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# On Identifiability of Bias-Type Actuator-Sensor Faults in Multiple-Model-Based Fault Detection and Identification

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## Abstract

This paper explores a class of multiple-model-based fault detection and identification (FDI) methods for bias-type faults in actuators and sensors. These methods employ banks of Kalman-Bucy filters to detect the faults, determine the fault pattern, and estimate the fault values, wherein each Kalman-Bucy filter is tuned to a different failure pattern. Necessary and sufficient conditions are presented for identifiability of actuator faults, sensor faults, and simultaneous actuator and sensor faults. It is shown that FDI of simultaneous actuator and sensor faults is not possible using these methods when all sensors have biases.

## 1 Introduction

Failures in control effectors and sensors can cause poor performance or instability in dynamical systems. In particular, faults in flight control systems for aircraft or spacecraft can lead to loss of control and serious incidents. Therefore, rapid fault detection and identification (FDI) in actuators and sensors is important for enhancing flight safety. One approach to actuator and sensor FDI is based on multiple-model methods [1], [2], which have been extended to detect faults and identify the fault pattern as well as the fault values [3], [4]. Such methods typically use banks of Kalman-Bucy filters (or Extended Kalman filters) and multiple hypothesis testing, and have been reported to be effective for bias-type faults such as aircraft control surfaces getting stuck at unknown values, or sensors (e.g., rate gyros) that develop unknown constant or slowly-varying biases. The underlying requirement for these methods is that the faults should be identifiable. Identifiability of bias-type faults was considered in [4] and preliminary results were presented. This paper focuses in greater detail on the identifiability of actuator faults, sensor faults, and simultaneous actuator and sensor faults.

## 2 Actuator Faults

Consider a linear time-invariant system:

$$\begin{aligned}\dot{x} &= Ax + Bu + w_p \\ y &= Cx + w_s\end{aligned}\tag{1}$$

where  $x \in R^n$ ,  $u \in R^m$ ,  $w_p \in R^n$ ,  $y \in R^l$ ,  $w_s \in R^l$  denote the state vector, control vector, process noise, output vector, and sensor noise respectively, and  $A$ ,  $B$ ,  $C$  are appropriately dimensioned matrices.  $w_p$  and  $w_s$  are usually assumed to be stationary zero-mean Gaussian processes having constant covariance intensities.

Of the  $m$  actuators in the system, some may fail at unknown time instants. The type of actuator failure addressed in this paper is a “stuck actuator” failure, which produces a constant unknown input value (which is zero in the case of complete actuator outage). Thus, in the  $k$ th failure pattern, if  $m_k$  of the  $m$  actuators fail, the

system dynamics becomes

$$\begin{aligned}\dot{x} &= Ax + \sum_{j \notin \mathcal{F}_{ak}} b_j u_j + \sum_{j \in \mathcal{F}_{ak}} b_j \bar{u}_j + w_p \\ &= Ax + B^k u^k + \bar{B}^k \bar{u}^k + w_p\end{aligned}\quad (2)$$

where  $\mathcal{F}_{ak}$  is the set of indices corresponding to the failed actuators, and  $\bar{u}_j$  denotes the corresponding failure value (for example, deflection of a stuck control surface in aircraft). There are up to  $2^m - 1$  possible failure patterns for  $m$  actuators.  $\bar{u}^k \in R^{m_k}$  denotes the failure value for the  $k$ th failure pattern;  $u^k \in R^{m-m_k}$  denotes the input vector corresponding to the functioning actuators;  $\bar{B}^k$  denotes the columns of  $B$  corresponding to the failed actuators  $B^k$  denotes the remaining columns of  $B$  corresponding to the functional actuators.

In multiple-model-based methods, up to  $2^m$  mathematical models (including the “no failures” case and up to  $2^m - 1$  failure patterns) are constructed from (2) and a corresponding bank of  $2^m$  Kalman-Bucy filters is designed. The fault is isolated and the fault values  $\bar{u}_j$  are estimated by augmenting (2) with:

$$\dot{\bar{u}}_j = w_{a_j} \quad (3)$$

where  $w_{a_j}$  is a fictitious zero-mean white noise process. Thus the augmented equation corresponding to model  $k$  (failure pattern  $k$ ) is

$$\frac{d}{dt} \begin{bmatrix} x \\ \bar{u}^k \end{bmatrix} = \begin{bmatrix} A & \bar{B}^k \\ 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ \bar{u}^k \end{pmatrix} + \begin{bmatrix} B^k \\ 0 \end{bmatrix} u^k + w^k \quad (4)$$

where  $w^k \in R^{n+m_k}$  denotes the input noise vector consisting of the process noise  $w_p$  and the fictitious noise corresponding to  $\bar{u}^k$ . Denoting

$$\xi^k = \begin{bmatrix} x \\ \bar{u}^k \end{bmatrix} \quad (5)$$

the system corresponding to failure pattern  $k$  is expressed as

$$\dot{\xi}^k = A_\xi^k \xi^k + B_\xi^k u^k + w^k \quad (6)$$

$$y = [C \ 0] \xi^k + w_s := C_\xi^k \xi^k + w_s \quad (7)$$

where  $A_\xi^k$ ,  $B_\xi^k$  are the augmented system- and input-matrices from (4).

The FDI approach employed in multiple-model-based methods is to design and implement a bank of Kalman-Bucy filters (KBF), one corresponding to each of the  $2^m$  models, and to determine (in real time) which model correctly represents the actual fault pattern, using criteria such as highest conditional probability or the smallest residual norm. The KBF corresponding to the correct model also gives an unbiased minimum-variance estimate of the fault values. The KBF corresponding to model  $k$  is given by

$$\hat{\xi}^k = A_\xi^k \hat{\xi}^k + B_\xi^k u^k + H^k (y - C_\xi^k \hat{\xi}^k) \quad (8)$$

where  $\hat{\xi}^k$  denotes the estimate of  $\xi^k$  and  $H^k$  (a function of time) is the KBF gain. From (6) and (8), the estimation error dynamics are given by

$$\tilde{\xi}^k = (A_\xi^k - H^k C_\xi^k) \tilde{\xi}^k + w^k - H^k w_s \quad (9)$$

where  $\tilde{\xi}^k = \xi^k - \hat{\xi}^k$ . The residual (which is used in decision-making to determine which fault model is “closest” to the actual system), is given by

$$r_k = y - C_\xi^k \hat{\xi}^k = C_\xi^k \tilde{\xi}^k + w_s \quad (10)$$

For the scheme to work, the error dynamics (9) must be asymptotically stable and unbiased, i.e., the mean of  $\tilde{\xi}^k$  should converge to zero and its covariance should be bounded  $\forall t \geq 0$ .

For failure pattern  $k$ , if  $\bar{B}^k$  is not of full rank, it is not possible to distinguish between fault values for actuators corresponding to linearly dependent columns of  $\bar{B}^k$ . For example, for some failure pattern, if

$$\bar{B} = [\bar{b}_1, \bar{b}_2, \bar{b}_3]$$

and if  $\bar{b}_3 = \alpha_1 \bar{b}_1 + \alpha_2 \bar{b}_2$  for some constant  $\alpha_1, \alpha_2$ , the input terms due to the failed actuators are

$$\begin{aligned} \bar{B}\bar{u} &= \bar{b}_1 \bar{u}_1 + \bar{b}_2 \bar{u}_2 + \bar{b}_3 \bar{u}_3 \\ &= \bar{b}_1 (\bar{u}_1 + \alpha_1 \bar{u}_3) + \bar{b}_2 (\bar{u}_2 + \alpha_2 \bar{u}_3) \end{aligned} \quad (11)$$

Thus only the aggregated fault values  $\bar{u}_1 + \alpha_1 \bar{u}_3, \bar{u}_2 + \alpha_2 \bar{u}_3$  can be estimated, and  $\bar{u}_1, \bar{u}_2, \bar{u}_3$  cannot be estimated individually. Therefore it is assumed henceforth that, for each failure mode, the fault inputs corresponding to linearly dependent columns of  $\bar{B}^k$  have been aggregated and that  $\bar{B}^k$  is of full rank. As a result, the number of distinguishable failure patterns (and the corresponding Kalman-Bucy filters) is usually less than  $2^m - 1$ .

## 2.1 Actuator Fault Identifiability

For the Kalman-Bucy filter to work correctly, observability of the augmented system (6), (7) is essential. Unobservability can result in grossly erroneous estimates of the augmented state vector and incorrect FDI. Furthermore, in the infinite duration case, a KBF gain (constant)  $H^k$  that stabilizes the system matrix in (9) exists only if  $(C_\xi, A_\xi)$  is detectable, which is possible only if the augmented zero-frequency modes of  $A_\xi$  are observable. Thus, the *observability* rather than *detectability* of  $(C_\xi, A_\xi)$  is important. The following theorem gives a necessary and sufficient condition for observability.

**Theorem 1.** The pair  $(C_\xi^k, A_\xi^k)$  is observable iff all of the following conditions are satisfied:

- (i)  $l \geq m_k$
- (ii) the pair  $(C, A)$  observable, and

(iii) the system  $(C, A, \overline{B}^k)$  has no zeros at the origin.

**Proof.** Applying the PBH rank test [5],  $[C_\xi^k, A_\xi^k]$  is observable iff

$$\text{rank} \begin{bmatrix} sI - A & -\overline{B}^k \\ 0 & sI_{m_k} \\ C & 0 \end{bmatrix} = n + m_k \text{ for } s = \lambda(A) \text{ and } s = 0 \quad (12)$$

where  $\lambda(\cdot)$  denotes eigenvalue. The first  $n$  columns of the PBH test matrix are independent (for all  $s$ ) iff  $(C, A)$  is observable. For  $s \neq 0$ , the last  $m_k$  columns are mutually independent as well as independent of the first  $n$  columns, thus the test matrix has a full rank. For  $s = 0$ , the rank condition (12) is satisfied iff  $l \geq m_k$  and  $(C, A, \overline{B}^k)$  has no system zeros at  $s = 0$ . ■

**Remark 2.1.** The zeros mentioned in Theorem 1 are *system zeros* of  $(C, A, \overline{B}^k)$ , which include transmission zeros and input decoupling zeros (idz) [6]. The idz's are simply the eigenvalues of  $A$  corresponding to the uncontrollable modes of  $(A, \overline{B}^k)$ . Because  $(C, A)$  is observable, there are no output decoupling zeros. Note that  $\overline{B}^k$  corresponds to failed actuators in failure pattern  $k$ ; therefore,  $(A, \overline{B}^k)$  may not be controllable for all  $k$ . For the corresponding models, Theorem 1 requires that the uncontrollable modes must not have zero eigenvalues. If  $(A, \overline{B}^k)$  is controllable, the system zeros are just the transmission zeros.

**Remark 2.2.** In practical implementation, the estimation is performed in a discrete-time setting using discrete-time Kalman filters. The observability conditions of Theorem 1 are very similar, the only difference being that, in item (iii), the phrase “no zeros at the origin” is replaced by: “no zeros at unity” (for the discretized version of  $(C, A, \overline{B}_\xi^k)$ ).

**Remark 2.3.** The unobservable subspace  $\overline{\mathcal{O}}_\xi^k$  of  $(C_\xi^k, A_\xi^k)$  can be obtained as follows (after some manipulation and using the fact that  $(\overline{C}, A)$  is observable)

$$\overline{\mathcal{O}}_\xi^k = \mathcal{N} \begin{bmatrix} C_\xi^k \\ C_\xi^k A_\xi^k \\ \vdots \\ C_\xi^k (A_\xi^k)^{(n+m_k-1)} \end{bmatrix} = \mathcal{N} \begin{bmatrix} A & \overline{B}^k \\ C & 0 \end{bmatrix} \quad (13)$$

where  $\mathcal{N}(\cdot)$  denotes the null space. Thus the unobservable subspace of  $(C_\xi^k, A_\xi^k)$  consists of the generalized eigenvectors of  $(C, A, \overline{B}^k)$  corresponding to the zeros at the origin. If  $A$  is nonsingular, it can be seen that

$$\overline{\mathcal{O}}_\xi^k = \left\{ \begin{bmatrix} -A^{-1}\overline{B}^k x_2 \\ x_2 \end{bmatrix}, x_2 \in R^{m_k} \right\} \quad (14)$$

For the case when  $A$  has one or more zero eigenvalues, the corresponding  $\overline{\mathcal{O}}_\xi^k$  is still non-trivial when  $(C, A, \overline{B}^k)$  has one or more zeros at the origin.

**Remark 2.4.** It is intuitively straightforward to see that a zero at the origin adversely affects the ability to estimate a constant fault value  $\bar{u}$ , because input frequency components corresponding to the system zeros do not appear in the output. Furthermore, a set of initial conditions exist such that  $y(t)$  is identically zero.

### 3 Sensor Bias

Suppose there are no actuator failures but  $q$  of the  $l$  sensors have unknown sensor biases, (or are known to be prone to developing biases). Denote the bias-free part and the biased part of the sensor output vector as  $y_1$  and  $y_2$  respectively, and the corresponding output matrices as  $C_1 \in R^{(l-q) \times n}$ ,  $C_2 \in R^{q \times n}$ . Then the sensor output equation is

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = Cx + w_s = \begin{bmatrix} C_1 x \\ C_2 x + \bar{y}_2 \end{bmatrix} + w_s \quad (15)$$

where  $\bar{y}_2 \in R^q$  is the sensor bias vector and  $C = [C_1^T \ C_2^T]^T$ . As was done in the case of linearly dependent columns of  $\bar{B}^k$ , it is assumed that linearly dependent sensor outputs have been combined and  $C$ ,  $C_1$ ,  $C_2$  all have full row ranks. (Biases corresponding to linearly dependent sensors cannot be estimated individually and must be aggregated).

For this sensor bias formulation, bias- and state-estimation does not require multiple models and a bank of KBFs, but requires only one KBF. Upon augmenting the sensor bias  $\bar{y}_2$  to the state vector, the system becomes

$$\begin{aligned} \dot{\eta} &:= \frac{d}{dt} \begin{bmatrix} x \\ \bar{y}_2 \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \eta + \begin{bmatrix} B \\ 0 \end{bmatrix} u + w' \\ &:= A_\eta \eta + B_\eta u \end{aligned} \quad (16)$$

$$y = \begin{bmatrix} C_1 & 0_{(l-q) \times n} \\ C_2 & I_q \end{bmatrix} \eta + w_s := C_\eta \eta + w_s \quad (17)$$

where  $w'$  denotes the augmented process noise vector.

#### 3.1 Sensor Fault Identifiability

The bias estimation approach involves constructing a KBF for the augmented system (16), (17). As in the case of actuator faults, observability of  $(C_\eta, A_\eta)$  is essential for the KBF to function correctly. The following theorem gives necessary and sufficient conditions for observability.

**Theorem 2.** The pair  $(C_\eta, A_\eta)$  is observable iff the following conditions are satisfied:

- (i) the pair  $(C, A)$  is observable, and
- (ii) all zero-frequency modes of  $A$  are observable with respect to the bias-free sensor outputs

**Proof.** Applying the PBH rank test,  $(C_\eta, A_\eta)$  is observable iff

$$\text{rank} \begin{bmatrix} sI - A & 0 \\ 0 & sI_q \\ C_1 & 0 \\ C_2 & I_q \end{bmatrix} = n + q \text{ for } s = \lambda(A) \text{ and } s = 0 \quad (18)$$

The first  $n$  columns of the PBH test matrix are linearly independent  $\forall s$  iff  $(C, A)$  is observable. For  $s \neq 0$ , the last  $q$  columns are mutually independent as well as independent of the first  $n$  columns. For  $s = 0$ , the rank of the test matrix is  $n + q$  iff

$$\text{rank} \begin{bmatrix} sI - A & 0 \\ C_1 & 0 \\ C_2 & I_q \end{bmatrix}_{s=0} = n + q \quad (19)$$

Since elementary column operations do not change the column rank,

$$\begin{aligned} \text{rank} \begin{bmatrix} sI - A & 0 \\ C_1 & 0 \\ C_2 & I_q \end{bmatrix}_{s=0} &= \text{rank} \left\{ \begin{bmatrix} sI - A & 0 \\ C_1 & 0 \\ C_2 & I_q \end{bmatrix}_{s=0} \begin{bmatrix} I_n & 0 \\ -C_2 & I_q \end{bmatrix} \right\} \\ &= \text{rank} \begin{bmatrix} sI - A & 0 \\ C_1 & 0 \\ 0 & I_q \end{bmatrix}_{s=0} \end{aligned} \quad (20)$$

Thus the rank of the PBH test matrix at  $s = 0$  is  $n + q$  iff the first  $n$  columns are linearly independent for  $s = 0$ , i.e., iff the zero-frequency modes of  $A$  are observable with respect to  $C_1$ . ■

When all sensors have biases, i.e.,  $q = l$ ,  $(C_\eta, A_\eta)$  is observable iff  $A$  has no zero eigenvalue, as stated below.

**Corollary 2.1.** If all sensors have biases,  $(C_\eta, A_\eta)$  is observable iff

- (i)  $(C, A)$  observable, and
- (ii)  $A$  has no zero eigenvalues.

**Remark 3.1.** For the case when all sensors have biases, the condition that  $A$  should not have zero eigenvalues is rather restrictive, since many engineering systems have free integrators in their dynamics. However, there does not appear to be an obvious way of getting around this problem. Consider the effect of using output feedback which moves the eigenvalues away from the origin, i.e.,

$$u = -Gy = -G(Cx + \bar{y}) \quad (21)$$

where  $G \in R^{m \times l}$ , which gives the following closed-loop system (including the augmented state  $\bar{y}$ );

$$\dot{\eta} = \begin{bmatrix} A - BGC & -BG \\ 0 & 0 \end{bmatrix} \eta + w := A'_\eta \eta + w \quad (22)$$

$$y = [C \ I] \eta + w_s \quad (23)$$

The pair  $(C_\eta, A'_\eta)$  is observable iff

$$\text{rank} \begin{bmatrix} sI - A + BGC & BG \\ 0_{l \times n} & sI_l \\ C & I_l \end{bmatrix} = n + l \text{ for } s = \lambda(A - BGC) \text{ and } s = 0 \quad (24)$$

Proceeding as in the proof of Theorem 2, the first  $n$  columns are linearly independent iff  $[C, (A - BGC)]$  is observable. For  $s \neq 0$ , the last  $l$  columns of the PBH test matrix are mutually independent as well as independent of the first  $n$  columns. For  $s = 0$ , since the rank of a matrix is unchanged by multiplication by an elementary matrix,

$$\text{rank} \left\{ \begin{bmatrix} I_n & -BG \\ 0 & I_l \end{bmatrix} \begin{bmatrix} sI - A + BGC & BG \\ C & I_l \end{bmatrix} \right\}_{s=0} = \text{rank} \begin{bmatrix} sI - A & 0 \\ C & I_l \end{bmatrix}_{s=0} \quad (25)$$

The rank is  $n + l$  iff  $A$  has no eigenvalue at the origin. Thus, using feedback to move the eigenvalue away from zero does not make the augmented closed-loop system observable.

**Remark 3.2.** If  $A$  has one or more zero eigenvalues, the unobservable subspace of  $(C_\eta, A_\eta)$  can be readily obtained. Defining  $\Gamma = [0_{q \times (l-q)} \ I_q]^T$ , the unobservable subspace is

$$\bar{\mathcal{O}}_\eta = \mathcal{N} \begin{bmatrix} C & \Gamma \\ CA & 0 \\ CA^2 & 0 \\ \vdots & \vdots \\ CA^{n+q-1} & 0 \end{bmatrix} \quad (26)$$

That is, if  $(x_1^T, x_2^T)^T \in \bar{\mathcal{O}}_\eta$ ,

$$Cx_1 + \Gamma x_2 = 0; \quad [\mathcal{O}]Ax_1 = 0 \quad (27)$$

where  $\mathcal{O} = [C^T, C^T A^T, \dots, C^T A^{n-1T}]^T$ , which yields (since  $(C, A)$  is observable):

$$Ax_1 = 0; \quad C_1 x_1 = 0; \quad C_2 x_1 + x_2 = 0 \quad (28)$$

That is,  $x_1$  must be an eigenvector of  $A$  corresponding to a zero-frequency mode of  $A$  that is unobservable with respect to  $C_1$ . If all zero-frequency modes of  $A$  are observable with respect to  $C_1$ , then  $x_1 = 0$ , therefore,  $x_2 = 0$ ,  $\bar{\mathcal{O}}_\eta = 0$ , and the augmented system is observable, which is consistent with Theorem 2. If some zero-frequency modes of  $A$  are not observable with respect to  $C_1$ ,  $\bar{\mathcal{O}}_\eta$  can be characterized as

$$\bar{\mathcal{O}}_\eta = \left\{ \begin{bmatrix} x_1 \\ -C_2 x_1 \end{bmatrix}, \ x_1 = \text{eigenvector of } A \text{ corresponding to unobservable 0-freq modes} \right\} \quad (29)$$

**Remark 3.3** In practice, the Kalman filter is implemented in a discrete-time setting, and the condition (ii) in Theorem 2 changes to: “all modes corresponding to  $\lambda(A) = 1$  are observable with respect to the bias-free sensor outputs” (for the discretized version of  $A$ ).

## 4 Simultaneous Actuator Faults and Sensor Bias

For the case with actuator fault pattern  $k$ , if  $q$  of the sensors have biases, the augmented system is given by

$$\frac{d}{dt} \begin{bmatrix} x \\ \bar{u}^k \\ \bar{y}_2 \end{bmatrix} := \dot{\varphi} = \begin{bmatrix} A & \bar{B}^k & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \varphi + \begin{bmatrix} B^k \\ 0 \\ 0 \end{bmatrix} u^k + w'' \quad (30)$$

$$:= A_\varphi \varphi + B_\varphi^k u^k + w'' \quad (31)$$

$$y = \begin{bmatrix} C_1 & 0 & 0 \\ C_2 & 0 & I_q \end{bmatrix} \varphi + w_s = C_\varphi \varphi + w_s \quad (32)$$

where  $w''$  is the augmented process noise vector. The following theorem gives necessary and sufficient conditions for observability.

**Theorem 3.** The pair  $(C_\varphi, A_\varphi)$  is observable iff

- (i)  $l \geq m_k + q$
- (ii) the pair  $(C, A)$  is observable, and
- (iii) the system  $(C_1, A, \bar{B}^k)$  has no zeros at the origin.

**Proof.** Applying the PBH rank test,  $(C_\varphi, A_\varphi)$  is observable iff

$$\text{rank} \begin{bmatrix} sI - A & -\bar{B}^k & 0 \\ 0 & sI_{m_k} & 0 \\ 0 & 0 & sI_q \\ C_1 & 0 & 0 \\ C_2 & 0 & I_q \end{bmatrix} = n + m_k + q \text{ for } s = \lambda(A) \text{ and } s = 0. \quad (33)$$

The first  $n$  columns of the PBH test matrix are linearly independent for all  $s$  iff  $(C, A)$  is observable. For  $s \neq 0$ , the last  $m_k + q$  columns are mutually independent as well as independent of the first  $n$  columns. For  $s = 0$ , the columns of the test matrix are linearly independent iff the columns of the following  $(n+l) \times (n+m_k+q)$  matrix (after applying elementary column operations as shown) are linearly independent:

$$\begin{bmatrix} sI - A & -\bar{B}^k & 0 \\ C_1 & 0 & 0 \\ C_2 & 0 & I_q \end{bmatrix}_{s=0} \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_{m_k} & 0 \\ -C_2 & 0 & I_q \end{bmatrix} = \begin{bmatrix} sI - A & -\bar{B}^k & 0 \\ C_1 & 0 & 0 \\ 0 & 0 & I_q \end{bmatrix}_{s=0} \quad (34)$$

The columns of the above matrix are linearly independent iff (i) and (iii) hold. ■

If all sensors have biases ( $q = l$ ), condition (i) cannot be satisfied in the presence of one or more actuator failures and the system is unobservable. This represents a major limitation of this approach to FDI when actuator faults and sensor biases are simultaneously present, and suggests that some alternate techniques should be considered (perhaps for sensor FDI).

## 5 Conclusions

This paper explored in detail a class of fault detection and identification (FDI) methods for bias-type faults in actuators and sensors, from the point of view of fault identifiability. The methods employ a bank of Kalman-Bucy filters (KBFs) to detect the faults, determine the fault pattern, and estimate the fault values. Each KBF is tuned to a different fault pattern. Necessary and sufficient conditions were presented for identifiability of bias-type actuator faults, sensor faults, and simultaneous actuator and sensor faults. The results indicate that caution should be exercised to ensure fault identifiability for different fault patterns when using such methods. It was shown that FDI of simultaneous actuator and sensor faults is not possible using these methods when all sensors have biases.

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