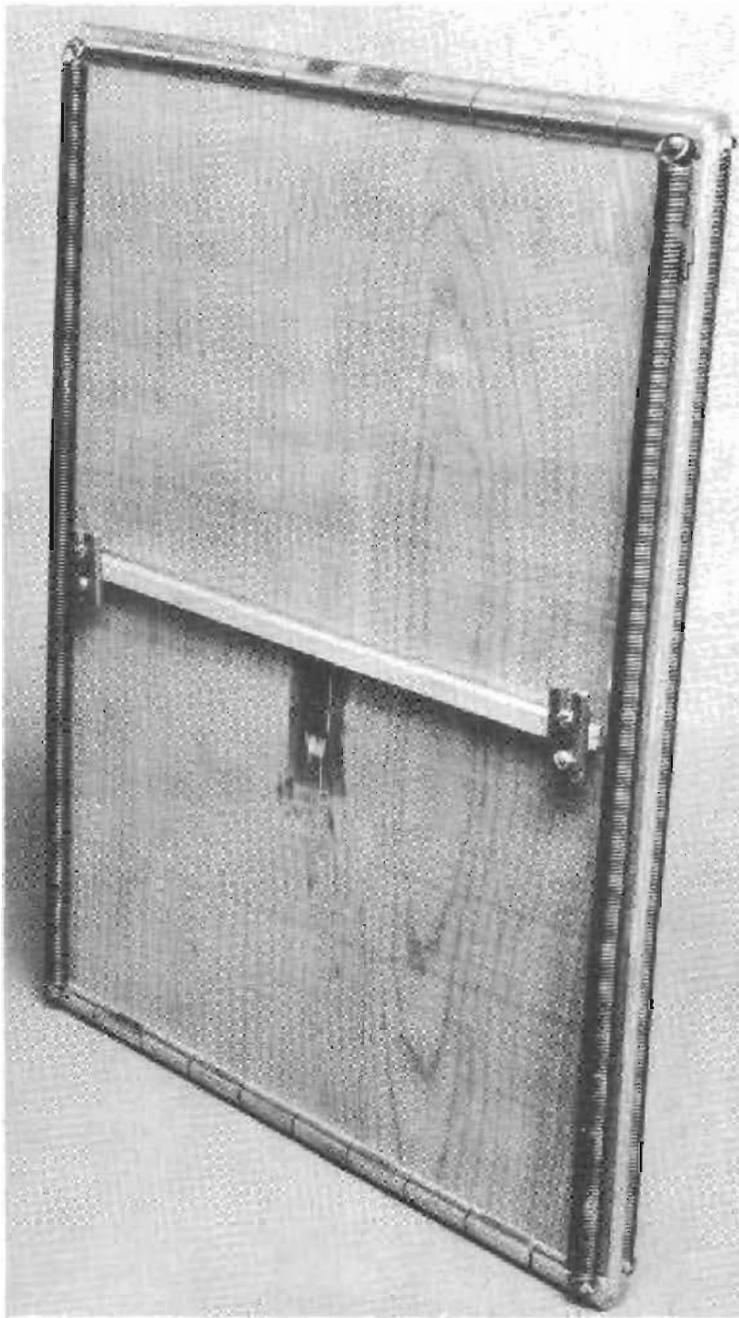


Figure 7.--Stiffened flat plywood plate mounted in segmented cylindrical steel loading rods prior to test. The method of restraining the ends of the stiffener to prevent separation at the ends, the metaelectric strain gage for measuring deformations, and the attachment of coil springs at the edges to reduce frictional restraint during test are shown.

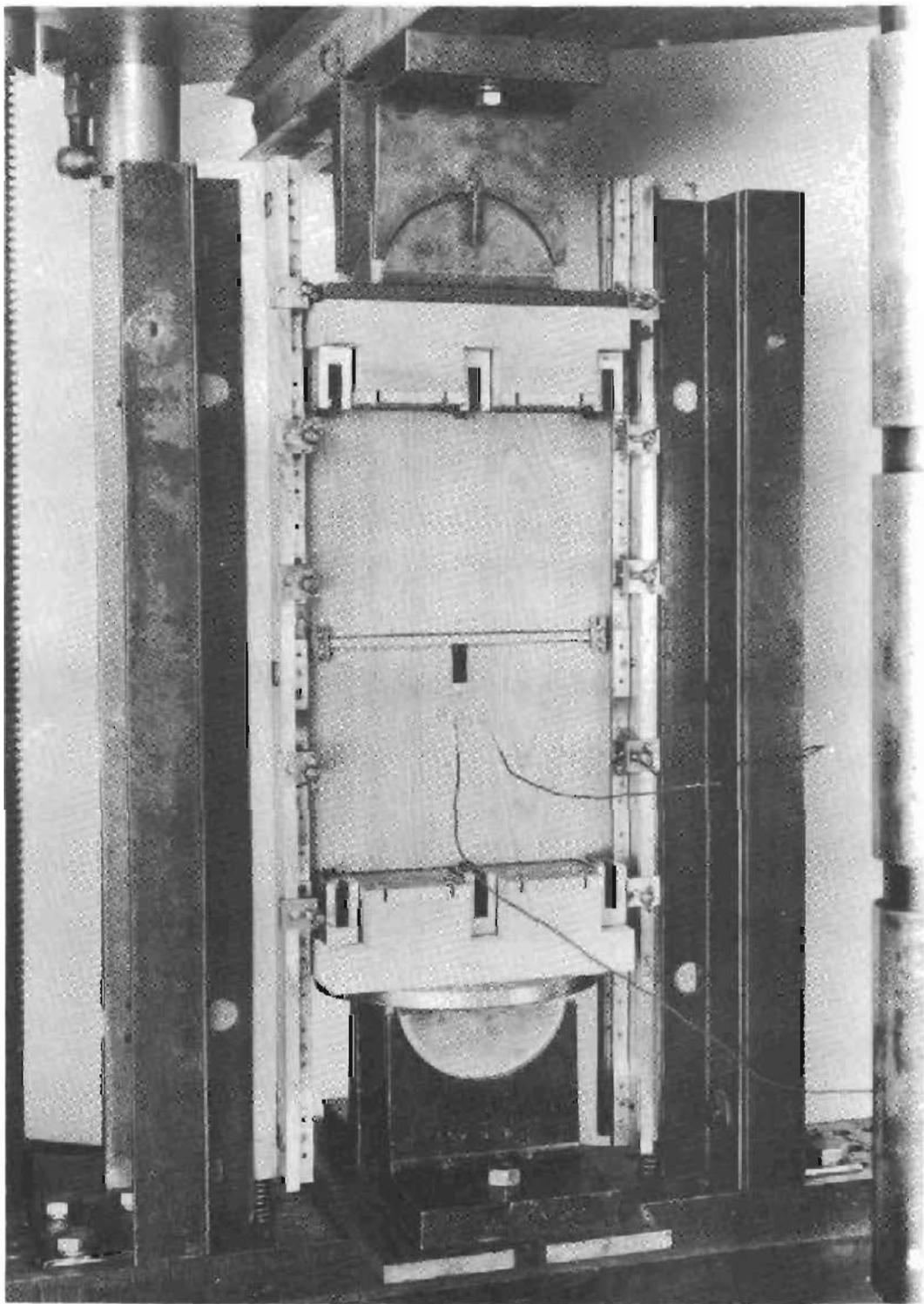


Figure 8.--Stiffened flat plywood plate ready for test.
Metallectric strain gage mounted below horizontal
stiffener is used in measuring deformations in the
specimen.

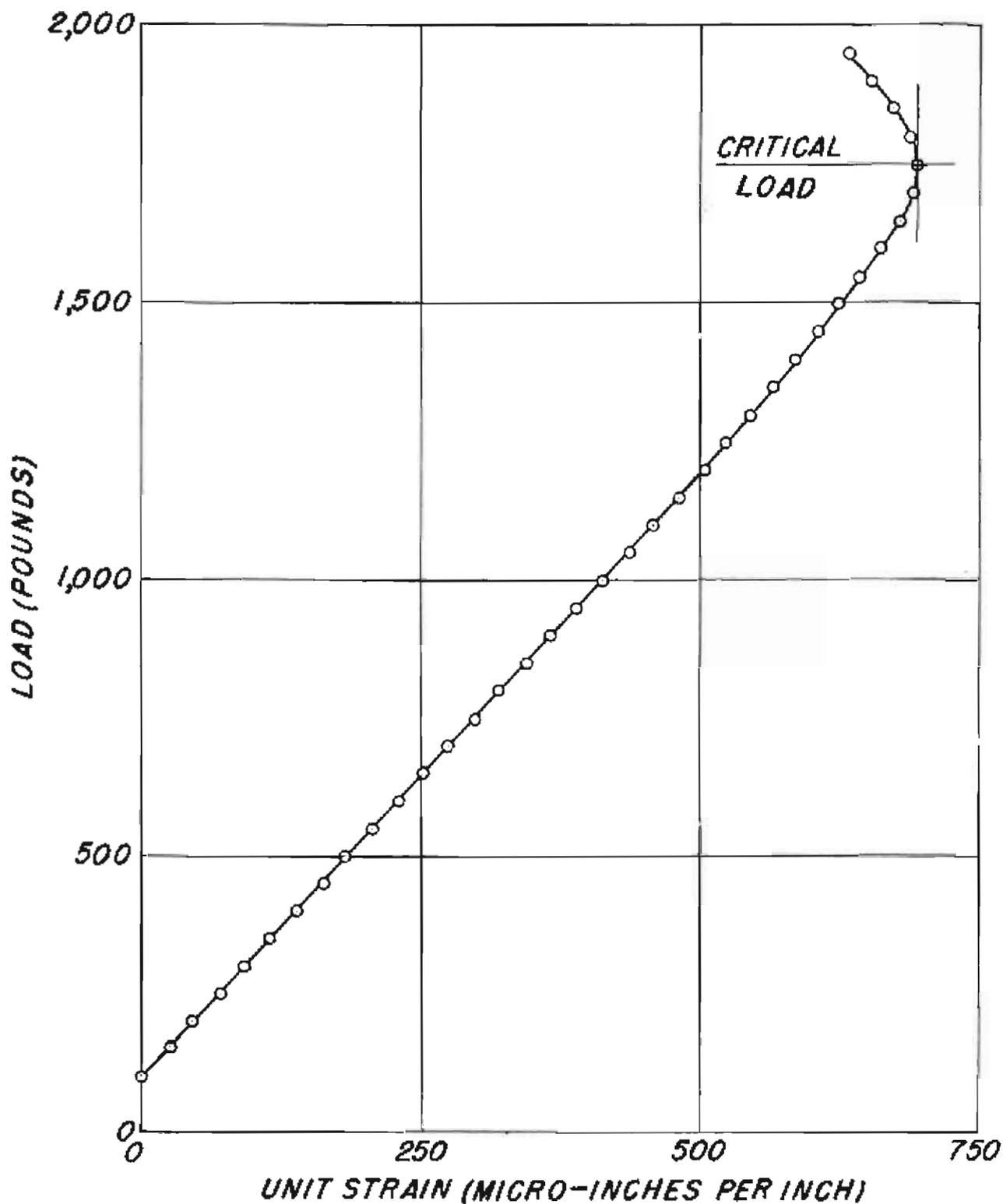


Figure 9.--Typical load-strain curve, plotted to determine critical load of a stiffened plywood plate in compression.

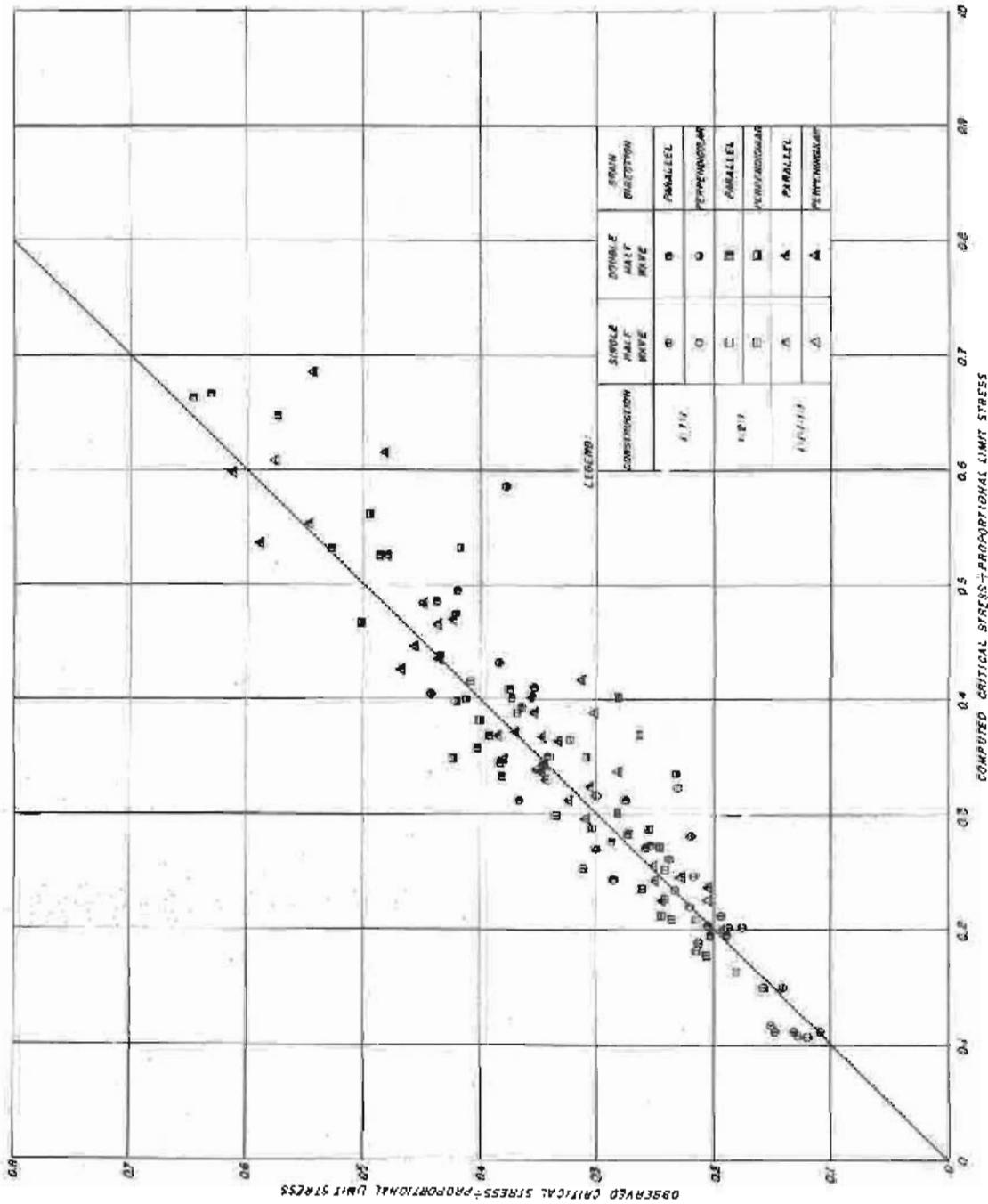


Figure 10.--Comparison of observed critical load with computed critical load -- both expressed as ratios to computed proportional limit load. Direction of face grain parallel or perpendicular to direction of compression.

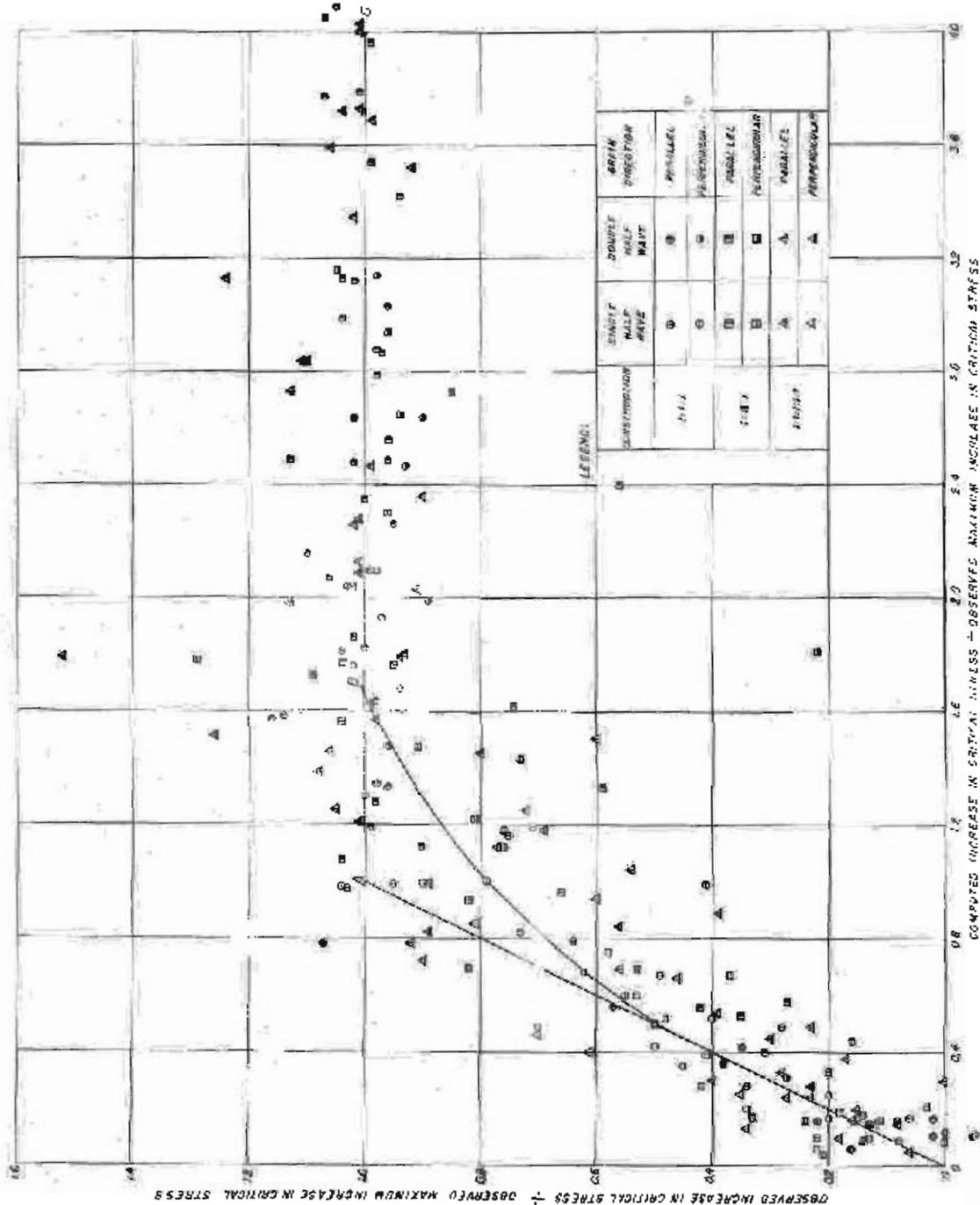


Figure 11.--Comparison of observed increase in critical stress with theoretical increase in critical stress -- both expressed as ratios to observed maximum increase in critical stress.

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