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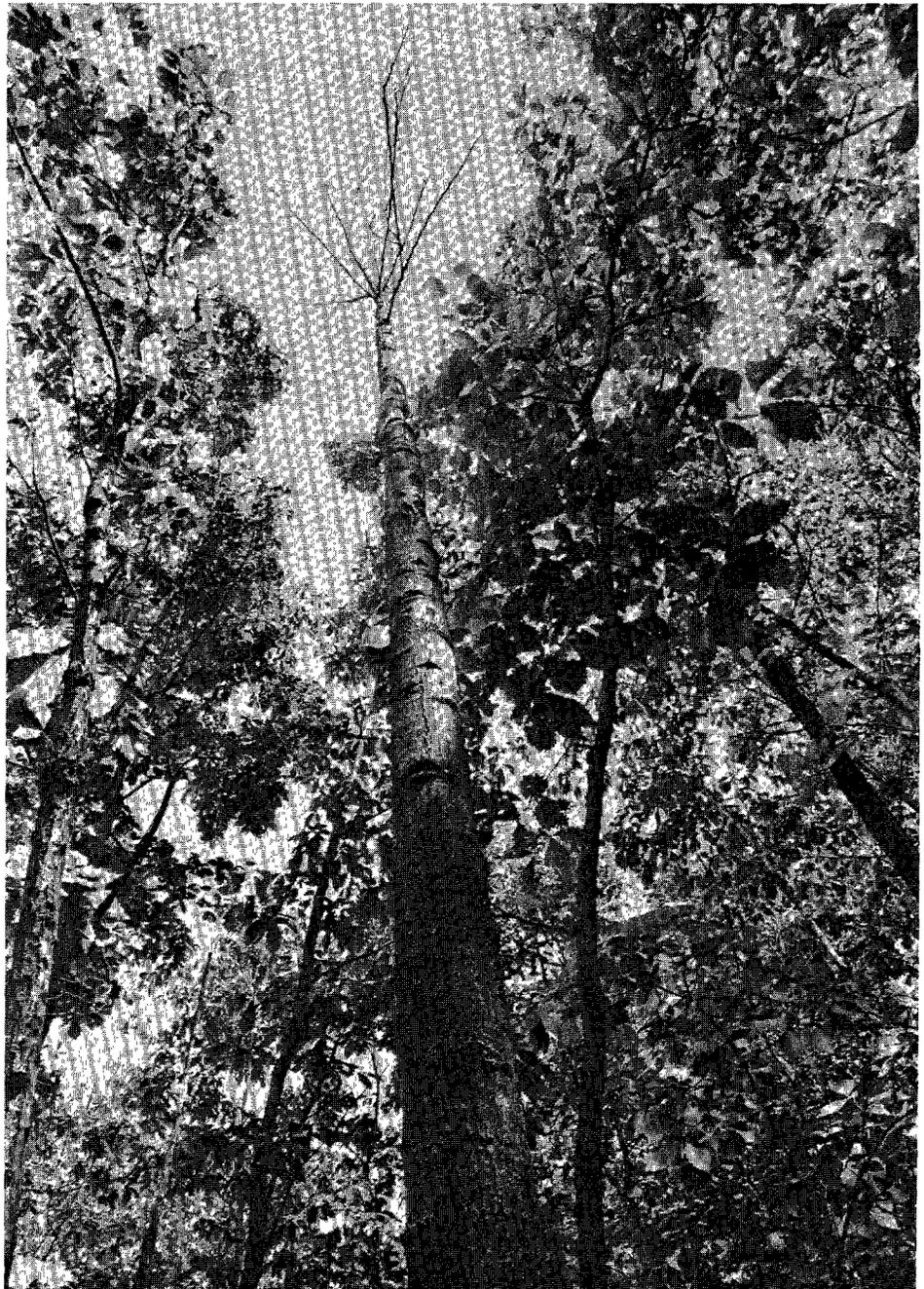
Northeastern Forest  
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Research Paper NE-642



# Individual-Tree Probability of Survival Model for the Northeastern United States

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## **Abstract**

Describes a distance-independent individual-tree probability of survival model for the Northeastern United States. Survival is predicted using a six-parameter logistic function with species-specific coefficients. Coefficients are presented for 28 species groups. The model accounts for variability in annual survival due to species, tree size, site quality, and the tree's competitive position within the stand. Model performance is evaluated using the chi-square goodness-of-fit test. Results are presented for the calibration data and an independent validation data set. The model has been incorporated into NE-TWIGS.

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## Introduction

More than 30 forest cover types occupy the Northeastern United States. Mixed-species stands dominate the region and a multitude of past cutting practices present resource managers with complex management decisions. Accurate and reliable growth and yield information improves our management capabilities. An individual-tree modeling approach is desirable when predicting growth under these diverse conditions because it provides the necessary information for tracking species, tree size, and tree quality, the three essential components for economic analyses in northeastern forest stands.

Models that predict the probability of survival of individual trees are an essential component of forest-growth prediction. Linking survival models with individual-tree diameter growth, height growth, and ingrowth allows us to predict forest stand development over time.

According to Waring (1987), "Trees die when they cannot acquire or mobilize sufficient resources to heal injuries or otherwise sustain life." The interaction of factors influencing individual-tree survival remains one of the least-understood elements of forest growth and yield estimation. Of the thousands of seedlings produced by a typical mature tree, only a few survive to full maturity. Most die as a direct or indirect consequence of failing to compete successfully for light, water, or soil nutrients (Peet and Christensen 1987). This type of mortality, commonly referred to as self-thinning (Lee 1971), can occur at any stage of stand development and is discussed in detail by Kramer and Kozlowski (1979). By contrast, catastrophic mortality is caused by major fires, windstorms, epidemic insect attacks, and other external agents. It is irregular in occurrence and more difficult to predict (Lee 1971). In this paper we address only the mortality caused by self-thinning, and endemic external agents such as insects and disease.

Here, we discuss the development and performance of an individual-tree survival model for the Northeastern United States. The model has been incorporated into the NE-TWIGS forest-growth projection system (Hilt and Teck 1989; Teck 1990), and is similar in form to those used to predict individual-tree survival in the Lake States (Buchman et al. 1983) and the Central States (Miner et al. 1988). Coefficients are presented for 28 species groups. The model was developed with USDA Forest Service Forest Inventory and Analysis (FIA) data from the following Northeastern States: Connecticut, Delaware, Kentucky, Maine, Maryland, Massachusetts, New Hampshire, New Jersey, New York, Ohio, Pennsylvania, Rhode Island, Vermont, and West Virginia.

## Data

Individual-tree measurements collected by the Northeastern Forest Experiment Station's FIA unit were used in developing the model. More than 4,400 1/5-acre permanent plots measured throughout the 14 Northeastern States were

used in this study. Data were collected in the 1960's, 1970's, and 1980's. Only one remeasurement period was available for each state except for Maine, which was remeasured twice. The remeasurement period averaged 12 years.

The data covered a wide range of age, site, and stocking conditions. Basal area ranged from 30 to 255 ft<sup>2</sup>/acre (Table 1). Site index (base age 50 years), recorded on each 1/5-acre plot for the dominant species, ranged from 30 to 90. Site-index conversion equations were used to assign the appropriate site index to each tree depending on its species. Quadratic mean stand diameter ranged from 5 to 13 inches, indicating a wide range in the age of the stands that were sampled.

Information recorded for each tree (more than 5 inches in d.b.h) included species, initial d.b.h (DBH), and a status code indicating whether the tree was alive or dead.

Plots that were cut heavily between remeasurements (residual basal area less than 30 percent of initial conditions) were excluded. Plots that showed levels of excessive mortality (more than 70 percent of initial basal area) also were eliminated to exclude episodes of catastrophic mortality from the data.

Every fourth plot was removed from the data set and reserved for validating the model. The final calibration data set, containing 59,465 trees, was divided into 28 species groups for analysis (Table 1). The validation data set contained 19,058 observations (Table 2).

## Methods

Each of the 28 species groups was analyzed separately. Our goal was to develop a single model form, and compute species-specific coefficients for that model. A single model form for all species is desirable because it simplifies model recalibration and localization of parameters.

Individual-tree survival/mortality models have been developed for various forest types and geographic regions. Many of these models predict survival as a function of tree size, and a combination of tree- and other stand-level variables such as tree vigor, basal area per acre, crown ratio, and site quality. It is desirable to model the annual survival rate with a function that can be defined between 0 and 1 (Buchman and Shifley 1983) since survival probability lies within this range (Hamilton 1986).

The sigmoid shape of the logistic function lends itself to modeling survival. Since Lee (1971) developed his linear survival model for lodgepole pine, most individual-tree survival/mortality models (Table 3) have been based on the logistic function, a nonlinear function bounded between 0 and 1, or some generalized form of the logistic function (Hamilton 1986). Although other mathematical functions have been used to model survival (Hett 1971; Moser 1972), Rennolls and Peace (1986) speculate that the flexibility of the logistic function is one of the primary reasons for its widespread use.

The general form of the logistic function is:

$$p(S) = [1 + \exp\{-(b_0 + b_1X_1 + \dots + b_nX_n)\}]^{-1} \quad (1)$$

where

$p(S)$  = probability of survival,  $0 \leq S < 1$ ,  
 $b_0$  = "intercept",  
 $b_1 \dots b_n$  = set of  $n$  regression coefficients,  
 $X_1 \dots X_n$  = set of  $n$  predictor variables, and  
 $\exp$  = base of the natural logarithm.

Many potential predictor variables were evaluated for inclusion in the model. Plot variables included site index (SI), basal area per acre, trees per acre, and quadratic mean stand diameter. Individual-tree variables included basal area and d.b.h.

Since a tree's survival is influenced by its competitive position within the stand, we analyzed three distance-independent competition indices: (1) the ratio of d.b.h to quadratic mean stand diameter, (2) ratio of tree basal area to plot basal area, and (3) basal area larger than or equal to

the subject tree (BAL). The latter had the highest correlation with survival.

We then compared the predictive capabilities of several models in Table 3 with a sample from our data. Not all of the models were evaluated since several contained predictor variables not available in our data. Results from this comparison showed that the Central States model (North Cent. For. Exp. Stn. 1983) was better at predicting mortality trends in the sample data set than the other models.

The data for each species group were then separated into DBH x BAL x SI cells. The upper and lower boundaries of each cell were selected so that there were approximately equal numbers of trees within each cell. The mean value for each of the three predictor variables and the annual survival rate within each cell were used in a preliminary analysis to select the model form.

Since a cell can contain trees with different remeasurement intervals, the annual survival rate for each cell was computed as (Buchman et al. 1983):

**Table 1.—Individual-tree and plot characteristics of the calibration data set**

Species group	No. of plots	Site index <sup>a</sup>			Plot TPA <sup>b</sup>			Plot basal area			DBH			BAL <sup>c</sup>			No. of trees	
		Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Total	Dead
								Ft <sup>2</sup> /acre			Inches			Ft <sup>2</sup> /acre				
1 American beech	607	32	90	57	40	661	230	32	245	102	5.0	35.6	9.4	4	245	68	2881	370
2 Balsam fir	703	30	82	50	70	705	325	36	237	115	5.0	18.4	7.1	2	224	76	5685	1228
3 Black cherry	283	39	90	68	42	621	280	32	210	105	5.0	32.4	8.5	4	199	70	984	172
4 Black oak	206	37	90	64	40	571	213	32	163	86	5.0	24.8	9.9	3	163	51	739	76
5 Chestnut oak	163	30	90	58	25	465	221	31	192	95	5.0	30.4	9.3	3	188	68	957	178
6 Eastern hemlock	484	30	83	50	50	806	309	32	255	131	5.0	29.2	9.0	3	255	85	3621	190
7 Hickory	279	35	88	61	25	806	229	32	255	92	5.0	24.9	8.6	4	242	68	1113	99
8 Loblolly pine	42	49	90	69	90	351	221	36	144	87	5.0	24.7	9.0	3	128	58	499	68
9 Noncommercial	345	30	80	55	45	690	258	32	234	98	5.0	24.9	6.4	5	234	82	1039	351
10 N. red oak	456	30	88	60	52	806	247	30	255	98	5.0	59.1	9.8	3	180	57	2155	163
11 N. white-cedar	309	30	70	42	92	716	354	38	234	130	5.0	28.0	8.7	3	224	74	3095	291
12 Other hardwoods	569	30	90	64	30	806	233	30	255	95	5.0	41.4	8.5	4	255	69	2668	835
13 Other pines	68	30	86	48	68	488	208	36	161	80	5.0	18.2	8.7	2	161	53	456	113
14 Paper birch	520	30	80	55	80	705	291	37	228	104	5.0	23.0	7.9	2	193	62	2003	173
15 Quaking aspen	330	36	90	61	50	806	292	32	255	95	5.0	24.5	7.9	3	248	52	1743	415
16 Red maple	1259	30	90	58	40	806	277	30	255	108	5.0	37.4	8.4	3	247	74	7238	647
17 Red pine	32	31	90	70	111	546	254	39	228	101	5.0	18.3	9.5	3	155	51	97	3
18 Red spruce	744	30	81	46	50	716	329	32	234	118	5.0	25.4	8.1	2	232	71	5447	479
19 Scarlet oak	184	36	90	59	52	473	196	32	174	78	5.0	32.5	9.5	4	158	51	757	113
20 Sugar maple	742	30	90	59	40	661	260	30	237	114	5.0	39.3	9.4	3	233	78	4476	280
21 Tamarack	57	30	69	50	105	705	355	36	234	106	5.0	14.8	7.7	2	156	53	292	49
22 Virginia pine	56	41	87	63	70	385	264	34	144	88	5.0	18.3	8.1	3	144	62	825	160
23 White ash	487	32	90	63	38	806	282	30	255	109	5.0	27.3	8.6	3	248	77	1585	192
24 White oak	376	31	88	59	25	806	229	37	255	90	5.0	37.9	9.3	3	201	62	1596	224
25 White pine	427	31	90	65	45	806	263	36	255	121	5.0	43.7	10.2	2	201	67	3233	319
26 White spruce	153	30	90	48	80	645	329	41	238	119	5.0	22.3	8.3	3	197	64	516	35
27 Yellow birch	782	30	90	58	45	806	265	32	255	112	5.0	33.8	9.3	3	242	74	2959	397
28 Yellow-poplar	188	38	90	69	25	401	181	30	219	94	5.0	32.8	9.9	4	182	67	806	64
Total	3003																59465	7684

<sup>a</sup>Total height (in feet) at age 50.

<sup>b</sup>Number of trees per acre.

<sup>c</sup>Basal area of trees larger than or equal to subject tree.

**Table 2.—Individual-tree and plot characteristics of the validation data set**

Species group	No. of plots	Site index <sup>a</sup>			Plot TPA <sup>b</sup>			Plot basal area			DBH			BAL <sup>c</sup>			No. of trees	
		Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Total	Dead
								Ft <sup>2</sup> /acre			Inches			Ft <sup>2</sup> /acre				
1 American beech	196	35	86	57	42	645	249	32	200	110	5.0	28.8	9.5	4	200	73	896	125
2 Balsam fir	225	30	75	51	70	659	334	40	240	117	5.0	17.6	7.2	3	200	77	1928	380
3 Black cherry	76	39	90	70	85	536	270	30	201	107	5.0	23.7	8.7	4	191	72	275	43
4 Black oak	80	33	86	60	25	476	213	36	187	92	5.0	41.0	10.9	3	177	53	329	31
5 Chestnut oak	61	30	80	55	80	618	267	33	225	106	5.0	26.5	8.9	3	196	73	525	67
6 Eastern hemlock	154	30	80	50	70	721	318	35	240	128	5.0	27.1	8.9	3	237	79	1210	34
7 Hickory	106	35	84	62	60	436	232	33	200	88	5.0	28.1	8.1	5	167	67	412	34
8 Loblolly pine	19	59	87	73	100	290	200	32	127	94	5.0	20.2	9.4	5	127	68	212	31
9 Noncommercial	103	30	77	54	65	645	241	34	225	93	5.0	13.2	6.3	6	225	79	319	110
10 N. red oak	159	30	89	60	25	618	264	32	225	105	5.0	29.6	9.7	3	225	60	666	60
11 N. white cedar	93	30	66	43	79	659	372	34	258	143	5.0	23.2	8.8	3	225	81	1052	111
12 Other hardwoods	218	30	90	62	45	558	220	30	225	96	5.0	25.8	8.8	4	204	70	758	215
13 Other pines	13	30	82	54	78	618	295	41	187	101	5.0	17.6	8.7	4	162	70	82	25
14 Paper birch	163	32	86	55	55	658	295	34	200	102	5.0	20.7	8.0	3	178	58	567	47
15 Quaking aspen	103	32	90	62	58	659	299	30	238	97	5.0	18.7	7.8	3	196	54	468	99
16 Red maple	398	30	86	58	40	802	292	30	242	110	5.0	31.6	8.3	3	242	74	2377	181
17 Red pine	9	34	80	65	102	619	399	50	187	133	5.1	17.8	9.1	4	152	56	63	2
18 Red spruce	249	30	72	47	70	802	350	34	258	122	5.0	21.5	7.9	3	258	74	1822	135
19 Scarlet oak	72	33	86	60	25	468	216	32	165	88	5.0	31.3	10.0	3	165	57	277	45
20 Sugar maple	241	31	86	60	40	721	248	35	232	110	5.0	37.2	9.6	3	232	74	1407	94
21 Tamarack	15	35	75	57	120	541	417	49	162	119	5.2	13.1	8.1	4	149	57	95	6
22 Virginia pine	21	37	90	66	78	363	208	35	127	84	5.0	16.4	9.0	5	127	56	163	26
23 White ash	146	36	90	64	42	558	250	32	225	101	5.0	26.7	8.7	4	197	70	443	52
24 White oak	129	30	83	58	45	618	227	32	192	94	5.0	32.4	9.3	4	192	67	608	100
25 White pine	127	34	90	64	70	802	271	42	242	118	5.0	39.1	10.0	3	193	66	813	83
26 White spruce	52	30	69	51	80	659	317	34	238	116	5.0	16.7	7.9	3	205	65	166	4
27 Yellow birch	277	32	86	57	55	564	238	34	258	104	5.0	29.7	9.6	4	228	68	931	112
28 Yellow-poplar	59	42	90	69	42	351	154	31	150	77	5.0	28.1	10.2	4	150	58	194	11
Total	1444																19058	2263

<sup>a</sup>Total height (in feet) at age 50.

<sup>b</sup>Number of trees per acre.

<sup>c</sup>Basal area of trees larger than or equal to subject tree.

**Table 3.—Previously developed survival/mortality models based on the logistic function**

Mathematical function	Citation	Region
1. $M = [1 + \text{EXP}-(b_1 + b_2 \cdot \text{PD} + b_3 \cdot \text{PDGR} + b_4 \cdot \text{CI})]^{-1}$	Monserud (1976)	Wisconsin
2. $M = [1 + \text{EXP}-(b_1 + b_2 \cdot \text{RBA} + b_3 \cdot \text{DBH} + b_4 \cdot \text{TPA})]^{-1}$	Krumland et al. (1978)	Northern California
3. $M = [1 + \text{EXP}-(b_1 + b_2 \cdot \text{DGR}^{b_3})]^{-1} + b_4$	Buchman (1979)	Lake States
4. $M = [1 + \text{EXP}-(b_1 + b_2 \cdot \text{DBH} + b_3 \cdot \text{DBH}^2)]^{-1}$	Wykoff et al. (1982)	Northern Rocky Mtns.
5. $S = b_1 - [1 + \text{EXP}-(b_2 + b_3 \cdot \text{DGR}^{b_4} + b_5 \cdot (\text{DBH} - 1)^{b_6} (b_7 \cdot (\text{DBH} - 1)))]^{-1}$	Buchman et al. (1983)	Lake States
6. $M = [1 + \text{EXP}-(b_1 + b_2 \cdot \text{DBH} + b_3 \cdot \text{DGR}^5)]^{-1}$	Hilt (1985)	Central hardwoods
7. $M = [1 + \text{EXP}-(b_1 + b_2 \cdot \text{DBH}^{0.5} + b_3 \cdot \text{BA}^{0.5} + b_4 \cdot \text{DGR} + b_5 \cdot \text{DBH}^{-1} + b_6 \cdot \text{X}_1 + b_7 \cdot \text{X}_2 + b_8 \cdot \text{X}_3 + b_9 \cdot \text{RDBH} + b_{10} \cdot \text{DDBH})]^{-1}$	Hamilton (1986)	Northern Idaho
8. $S = 1 - [1 + \text{EXP}(b_1 + b_2 \cdot (\text{DBH} + 1)^{b_3} \cdot (b_4 \cdot \text{DBH} + b_5 \cdot \text{BAL}))]^{-1}$	North Cent. For. Exp. Stn. (1983)	Central States
9. $M = [1 + \text{EXP}-(b_1 + b_2 \cdot \text{TAB}_1 + b_3 \cdot \text{TPA} + b_4 \cdot \text{PLC} + b_5 \cdot \text{DBH} + b_6 \cdot \text{SI})]^{-1}$	Bolton and Meldahl (unpublished)	Georgia

S = predicted survival rate; M = predicted mortality rate; PD = predicted diameter; PDGR = predicted diameter growth rate; CI = competition index; RBA = relative basal area; TPA = trees per acre; DGR = diameter growth rate; DBH = diameter at breast height; DGR5 = 5 year periodic annual diameter growth; BA = stand basal area; SI = site index; X<sub>1</sub>, . . . , X<sub>3</sub> = species-specific constants; QMD = mean stand diameter; RDBH = DBH/QMD; DDBH = DGR/DBH; BAL = basal area per acre larger than or equal to subject tree; TAB<sub>1</sub> = number of trees larger than subject tree; PLC = predicted live crown ratio; b<sub>1</sub>, . . . , b<sub>10</sub> = species-specific coefficients.

$$S = \left[ \frac{\sum_i X_i}{\sum_i N_i} \right]^{\left[ \frac{\sum_i N_i}{\sum_i T_i N_i} \right]} \quad (2)$$

where S is the annual survival rate,  $N_i$  is the total number of trees in the  $i^{\text{th}}$  group at the first measurement;  $X_i$  is the number of re-measured live trees in the  $i^{\text{th}}$  group; and  $T_i$  is the number of years between re-measurements in the  $i^{\text{th}}$  group.

The final model chosen was:

$$S = 1 - [1 + e^n]^{-1} \quad (3)$$

where

S = the tree's annual probability of survival

$$n = b_1 + b_2(\text{DBH} + 1)^{b_3} \exp[b_4(\text{DBH}) - b_5(\text{BAL}) - b_6(\text{SI})] \quad (4)$$

DBH = diameter at breast height (inches),  
 BAL = basal area per acre, in square feet, greater than or equal to the basal area of the subject tree,  
 SI = species-specific site index (base age = 50),  
 exp = base of the natural logarithm, and  
 $b_1$ – $b_6$  = parameters to be estimated.

Equation (3) was fit to the cell data for each species group using weighted nonlinear least squares regression (SAS NLIN procedure). Each cell was weighted by the number of observations within the cell. The survival model uses the same form of the logistic function as the model developed by Shifley for the Central States (North Cent. For. Exp. Stn. 1983). However, our model also includes site index as a predictor variable.

## Results

Species-specific coefficients for the 28 species groups are listed in Table 4. The survival rates predicted by this model form show the following trends: (1) survival rates decrease with increasing competition (BAL) for a given DBH and SI (Fig. 1); (2) survival rates increase with increasing DBH for a given value of BAL, reach a maximum, and then begin to decrease as DBH continues to increase (Fig. 2); the diameter at which survival peaks is species-specific and is dependent upon the interaction of B2, B3, and B4; (3) survival rates decrease with increasing SI for a given DBH and BAL (Fig. 3); (4) survival rates for a given DBH and SI

**Table 4.—Individual-tree probability of survival model coefficients**

Species group	B1	B2	B3	B4	B5	B6
1 American beech	1.33064	1.57768	0.52680	0.01996	0.00284	0.00262
2 Balsam fir	-0.83985	6.28167	0.20323	0.01438	0.00138	0.00324
3 Black cherry	0.76782	2.57168	0.52955	0.02286	0.00250	0.00324
4 Black oak	1.46779	1.68294	0.58186	0.02411	0.00354	0.00377
5 Chestnut oak	1.36843	1.62893	0.52723	0.02292	0.00303	0.00368
6 Eastern hemlock	0.80051	4.89879	0.19495	0.01319	0.00174	0.00396
7 Hickory	2.12479	0.98254	0.80490	0.04604	0.00385	0.00425
8 Loblolly pine	1.34643	5.14789	0.22382	0.01357	0.00245	0.00473
9 Non commercial	1.59314	3.68899	0.20783	0.01528	0.00487	0.00345
10 N. red oak	1.96783	0.89220	0.82229	0.03205	0.00426	0.00474
11 N. white-cedar	1.56217	3.53879	0.49115	0.04506	0.00245	0.00640
12 Other hardwoods	3.13639	0.23782	1.50877	0.06993	0.00391	0.01978
13 Other pines	0.56126	2.52989	0.50346	0.02338	0.00178	0.00374
14 Paper birch	1.61456	3.67935	0.21853	0.01602	0.00488	0.00404
15 Quaking aspen	1.99373	1.09417	0.78211	0.03903	0.00372	0.00492
16 Red maple	1.40846	1.70343	0.58927	0.02694	0.00304	0.00458
17 Red pine	1.34643	5.14789	0.22382	0.01357	0.00245	0.00473
18 Red spruce	0.84085	5.12887	0.19698	0.01306	0.00178	0.00518
19 Scarlet oak	1.96799	0.84200	0.82255	0.03902	0.00384	0.00432
20 Sugar maple	2.17938	0.98263	0.78400	0.04011	0.00352	0.00517
21 Tamarack	-0.83985	6.28167	0.20323	0.01438	0.00138	0.00324
22 Virginia pine	0.56126	2.52989	0.50346	0.02338	0.00178	0.00374
23 White ash	0.78229	2.55544	0.56692	0.02881	0.00257	0.00417
24 White oak	1.33389	1.56947	0.52566	0.02083	0.00271	0.00305
25 White pine	1.34643	5.14789	0.22382	0.01357	0.00245	0.00473
26 White spruce	2.78437	3.01029	0.68225	0.05261	0.00492	0.01161
27 Yellow birch	3.10893	0.22995	1.46938	0.07211	0.00598	0.00579
28 Yellow-poplar	0.77006	2.53447	0.62952	0.03124	0.00306	0.00507

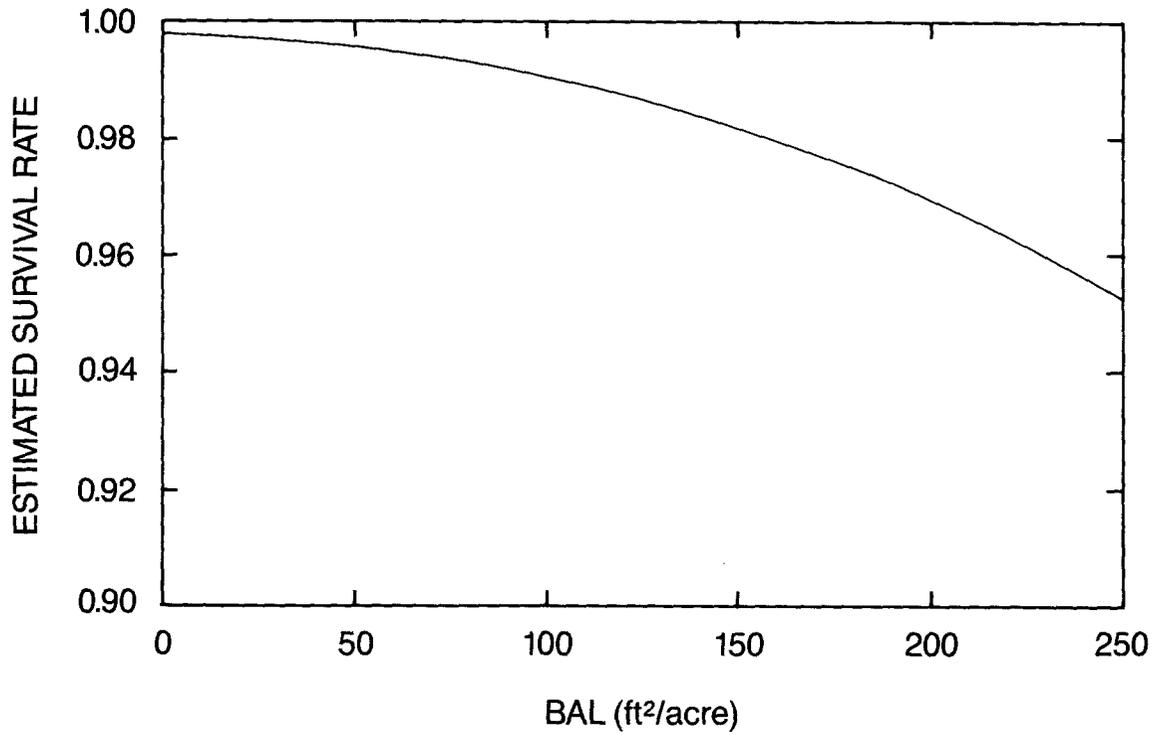


Figure 1.—Estimated annual survival rates for red spruce (SI = 50, d.b.h. = 10 inches), demonstrating effect of BAL on survival rate.

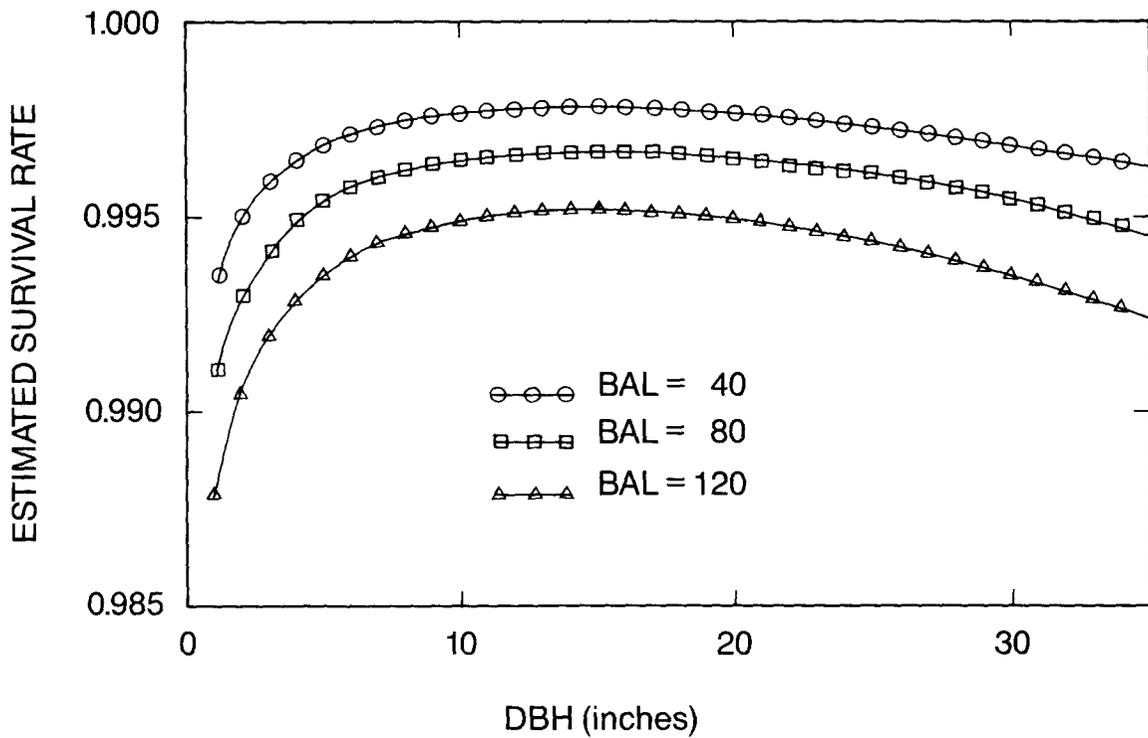


Figure 2.—Estimated annual survival rates for red spruce (SI = 50), demonstrating effect of DBH on survival rate.

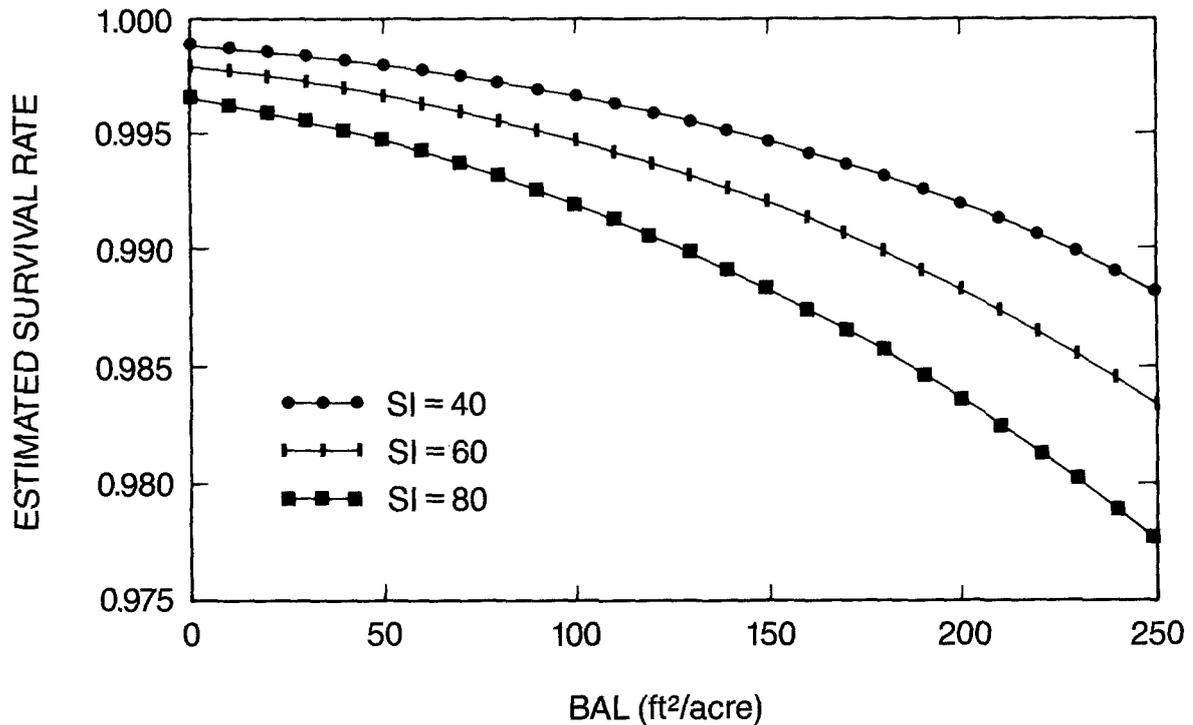


Figure 3.—Estimated annual survival rates for red spruce (d.b.h. = 10 inches), demonstrating effect of SI on survival rate.

approach a minimum as inter-tree competition (BAL) increases.

Observed and estimated number of survivor trees by species group and their associated annual survival rates are given in Table 5 for the calibration data set. The predicted survival rate for 19 of the 28 species groups was within 1 percent of the observed survival rate. The absolute difference between the predicted annual survival rate for the other five species groups was between 1 and 2.2 percent of the observed annual survival rate. For example, the observed annual survival rate for paper birch is 99.19 percent, and the predicted annual survival rate is 99.01 percent. Therefore, the absolute difference between the observed and predicted annual survival rate is 0.18 percent. The model underpredicted survival for 7 species groups and overpredicted survival for 21 groups.

A chi-square goodness-of-fit statistic was used to test the model. The  $X^2$  statistic is:

$$X^2 = \sum_i^n [O_i - E_i]^2 / E_i \quad (5)$$

where

$X^2$  = chi-square statistic,  
 $O_i$  = observed number of survivor trees in the  $i^{\text{th}}$  cell,

$E_i$  = expected number of survivor trees in the  $i^{\text{th}}$  cell, and  
 $n$  = number of cells.

With the exception of balsam fir, the  $X^2$  test showed no significant difference between the observed and predicted survival distributions.

## Validation

Observed and estimated number of survivor trees by species group and their associated annual survival rates for the validation data set are shown in Table 6. The predicted survival rate was within 1 percent of the observed survival rate for 23 of the 28 species groups. The absolute difference between the predicted and observed survival rates for the other five species groups was less than 2.6 percent. The model underpredicted annual survival for 9 species groups and overpredicted annual survival for 18 groups.

Chi-square goodness-of-fit statistics by species group for the validation data are shown in Table 6. There were no significant differences between predicted and observed survival rates for 27 of the 28 species groups. The exception was balsam fir.

**Table 5.—Performance of probability of survival model for calibration data set, by species group**

Species group	No. of trees	Mean interval	Survival				Absolute difference <sup>a</sup>	Degrees of freedom <sup>b</sup>	Chi-square <sup>c</sup>	Critical value ( $\alpha = .05$ )	Significance <sup>d</sup>
			Observed		Estimated						
		Years	No. of trees	Annual rate	No. of trees	Annual rate					
American beech	2881	11.6	2511	0.9882	2474	0.9870	-0.0012	62	54.4	81.4	NS
Balsam fir	5685	11.4	4456	0.9790	5470	0.9966	0.0176	55	233.0	73.3	**
Black cherry	984	12.0	812	0.9841	919	0.9943	0.0102	58	22.3	76.8	NS
Black oak	739	12.0	662	0.9909	665	0.9913	0.0004	60	8.5	79.1	NS
Chestnut oak	957	12.1	778	0.9830	798	0.9851	0.0021	58	20.9	76.8	NS
Eastern hemlock	3621	11.9	3431	0.9955	3445	0.9958	0.0003	61	4.6	80.2	NS
Hickory	1113	11.8	1013	0.9921	966	0.9881	-0.0040	57	10.8	75.6	NS
Loblolly pine	499	10.4	430	0.9858	486	0.9974	0.0116	42	17.0	58.1	NS
Non commercial	1039	11.6	684	0.9647	893	0.9870	0.0223	41	69.0	56.9	NS
N. red oak	2155	12.1	1992	0.9935	1869	0.9883	-0.0052	65	20.9	84.8	NS
N. white cedar	3095	11.3	2804	0.9913	3009	0.9975	0.0062	58	23.9	76.8	NS
Other hardwoods	2668	11.7	1832	0.9683	2123	0.9806	0.0123	65	65.0	84.8	NS
Other pines	456	13.0	341	0.9779	427	0.9949	0.0170	38	34.9	53.4	NS
Paper birch	2003	11.1	1830	0.9919	1793	0.9901	-0.0018	60	9.5	79.1	NS
Quaking aspen	1743	11.6	1327	0.9768	1545	0.9897	0.0129	51	50.3	68.7	NS
Red maple	7238	11.8	6591	0.9921	6217	0.9782	-0.0049	64	67.3	83.7	NS
Red pine	97	11.7	91	0.9946	94	0.9946	0.0000	32	0.9	46.2	NS
Red spruce	5447	11.4	4968	0.9919	5239	0.9966	0.0047	63	22.7	82.5	NS
Scarlet oak	757	12.0	643	0.9865	649	0.9873	0.0008	62	12.6	81.4	NS
Sugar maple	4476	11.8	4195	0.9945	3846	0.9872	-0.0073	63	53.4	82.5	NS
Tamarack	292	11.0	238	0.9815	284	0.9973	0.0158	36	15.5	51.0	NS
Virginia pine	825	10.7	664	0.9800	762	0.9927	0.0127	43	26.0	59.3	NS
White ash	1585	11.9	1393	0.9892	1463	0.9933	0.0041	63	10.3	82.5	NS
White oak	1596	12.0	1372	0.9875	1351	0.9862	-0.0013	60	13.5	79.1	NS
White pine	3233	12.0	2914	0.9914	3129	0.9973	0.0059	65	30.5	84.8	NS
White spruce	516	11.3	481	0.9938	503	0.9978	0.0040	43	4.1	59.3	NS
Yellow birch	2959	11.6	2562	0.9876	2628	0.9898	0.0022	62	22.2	81.4	NS
Yellow-poplar	806	11.0	742	0.9925	757	0.9943	0.0018	61	5.0	80.2	NS

<sup>a</sup>Estimated survival rate minus the observed survival rate.

<sup>b</sup> $n-p-1$  where  $n$  = number of cells and  $p$  = number of model parameters (6).

<sup>c</sup>Sum of chi-square statistics for all cells within a species group based on number of observed and predicted survival trees.

<sup>d</sup>\*\* = Significant at  $\alpha = 0.5$ ; NS = not significant at  $\alpha = 0.5$ .

## Discussion

The model described predicts the annual survival rates for 28 major species groups indigenous to the Northeast. The model accounts for variability in annual survival due to species, tree size, site quality, and the tree's competitive position within the stand.

Since the data contained only trees greater than or equal to 5 inches d.b.h., predicted survival rates for smaller trees are extrapolations contingent upon model form and the parameters associated with larger trees. This does not preclude the use of this model for predicting survival rates of trees in the smaller d.b.h classes. However, until data are available for validating the survival rates for these smaller classes, we recommend that users be cautious when applying the model to very young stands.

Efforts were taken to eliminate plots that experienced episodes of catastrophic mortality during the

remeasurement period. If the survival model is applied to forest stands in which some catastrophic mortality has occurred, the model may have a tendency to overpredict survival. We recommend that users take this into consideration.

Users also should recognize that a very small change in predicted survival rates can result in a substantial difference in the number of trees predicted to survive for a given length of time. For example, suppose we are interested in predicting how many American beech and black cherry trees will survive in a 30-year projection. Using the compound survival formula (equation 6) and assuming an initial starting condition of 1,000 trees per acre for each species, and an overall mean annual survival rate for each species of 98.70 and 99.43 percent, respectively (Table 5), we would predict that 675 American beech and 842 black cherry trees per acre will be alive after 30 years:

$$NS_n = SR^n(N_i) \quad (6)$$

where  $NS_n$  is the predicted number of survivors after  $n$  years;  $SR$  is the annual survival rate;  $N_i$  is the initial number of trees; and  $n$  is the number of years in the projection.

In this example, 84.2 percent of the black cherry survived compared with 67.5 percent of the American beech, a difference of 16.7 percent. The small difference of 0.73 percent in the predicted annual survival rate between the two species resulted in a 16.7 percent difference in the predicted number of survivors. This difference will increase as the length of the projection increases due to the compound nature of survival prediction.

The survival model has several desirable properties. Survival is dependent on a tree's competitive position within a stand. As BAL increases, the ability of that tree to compete successfully for moisture, sunlight, and nutrients

decreases, resulting in a decreased likelihood of survival. Suppressed trees (small d.b.h.) in crowded young stands and overmature trees have higher probabilities of mortality. Finally, survival rates decrease with increasing site quality since more productive sites usually have higher stand volumes but fewer trees per acre than less productive sites. A three-dimensional response surface of the survival model for red spruce and three major species of the northern hardwood forest type is shown in Figure 4. We are satisfied that the predicted survival rates exhibit biologically reasonable trends when the models are extrapolated for use under conditions not encountered in the data.

This survival model has been incorporated into NE-TWIGS, an individual-tree growth and yield projection system for mixed-species forests of the Northeastern United States (Hilt and Teck 1989; Teck 1990).

**Table 6.—Performance of probability of survival model for validation data set, by species group**

Species group	No. of trees	Mean interval	Survival				Absolute difference <sup>a</sup>	Degrees of freedom <sup>b</sup>	Chi-square <sup>c</sup>	Critical value ( $\alpha = .05$ )	Significance <sup>d</sup>
			Observed		Estimated						
			No. of trees	Annual rate	No. of trees	Annual rate					
		<i>Years</i>									
American beech	896	11.7	771	0.9873	764	0.9865	-0.0008	57	29.4	75.6	NS
Balsam fir	1928	11.3	1546	0.9806	1854	0.9966	0.0160	45	72.3	61.7	**
Black cherry	275	11.7	230	0.9848	257	0.9941	0.0093	48	10.1	65.2	NS
Black oak	329	12.1	297	0.9916	297	0.9917	0.0001	57	3.8	75.6	NS
Chestnut oak	525	11.4	458	0.9881	439	0.9844	-0.0037	55	8.3	73.6	NS
Eastern hemlock	1210	11.6	1176	0.9976	1155	0.9960	-0.0016	51	3.8	68.7	NS
Hickory	412	11.8	378	0.9927	356	0.9876	-0.0051	49	5.0	66.3	NS
Loblolly pine	212	10.7	181	0.9853	205	0.9968	0.0115	28	8.5	41.3	NS
Non commercial	319	11.5	206	0.9626	277	0.9878	0.0252	31	31.6	45.0	NS
N. red oak	666	12.1	606	0.9922	575	0.9879	-0.0043	57	8.6	75.6	NS
N. white cedar	1052	11.4	941	0.9903	1018	0.9972	0.0069	47	15.0	64.0	NS
Other hardwoods	758	11.9	541	0.9721	607	0.9815	0.0094	56	20.4	74.5	NS
Other pines	82	12.8	54	0.9679	75	0.9932	0.0253	22	11.5	33.9	NS
Paper birch	567	11.0	518	0.9918	511	0.9906	-0.0012	44	4.5	60.5	NS
Quaking aspen	468	11.9	367	0.9797	413	0.9895	0.0098	43	12.7	59.3	NS
Red maple	2377	11.7	2195	0.9932	2043	0.9871	-0.0061	55	27.8	73.3	NS
Red pine	63	11.7	60	0.9959	61	0.9977	0.0018	14	0.6	23.7	NS
Red spruce	1822	11.3	1686	0.9932	1745	0.9962	0.0030	54	6.2	72.2	NS
Scarlet oak	277	11.5	223	0.9813	238	0.9869	0.0056	54	11.9	72.2	NS
Sugar maple	1407	11.8	1313	0.9942	1215	0.9877	-0.0065	56	16.6	74.5	NS
Tamarack	95	11.1	85	0.9900	91	0.9966	0.0066	24	1.4	36.4	NS
Virginia pine	163	11.2	128	0.9786	151	0.9931	0.0145	30	7.3	43.8	NS
White ash	443	12.2	389	0.9894	411	0.9938	0.0044	50	6.1	67.5	NS
White oak	608	12.3	508	0.9855	508	0.9856	0.0001	57	11.4	75.6	NS
White pine	813	12.0	730	0.9910	788	0.9974	0.0064	60	13.0	79.1	NS
White spruce	166	11.5	162	0.9979	161	0.9975	-0.0004	33	0.8	47.4	NS
Yellow birch	931	11.7	817	0.9889	834	0.9906	-0.0017	53	7.1	71.0	NS
Yellow-poplar	194	11.0	182	0.9942	183	0.9948	0.0006	42	1.8	58.1	NS

<sup>a</sup>Estimated survival rate minus the observed survival rate.

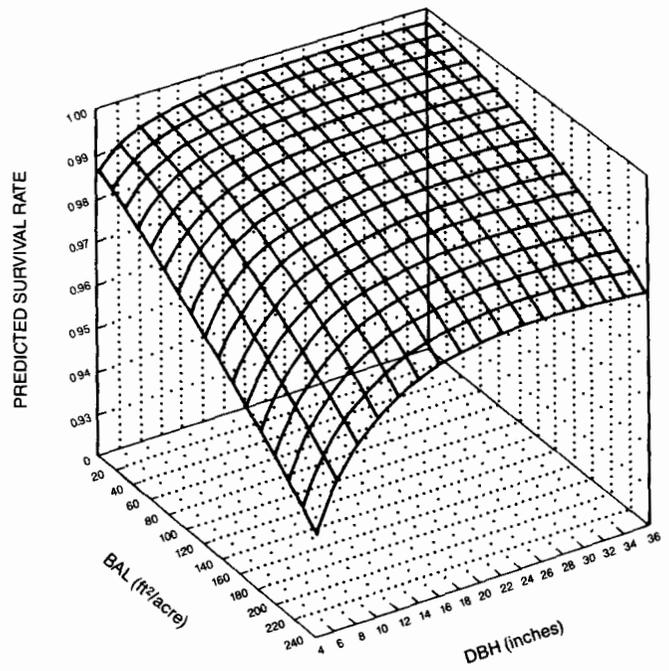
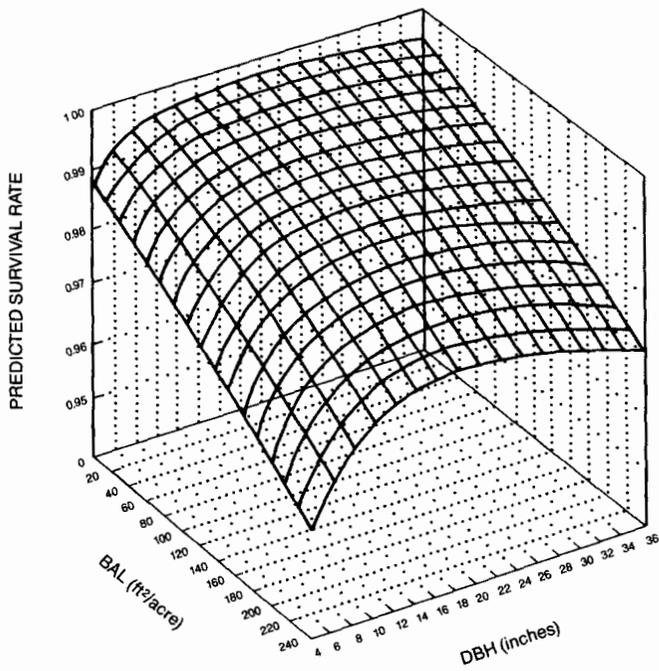
<sup>b</sup> $n-p-1$  where  $n$  = number of cells and  $p$  = number of model parameters (6).

<sup>c</sup>Sum of chi-square statistics for all cells within a species group based on number of observed and predicted survival trees.

<sup>d</sup>\*\* = Significant at  $\alpha = 0.5$ ; NS = not significant at  $\alpha = 0.5$ .

Sugar Maple (Site Index = 56)

American Beech (Site Index = 56)



Yellow Birch (Site Index = 56)

Red Spruce (Site Index = 50)

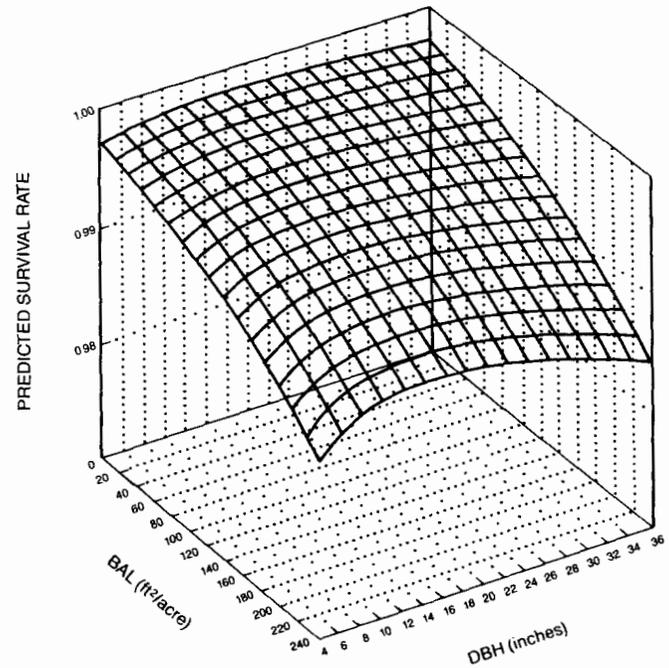
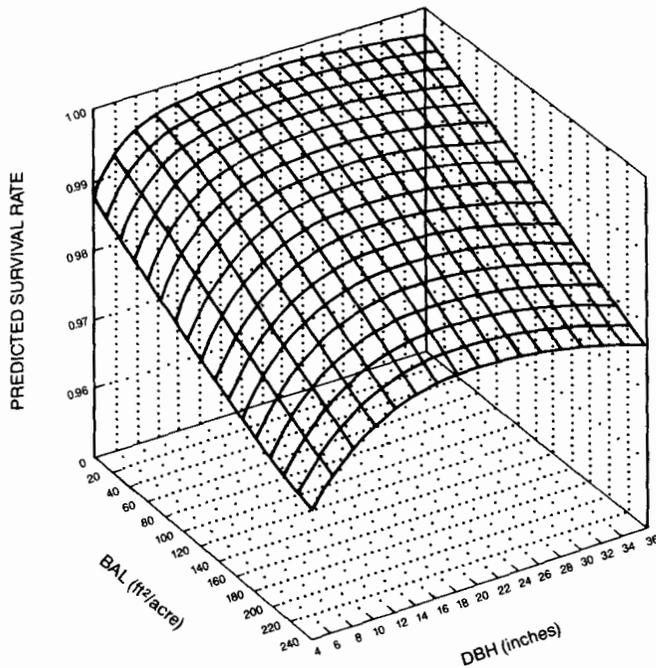


Figure 4.—Three-dimensional response surface of survival model for red spruce and three major species of northern hardwood forest type on average site.

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Teck, Richard M.; Hilt, Donald E. 1990. **Individual-tree probability of survival model for the Northeastern United States**. Res. Rep. NE-642. Radnor, PA: U.S. Department of Agriculture, Forest Service, Northeastern Forest Experiment Station. 10 p.

Describes a distance-independent individual-tree probability of survival model for the Northeastern United States. Survival is predicted using a six-parameter logistic function with species-specific coefficients. Coefficients are presented for 28 species groups. The model accounts for variability in annual survival due to species, tree size, site quality, and the tree's competitive position within the stand. Model performance is evaluated using the chi-square goodness-of-fit test. Results are presented for the calibration data and an independent validation set. The model has been incorporated into NE-TWIGS.

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**Keywords:** Logistic function; mortality; survival model; survival prediction

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