

International Finance Discussion Papers

Number 294

December 1986

AN INTERNATIONAL ARBITRAGE PRICING MODEL
WITH
PPP DEVIATIONS

by

Ross Levine

NOTE: International Finance Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to International Finance Discussion Papers (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author or authors.

ABSTRACT

This paper develops an intertemporal, international asset pricing model for use in applied theoretical and empirical research. An important feature of the model is that it incorporates both stochastic inflation rates and stochastic Purchasing Power Parity deviations (PPP). The model derives the equilibrium real return on assets, and obtains empirically tractable reduced form equations which can be used to examine such issues as capital market segmentation, currency substitution, exchange rate volatility, and the forward exchange market's risk premium. Mechanically, the model begins as a system of stochastic differential equations which describe the dynamic paths of a vector of state variables, prices, and PPP deviations. The state variables' intertemporal development determines the production and credit opportunities, and provides the model's fundamental dynamic nature. The model is shown to be consistent with the domestic-general equilibrium asset pricing models of Cox, Ingersoll, and Ross (1985) and Brock (1982). The model is applied to pricing forward exchange, and an empirically tractable equation of the risk premium is derived which will allow researchers to uncover the risk premium's economic determinants.

**AN INTERNATIONAL ARBITRAGE PRICING MODEL
WITH
PPP DEVIATIONS**

by

Ross Levine*

I. Introduction

This paper develops an intertemporal, international asset pricing model for use in applied theoretical and empirical research. An important feature of the model is that it incorporates both stochastic inflation rates and stochastic Purchasing Power Parity (PPP) deviations. The model derives the equilibrium real return on assets, and obtains empirically tractable reduced form equations which can be used to examine such issues as capital market segmentation, currency substitution, exchange rate volatility, and the risk premium in forward foreign exchange markets. The model is consistent with the domestic-general equilibrium asset pricing models of Cox, Ingersoll, and Ross (1985) and Brock (1982). This paper's international emphasis, however, focuses attention on different issues.

In order to distinguish domestic from international asset pricing paradigms, an economic concept of statehood is necessary. The Ricardian model identifies nations by their technologies, while the Heckscher-Ohlin theory defines countries as areas within which physical factors of production are confined. In monetary models and some asset pricing models (Hodrick (1981),

*The author is a student intern in the International Finance Division of the Board of Governors, and a graduate student at the University of California, Los Angeles. This paper represents the views of the author and should not be interpreted as reflecting those of the Board of Governors of the Federal Reserve System or other members of its staff. I have benefited from valuable discussions with Maria Carkovic, Michael Darby, Sebastian Edwards, Mark Grinblatt, Arnold Harberger, Giovanna Mossetti, and Guido Tabellini.

Kouri (1979), Fama and Farber (1979), and Grauer, Litzenberger, and Stehle (1976)), nations are defined as economic units holding the same currency as a means of payment.

This paper follows Solnik (1974) and defines a country as a place where a subset of agents uses the same price index to deflate nominal returns. Nations are delineated by deviations from PPP which cause residents to evaluate differently the real return on the same asset. PPP deviations may exist because of differences in consumption tastes, or from differences in the price of individual commodities arising from transactions costs, storage costs, tariffs, etc. "This heterogeneity in individuals' evaluation of returns plays havoc with the standard Separation, Aggregation, and Asset Pricing results of Portfolio Theory."¹ The majority of this paper will be devoted to resolving the problem of portfolio choice when investors' real returns differ.

Resolving the problem of portfolio choice and asset pricing when investors' evaluations of real returns differ internationally is a necessary first step towards a truly international theory of finance. Solnik (1974) takes this first step. In a model with PPP deviations, no inflation, and no correlation between PPP deviations and local real asset returns, Solnik derives an international capital asset pricing model. This model is extended by Sercu (1980) to permit correlation between exchange rates and local real asset returns, and by Kouri and de Macedo (1978) to allow for non-stochastic inflation. Each of these models is an extension of Merton's (1973) intertemporal CAPM.

¹Adler and Dumas (1983, p. 926). Much of this introduction owes its origin to their survey of the international finance literature.

This paper further extends Solnik's (1974) model by allowing stochastic inflation rates, stochastic PPP deviations, and correlation between PPP deviations and real asset returns. In order to accomplish this objective, I follow Ross and Walsh (1983) in extending Ross' (1976, 1977) arbitrage pricing theory (APT) to an international environment with PPP deviations.² This new technique is necessary because, given this paper's extensions, international capital asset pricing models (ICAPM) cannot price assets without severe restrictions on utility functions and correlation matrices; and because, even granting those restrictions, the ICAPMs are empirically intractable.³

The arbitrage pricing approach alone, however, is insufficient because it does not yield an optimal portfolio equation, which is necessary for many extensions of the model. The arbitrage asset pricing approach must be applied in conjunction with the representative investor utility maximization approach to obtain both equilibrium asset prices and optimal portfolio shares.

The model's structural equations are discussed in Section II. Section III uses these equations to derive an international arbitrage pricing model (IAPM). Section IV rigorously analyzes the price of systematic risk derived in Section III. In the process, we obtain the optimal portfolio for a representative individual. This is important for extensions of the model. Section V highlights the dynamic nature of the IAPM and describes how to implement empirically the model. The forward exchange market's risk premium

²Solnik (1983) extends Ross' APT to an international setting where PPP holds. This model is estimated by Cho, Eun, and Senbet (1986).

³These models are empirically suspect because (1) it would be difficult to construct portfolios which mimic the world state variable; (2) Roll's (1977) critique of the CAPM applies to the ICAPM; and (3) even if a world market portfolio could be constructed and state variable portfolios assembled, Stulz (1981b) shows that in a world with differential tax structures the world market portfolio is inefficient for all individuals.

is derived in Section VI. A brief summary is presented in Section VII.

II. The Model

The world economy consists of n countries each with its own currency. Every investor is able to issue and purchase assets in any country through freely floating international exchange markets. It is assumed that each country, for example country i , has N_i assets, where $n \ll \sum_{i=1}^n N_i = N$.

Although all goods are available in each country, residents of country i can consume only goods purchased in country i . The existence of differential transactions costs, storage costs, tariffs, tastes, etc., make the relative price of representative consumption baskets differ internationally. Given these deviations from Purchasing Power Parity, one cannot trade a basket of goods in country i for an identical basket in country j at a 1:1 ratio. Consequently, the real return on any asset will depend upon the investor's nationality.

It is assumed that there exists a $K \times 1$ vector of state variables, θ , which describes the state of the world and provides the fundamental dynamic relations of the model. I assume that the state vector, θ , follows a continuous time vector Markov process of the Ito type,

$$(1) \quad d\theta = \mu_{\theta}(\theta(t), t)dt + \Sigma_{\theta}(\theta(t), t)dZ(t)$$

where throughout this paper, $\mu_{\ell}(\theta(t), t)$, represents the expected rate of change in variable ℓ at time t , given that the state vector is $\theta(t)$.

$\Sigma_{\theta}(\theta(t), t)$ is a $K \times K$ diagonal matrix of instantaneous standard deviations.

The main diagonal consists of: $\sigma_1(\theta(t), t), \dots, \sigma_K(\theta(t), t)$; where $\sigma_s(\theta(t), t)$

is state variable s 's instantaneous standard deviation at time t when the

state of the world is described by $\theta(t)$. The $dZ(t)$ variables are

correlated Wiener Processes having zero mean and unit variance per unit of

time.⁴ Thus, the unanticipated change in state variable s at time t is equal to $\sigma_s(\theta(t),t)dz_s(t)$.

The intertemporal development of the state variables determines the production and credit opportunities available to the economy. They include such variables as technological growth, the money supply, institutional arrangements, tax and tariff structures, the price of intermediate inputs, etc. The state variables provide the dynamic nature of the model. More specifically, the probability distribution of important variables such as aggregate price levels, exchange rates, and deviations from PPP, depends on the current level of the state variables, θ , which are themselves changing randomly over time.

Remembering the notational convention described above the remaining structural equations may be specified.

Each country's inflation rate is assumed to follow an Ito-type continuous time Markov process.

$$(2) \quad \frac{dP_i}{P_i} = \mu_{P_i}(\theta(t),t)dt + \sum_{s=1}^K b_{P_i}^s(\theta(t),t) \sigma_s(\theta(t),t)dz_s(t) \quad i = 1, \dots, n$$

where P_i is country i 's domestic price level, and where throughout $b_{P_i}^s(\theta(t),t)$ quantifies the sensitivity of variable P_i 's rate of change to unanticipated movements in state variable s at time t , when the state of the world is $\theta(t)$. Note that the expected change in country i 's domestic price level at time t , $\mu_{P_i}(\theta(t),t)$, is made conditional on all information

⁴A real valued function z on $[t,t']$ is a Wiener process if: (i) z is a continuous process with independent increments, and (ii) $z(t') - z(t)$ has a normal distribution with mean zero and variance $t' - t$. For a more rigorous definition of Wiener processes and their applications in economics and finance see Malliaris and Brock (1982).

available up until time t .

The instantaneous nominal return on the N nominally risky assets is⁵

$$(3) \quad \frac{dQ_i}{Q_i} = \mu_{Q_i}(\theta(t), t) dt + \sum_{s=1}^K b_{Q_i}^s(\theta(t), t) \sigma_s(\theta(t), t) dz_s(t) \quad i = 1, \dots, N$$

where Q_i is the nominal price of asset i in home currency terms.

The nominal exchange rate is assumed to reflect not only relative price differences but PPP deviations as well:

$$(4) \quad S_{ij} = \frac{P_i}{P_j} D_{ij},$$

where S_{ij} is the amount of currency i exchangeable for a unit of currency j on the spot exchange market, and D_{ij} represents PPP deviations between countries i and j .

There are three important stylized facts about PPP deviations which should be incorporated into an international model. First, PPP deviations are the rule rather than the exception. Second, PPP deviations between, for example, the U.S. and France are different from PPP deviations between the U.S. and Japan. Third, although Darby (1983) finds a significant MA term, much empirical work (Adler and Dumas (1983)) suggests that PPP deviations are not statistically very different from a martingale.

This paper recognizes the empirical evidence and models PPP deviations accordingly. PPP deviations are modeled as a sub-system of stochastic differential equations.

$$(5) \quad \frac{dD_{ij}}{D_{ij}} = \mu_{D_{ij}}(\theta(t), t) + \sum_{s=1}^K b_{D_{ij}}^s(\theta(t), t) \sigma_s(\theta(t), t) dz_s(t)$$

⁵Note that a nominally risk free bond is also representable within equation (3). Simply let the uncertain component of dQ/Q equal zero for the nominally risk free return, i.e., set the b 's equal to zero.

Note that $dD_{ii}/D_{ii} = 0$ by definition, and each country has a different stochastic PPP relationship with each other country. The expected change in PPP deviations is not necessarily zero. Thus, while a martingale is not inconsistent with the above formulation, I do not restrict PPP deviations to be a random walk.

Assuming identical consumption bundles, D_{ij} can be interpreted as the real exchange rate -- by which I mean the relative cost of the same bundle of goods in two different countries, e.g., let $i = \text{Germany}$ and $j = \text{U.S.}$, so that $S_{j,j} = \text{D.M./\$}$, $P_i = \text{D.M./bundle(Germany)}$, $P_j = \text{\$/bundle(U.S)}$ then $D_{ij} = \text{bundle(Germany)/bundle(U.S.)}$. The real exchange rate may be different if a different representative bundle of goods is chosen.⁶

This model is partial equilibrium in the sense that unspecified state variables provide the fundamental dynamic relations of the model. It is not general equilibrium in the sense of Arrow-Debreu because technological sources of uncertainty are not explicitly related to the equilibrium prices. Cox, Ingersoll, and Roll (1985) (CIR), and Brock (1982) construct general equilibrium asset pricing models in a domestic setting. Production possibilities are explicitly modelled as a set of linear stochastic activities where it is these direct technological shocks that ultimately induce stochastic contingent claim prices. They, however, construct their models in a completely real setting with no aggregate price level. Since the model used in this paper incorporates stochastic inflation rates in each of n countries, a money market and aggregate price level would have to be added to the CIR and Brock models: a non-trivial task left for future work.

⁶The assumption of identical consumption bundles is made for expositional purposes. It plays no role in this paper.

It is not necessary, however, to model explicitly the microeconomic components of risk for the framework to be consistent with a general equilibrium model. In order to be consistent with general equilibrium, prices must be endogenously determined through the equilibrium of supply and demand. Since all random shocks are captured as elements of the state vector, θ , and assuming asset supply and demand schedules are functions of the state variables which follow Ito processes, then the resulting equilibrium prices will also follow Ito processes.⁷ Thus, the economic model presented is consistent with endogenously determined prices.

III. An International Arbitrage Pricing Model with PPP Deviations

This section follows Ross and Walsh (1983) and applies the APT developed by Ross (1976, 1977) to the model presented in Section II in order to price internationally traded assets in a world with PPP deviations.

This section is divided into three subsections. The first subsection demonstrates that the framework presented in Section II is consistent with real asset returns following a linear return generating process, which is a necessary component in any APT approach. The second subsection derives an intertemporal, international asset pricing equation using the arbitrage pricing approach. This equation specifies that the real return on asset i evaluated in country j , for example, is a linear combination of asset i 's sensitivity to unanticipated movements in the state variables. The third subsection gives an intuitive explanation behind the "pricing" of an asset's sensitivity to specific shocks.

⁷This section borrows heavily from Breeden (1979) and Richard (1979).

III.A The Linear Return Generating Process

To facilitate exposition, let:

$$\delta_s = \sigma_s(\theta(t), t) dz_s(t) \quad s = 1, \dots, K$$

$$b_j^s = b_j^s(\theta(t), t) \quad \text{for all } s \text{ and } j$$

and

$$\mu_j = \mu_j(\theta(t), t)$$

It should be remembered, however, that (1) b_j^s is a time varying, state dependent parameter, and (2) δ_s is the unanticipated movement in state variable s , and is distributed normally with mean zero, and variance $\sigma_s(\theta(t), t) dt$.

Equation (3) then becomes

$$(3') \quad \frac{dQ_i}{Q_i} = \mu_{Q_i} dt + \sum_{s=1}^k b_{Q_i}^s \delta_s$$

For simplicity, assume that asset i 's home country is country i . Then, the instantaneous real return on asset i in country j is (after using Ito's Lemma and collecting terms)

$$(6) \quad \tilde{\alpha}_j^i = \alpha_j^i + \sum_{s=1}^k B_s^{i,j} \delta_s$$

where $\tilde{\alpha}_j^i$ is the real return of asset i in country j , α_j^i is the expected real return of asset i in country j , $B_s^{i,j} (= b_{Q_i}^s - b_{P_i}^s - b_{D_{ij}}^s)$ is the sensitivity of the real return of asset in country j to an unanticipated movement in the state variable s . (See Appendix A for the derivation of equation (6)).

Note that $B_s^{i,j}$ consists of three components. The first component, $b_{Q_i}^s$, signifies the sensitivity of the nominal return of asset i to changes in state variable s . The second component, $b_{P_i}^s$ represents the sensitivity of

asset i 's home country's price level to changes in state variable s . These two components are the same regardless of the country in which final evaluation is to occur. The third term, $b_{D_{ij}}^s$ indicates the sensitivity of purchasing power parity deviations between asset i 's home country and country j , the country in which the real return of asset i is being evaluated. This term is what causes the real return of assets to be evaluated differently internationally. Models in which PPP is assumed to hold are special cases of the above formulation (see Hodrick (1981), Kouri (1979)).

The real return on any asset computed by any country is thus seen to follow a linear return generating model. This satisfies the quintessential assumption of the APT. The other assumptions necessary to use the arbitrage pricing approach are perfectly competitive capital markets, frictionless asset markets, and that the number of stochastic state variables affecting asset returns is relatively small, i.e., $K \ll N$.⁸ In the parlance of the APT literature, there are only a few systematic components of risk existing in nature, $\delta_1, \dots, \delta_K$; these are called systematic risk factors.

III.B The IAPM

Given these assumptions, it is possible to derive an international arbitrage pricing model with PPP deviations. Following Ross (1976) and Roll and Ross (1980), consider an individual in country j who is considering a change in his portfolio. Let x_j^i ($i = 1, \dots, N$) equal the amount purchased or sold of asset i as a fraction of total real wealth, where "real" is defined in country j terms. Since purchases of assets must be financed by sales of

⁸There is an entire literature on the APT's "necessary" conditions. See Ross (1976) and especially Ross (1977) for the original presentation, and see Dybvig and Ross (1985) for a recent list of relevant citations.

other assets, the new portfolio must have the same real value as the old portfolio in equilibrium. Thus,

$$\sum_{i=1}^N x_j^i = 0$$

Portfolios that use no wealth such as $X_j = (x_j^1, \dots, x_j^N)$ are called arbitrage portfolios.

Consider an arbitrage portfolio, X_j , chosen to have no systematic risk, i.e., unanticipated changes in the state variables will not affect the real return to residents of country j holding X_j . Formally this is written:

$$\sum_{i=1}^N B_s^{i,j} X_j^i = 0 \quad \text{for all } s$$

This arbitrage portfolio, X_j , will have real return to residents of country j ,

$$X_j' \tilde{\alpha}_j = X_j' \alpha_j + X_j' B_1^j \delta_1 + \dots + X_j' B_K^j \delta_K = X_j' \alpha_j,$$

where

$$\tilde{\alpha}_j = \begin{vmatrix} \tilde{\alpha}_j \\ \vdots \\ \alpha_j^N \end{vmatrix}, \quad \alpha_j = \begin{vmatrix} \alpha_j^1 \\ \vdots \\ \alpha_j^N \end{vmatrix}, \quad B_s^j = \begin{vmatrix} B_s^{1,j} \\ \vdots \\ B_s^{N,j} \end{vmatrix}$$

Notice that X_j is a special arbitrage portfolio: it has no risk and uses no wealth. Therefore, the real return on X_j must equal zero in equilibrium or, a riskless and costless pump of consumption goods would exist!

Using simple linear algebra we can now solve for the equation of expected real asset returns from the vantage point of a country j resident. Any vector, X_j , which is orthogonal to the constant vector and to each of the coefficient vectors, B_1^j, \dots, B_K^j , must also be orthogonal to the vector of expected returns. The algebraic consequence of this statement is that the

expected return vector, α_j , must be a linear combination of the constant vector and the B^j vectors. Thus, there exist $K + 1$ weights such that

$$(7) \quad \alpha_j = \lambda_0^j + \lambda_1^j B_1^j + \dots + \lambda_K^j B_K^j$$

with the understanding that if a riskless real asset exists for residents of country j it would have real return λ_0^j , with an expected value of $\alpha_j^0 = \lambda_0^j$.

For asset i in particular:

$$(8) \quad \alpha_j^i = \alpha_j^0 + \lambda_1^j B_1^{i,j} + \dots + \lambda_K^j B_K^{i,j}$$

Intuitively, equations (6) and (8) are easily explained. Equation (6) expresses the notion that the real return on asset i evaluated in country j is equal to its expected value plus the effect of random shocks. The magnitude of these unanticipated shocks is measured by $\delta = (\delta_1, \dots, \delta_K)$ and the sensitivity of the real return of asset i in j is captured by $B^{i,j} = (B_1^{i,j}, \dots, B_K^{i,j})$. Due to stochastic PPP deviations these sensitivities depend on the asset's home country and the country in which the real return of that asset is being evaluated. For example, unanticipated changes in the U.S. money supply may affect the real return on U.S. securities evaluated in Germany differently from the real return evaluated in Italy. Equation (8) states that the expected real return of an asset will depend upon the sensitivity of that asset's real return to shocks. That is, a risky asset -- one whose real return is expected to be very sensitive to unexpected changes in fundamental state variables -- would also be expected to have a compensating higher real rate of return than a less risky asset.

The effect on the expected real return of an asset due to that asset's exposure to unanticipated shocks is captured by the λ 's. These λ 's are called factor risk premiums. They indicate the market price of any asset's sensitivity to various shocks. An asset's sensitivity to these common shocks

is called the asset's systematic risk, and is signified by B_1^j, \dots, B_K^j . The relevant shocks in the economy are $\delta_1, \dots, \delta_K$ and are called systematic risk factors.

III.C Factor Risk Premia: Intuition

To see why the factor risk premiums are the market price of systematic risk consider a world with only one factor. The international arbitrage pricing model derived above tells us that

$$\alpha_j^i - \alpha_j^o = \lambda_1^j B_1^{i,j}$$

The expected excess real return on asset i in country j above the country j risk-free rate depends upon the exposure of asset i in j to the lone risk factor times a constant, λ_1^j -- constant in the sense of not changing across assets evaluated in country j . The factor risk premiums must be equal across assets to rule out riskless arbitrage profits. This constant, λ_1^j , is the market price of a unit of systematic risk. That is, a unit increment in asset i 's systematic risk increases the expected real return of asset i in j by λ_1^j .

This unifactor example can be extended by forming portfolios with unit systematic risk on each factor and no risk on other factors. Each λ may then be interpreted as

$$(9) \quad \lambda_s^j = \alpha_j^s - \alpha_j^o \quad s = 1, \dots, K$$

which is the excess real return, or factor risk premium, on portfolios with only systematic risk factor s . Equation (8) can then be rewritten as

$$(10) \quad \alpha_j^i - \alpha_j^o = (\alpha_j^1 - \alpha_j^o) B_1^{i,j} + \dots + (\alpha_j^K - \alpha_j^o) B_K^{i,j}.$$

This section derived an intertemporal, international arbitrage pricing model with PPP deviations. Equation (8) is the intertemporal arbitrage

pricing equation. It indicates that the expected real return on asset i evaluated in country j is a linear function of asset i 's exposure to a set of systematic risk factors. These exposures, or sensitivities, are then weighted by the factor risk premiums to yield the expected real return of asset i in j .

IV. The Optimal Portfolio and the Risk Factor Premiums

This section has two related objectives. The first is to derive the factor risk premium's economic determinants. This bolsters the economic foundation of the IAPM, and enhances its empirical usefulness. The section's second objective is to solve for the optimal portfolio of country j 's representative individual. This is important because it allows the IAPM to be extended to such issues as currency substitution and exchange rate volatility.

It is important to note that the arbitrage approach alone cannot be used to obtain an optimal portfolio equation. Therefore, we must hypothesize a representative individual and solve for his optimal portfolio. This utility maximization approach, however, is also insufficient because assets cannot easily be priced within the context of the above model. The two approaches used together are necessary to derive a meaningful optimal portfolio equation.

IV.A The Individual Investor Choice Problem

It is assumed that each investor chooses that consumption and portfolio strategy that maximizes the expected value of utility over an infinite time horizon. The instantaneous utility function is assumed to be a strictly concave function of consumption. The choice problem for the representative investor of country j is

$$(11) \quad \text{Max}_{\{C_j, q_j\}} E_0 \left[\int_0^{\infty} e^{-\rho t} U^j(C_j(t)) dt \right]$$

s.t.

$$(12a) \quad dW_j = W_j q_j' \alpha_j dt - C_j dt + W_j q_j' \psi_j dZ$$

$$(12b) \quad q_j' \cdot \underline{1} = 1,$$

where: W_j is the real wealth held by the representative individual of country j .

q_j is the vector of representative investor j 's invested wealth shares, i.e., the first component of q_j is q_j^1 and it signifies the optimal proportion of j 's wealth invested in asset 1.

C_j is j 's instantaneous real consumption

ψ_j is the matrix containing country j 's real asset return sensitivities to shocks. It is derived in Appendix B.

E_0 is the expectation operator evaluated at $t = 0$.

ρ is a discount factor.

$\underline{1}$ is an $N \times 1$ vector of ones.

The first constraint, equation (12a), specifies the intertemporal budget constraint. The accumulation of real wealth occurs when real capital gains on assets exceed instantaneous real consumption, C_j . It is assumed that no other source of income exists. The second constraint, equation (12b), merely states that the summation of portfolio shares over assets is equal to one.

This utility maximization problem can be solved using stochastic optimal control theory.⁹ For simplicity, I will drop the j subscripts, indicating the home country of the representative individual. The subscript will reappear at important times.

⁹For a mathematical explanation of stochastic optimal control see Fleming and Rishel (1975), Deterministic and Stochastic Optimal Control. For a more heuristic presentation with economic applications see Malliaris and Brock (1982), Stochastic Methods in Economics and Finance.

Let $J(W, \theta)$ represent the value function, i.e., J is the solution to equation (11) for given levels of real wealth and state variables. The fundamental principle of stochastic dynamic programming lets us transform the problem given in equation (11) to

$$0 = \text{Max}_{\{C, q\}} [U(C) - \rho J + J_W [W(q' \alpha) - C] + 1/2 J_{WW} W^2 [q' \Omega q] + J_{W\theta} W [q' \Phi] + G [1 - q' \underline{1}]]$$

where G is the Lagrange multiplier. Ω is the covariance matrix of real asset returns. Φ is the covariance matrix of real asset returns with the K state variables. Ω and Φ are country specific and implicitly have j subscripts. They are defined more rigorously in Appendix B. J_W and J_{WW} are the first and second partial derivatives of the value function with respect to real wealth. $J_{W\theta}$ is a $K \times 1$ vector of second partial derivatives of the value function with respect to real wealth and the K state variables.

The first order conditions for the problem described above are:

$$(13) \quad U_C - J_W = 0$$

$$(14) \quad J_W W \alpha + J_{WW} W^2 \Omega q^* + J_{W\theta} W \Phi - G \underline{1} = 0$$

Equation (13) states that at an optimum consumption will be withdrawn until its marginal utility equals the marginal utility of wealth.

Equation (14) can be solved for the real asset return vector on optimally invested wealth.

$$(15) \quad \alpha = R \Omega q^* + \Phi A + G / W J_W \cdot \underline{1}$$

where: $R = -J_{WW} / J_W$; the coefficient of relative risk aversion

$A = -J_{W\theta} / J_W$; a $k \times 1$ vector

$\underline{1} = N \times 1$ vector of ones

and, q^* = vector of optimally invested shares of real wealth.

Now, form a minimum variance portfolio with shares q^0 , such that it has zero covariance with optimally invested wealth, and zero covariance with the state variables. The real rate of return on this portfolio is

$$\alpha^0 = q^{0'} \alpha = WRq^{0'} \Omega q^* + q^{0'} \Phi A q^* + G/WJ_w q^{0'} \cdot \underline{1}$$

$$\alpha^0 = G/WJ_w$$

Substitute α^0 into equation (15) and reintroduce the j -subscripts to obtain:

$$(15a) \quad \alpha_j - \alpha_j^0 \cdot 1 = R_j \Omega_j q_j^* + \phi_j A_j.$$

Solving for optimal portfolio shares, q_j^* , yields

$$(16) \quad q_j^* = \frac{1}{R_j} \Omega_j^{-1} (\alpha_j - \alpha_j^0) - 1/R_j \Omega_j^{-1} \phi_j A_j$$

Equation (16) expresses the optimal portfolio shares for a representative investor from country j . The expected rates of return in equation (16) can be obtained from an IAPM of Section III, equation (8).¹⁰

IV.B The Factor Risk Premiums

The factor risk premiums may now be obtained from the existing equations. Let q^s be the vector of portfolio weights with unit systematic risk on the s^{th} factor, i.e., q^s is the vector of portfolio shares whose expected rate of return, α^s , depends only on state variable s . Assuming, with some loss of generality but with worthwhile gains in clarity, that the state variables follow independent diffusion processes, the following expression can be derived. (Appendix C derives equation (17) and also solves for the factor risk

¹⁰ Assuming asset returns follow a linear return generating process, market equilibrium implies that the no-arbitrage condition holds. Therefore, the IAPM and the optimal representative investor model are consistent, and the expected real returns generated from the IAPM can be used in the optimal portfolio equation.

premium without assuming the state variables are independent)¹¹

$$(17) \quad \alpha_j^s - \alpha_j^o = (R_j) \left(\frac{dC_j}{d\theta_j C_j} \right) \sigma_s^2$$

where R_j represents investor j 's coefficient of relative risk aversion, $\frac{dC_j}{d\theta_j C_j}$ is the partial elasticity of consumption with respect to the s^{th} state variable, and σ_s^2 is the variance of factor s .

The left hand side of equation (17) is the same as the right hand side of equation (9). Both equations identify the risk factor premium. The equilibrium risk factor premiums are seen to depend positively on the individual's level of relative risk aversion, and on the factor's own variance. The sign of the risk premium can be positive or negative depending on the sign of the partial elasticity of wealth with respect to the relevant state variable. Since the sign of the risk premium, λ , can be reversed by reversing the sign of the state variable, I will assume that λ is always positive.

V. The IAPM's Dynamic Nature and Its Empirical Applicability

This section is divided into two parts. The first subsection describes the intertemporal nature of the international arbitrage pricing model derived in Section III. The second subsection briefly outlines how the IAPM can be empirically implemented.

¹¹This section borrows heavily from Roll and Ross (1980) and Cox, Ingersoll, and Ross (1985). They derive different expressions for λ . This is because they make different assumptions regarding the independence of the state variables. In the text, I follow Roll and Ross while, in Appendix C I follow Cox, Ingersoll and Ross (1985). Both of these papers derive the factor risk premiums in a domestic setting but, the familiar reader will recognize that the extension to the international setting is trivial.

V.A. The Dynamic Nature of the IAPM

Up until now, I have been lax in distinguishing ex ante from ex post perceptions. In order to apply the international arbitrage pricing equation, however, this distinction is important. Therefore a little backtracking and clarification is necessary before explaining how the IAPM can be used.

Given the model specified in Section II and assuming rational expectations, equation (6) describes both the ex ante perceptions of the return generating process and the ex post realization. The real return on an asset is its expected value plus the effect of shocks on the asset's real return.

Equation (8), on the other hand, is an equation describing an asset's expected real rate of return. The B's in equation (8) are expected values; they are the expected sensitivity of the asset's real return to unanticipated movements in the state variables. The expectation of $B_s^{i,j}(t)$ is formed conditional on the expected state of the world at time t . (Recall that $B_s^{i,j} = b_{Q_i}^s(\theta(t), T) - b_{P_i}^s(\theta(t), T) - b_{D_{ij}}^s(\theta(t), t)$).

A more rigorous characterization of "expected state of the world" can be obtained by integrating equation (1)

$$(18) \quad \theta(t) = \theta(0) + \int_0^t \mu_\theta(\theta(v), v) dv + \int_0^t \sum_{\theta} (\theta(v), v) dZ(v),$$

where equation (18) represents the actual state of the world at time t . It obviously includes the unanticipated movements happening the instant before time t .

The expectation of $\theta(t)$ at time $t-h$ is

$$E_{t-h} [\theta(t) / I_{t-h}] = \theta(0) + \int_0^t \mu_\theta(\theta(v), v) dv + \int_0^{t-h} \sum_{\theta} (\theta(v), v) dZ(v)$$

where h is positive, and I_{t-h} represents information available at time $t-h$. The expectation of $B_s^{i,j}(t)$ is formed conditional on an information set

that does not include the realization of shocks between $t-h$ and t , i.e., the expectation of $B_s^{i,j}(t)$ is formed conditional on $E_{t-h}[\theta(t)/I_{t-h}]$ not $\theta(t)$.

These expectations of systematic risk are then weighted by the factor risk premiums to yield the expected real rate of return (equation (8)). As the expectations of systematic risk, $B_s^{i,j}$, vary intertemporally the expected real return on assets will also vary.

In conclusion to this clarifying subsection on the intertemporal behavior of the IAPM, I pose a basic question: Why do expected real returns vary within the context of an intertemporal international arbitrage pricing model? By looking at equation (8) and using Section IV, the answer is clear. The expected real rate of return on an asset will vary through time as the asset's expected exposure to systematic risk varies. For example, as the expected values of $B_1^{1,j}, \dots, B_s^{1,j}$ change, the expected real return on asset 1 in country j will change. Additionally, changes in attitudes toward risk, and perceived alterations in probability distributions governing unanticipated movements in the state variables, i.e., changes in factor risk premiums will also induce changes in the expected real returns on assets. These dynamic considerations must be accounted for in any meaningful econometric work utilizing the IAPM.

V.B. Empirical Application of International APT

In order to estimate an asset's expected real return within the context of the IAPM, one can either specify the systematic risk factors believed to induce risk, or use factor analysis. This subsection will briefly outline the econometric methodology required for using the IAPM when systematic risk

factors are prespecified.¹²

In order to determine which economic state variables -- systematic risk factors -- influence real returns, and in order to construct estimated real returns using the IAPM, it is necessary to:

- (1) choose a set of assets and systematic risk factors, δ 's;
- (2) estimate each asset's real return sensitivity to the chosen systematic risk factors, i.e., collect a time series of B 's for each asset from equation (6);
- (3) estimate equation (8) in each time period, i.e., using the estimated systematic risk coefficients, the B 's, from step (2) as predetermined variables and realized real rates as proxies for expected real rates as the dependent variables, equation (8) may be estimated and the risk factor premia, the λ 's, retrieved.

The results are a time series of systematic risk estimates, B 's, a time series of risk factor premia, λ 's, and hence a time series of estimated expected real rates of return, α 's.

The most difficult econometric step in the above procedure is step 2. Estimating the exposure of each asset's return to the chosen systematic risk factors involves the collection of a time series of B 's from equation (6). Econometrically this entails recovering a series of time-varying parameters, and using this series as the explanatory variables in equation (8). Equation (8) is then estimated in order to determine which systematic risk factors are

¹²It should be emphasized that use of the IAPM is a separate issue from the debate surrounding the APT's testability. The seminal article in this area is Roll and Ross (1980), where they conclude that the APT is testable. After using factor analysis, they find that about five risk factors seem to explain stock price movements in the U.S. For criticisms of the APT and its testability see Shanken (1982) and Dhrymes, Friend, and Gultekin (1984). For a response to these papers, see Dybvig and Ross (1985).

Similarly, the difference between the expected real return of asset i evaluated in country i and the expected real return of asset j evaluated in country i can be expressed as either

$$(22') \quad \alpha_i^i - \alpha_j^j = (\mu_{Q_i} - \mu_{Q_j}) - (\mu_{P_i} + \mu_{P_{ij}} - \mu_{P_j}) + \text{Var}(P_i) \\ - \text{Var}(P_j) + \text{Cov}(P_j, D_{ij}),$$

or

$$(23') \quad \alpha_i^i - \alpha_j^j = \sum_{s=1}^K \{ \lambda_s^i (B_s^{i,i} - B_s^{j,i}) \}.$$

These two sets of equations (equations (22)-(23) and equations (22')-(23')) are needed to derive the two versions of the risk premium.

Substituting equation (22) [(22')] into (21), rearranging, and using equation (19) we obtain

$$(24) \quad \text{RP} = \text{Cov}(P_i, P_j) + \text{Cov}(P_j, D_{ij}) - \text{Cov}(P_i, D_{ij}) - \text{Var}(P_i) - \mu_{D_{ij}} + (\alpha_i^i - \alpha_j^j)$$

$$(24') \quad \text{RP} = \text{Cov}(P_i, P_j) + \text{Cov}(P_j, D_{ij}) - \text{Cov}(P_i, D_{ij}) - \text{Var}(P_i) + (\alpha_i^i - \alpha_j^j).$$

Now we can express the risk premium using the IAPM by substituting (23) [(23')] into (24) [(24')] to get

$$(25) \quad \text{RP} = \text{Cov}(P_i, P_j) + \text{Cov}(P_j, D_{ij}) - \text{Cov}(P_i, D_{ij}) - \text{Var}(P_i) - \mu_{D_{ij}} \\ + (\alpha_i^o - \alpha_j^o) + \sum_{s=1}^K \{ \lambda_s^i B_s^{i,i} - \lambda_s^j B_s^{j,j} \}$$

$$(25') \quad \text{RP} = \text{Cov}(P_i, P_j) + \text{Cov}(P_j, D_{ij}) - \text{Cov}(P_i, D_{ij}) - \text{Var}(P_i) \\ + \sum_{s=1}^K \lambda_s^i \{ B_s^{i,i} - B_s^{j,i} \}.$$

Equations (24), (24'), (25), (25') are all expressions for the forward foreign exchange market's risk premium between countries i and j . Equation (24) states that the risk premium is a function of the expected real return differential evaluated in the home country, the expected change in the real

exchange rate, and four terms which capture the explicitly stochastic nature of the model employed. Equation (24') is the same as equation (24) except that the real return differential is specified using the IAPM developed above. In equation (25), the expected change in the real exchange rate is embodied in the expected real return differential. The expected real return differential in equation (25) is evaluated solely in country i terms, and thus considers expected changes in the real exchange rate. The difference between equation (25) and equation (25') is that the expected real return differential is specified using the IAPM.

Korajczyk (1985) estimates a model that closely resembles equation (24). He, however, does not construct an explicitly stochastic model, and thus, he does not derive an expression for the risk premium that contains the first four terms on the right hand side of equation (24). Korajczyk (1985) assumes that ex ante PPP deviations follow a martingale process. This assumption has two important implications: (1) real asset returns are independent of residence, and (2) the expected change in the real exchange rate is zero. Given these assumptions, Korajczyk (1985) estimates an equation of the form $RP = a + b(\alpha_1^i - \alpha_j^j) + u$. Using three stage least squares, he finds that the expected real return differential is useful in predicting the forward forecast error. This represents an encouraging step toward the conclusion that the forward bias is due to a risk premium rather than market inefficiency.

Although Korajczyk's (1985) paper is a seminal contribution, it can be improved upon in at least two ways. First, given the IAPM developed above, it is possible to test for the risk premium's existence within the context of a specific asset pricing model. Using the IAPM to analyze the risk premium's existence is an improvement over past work because now the risk premium's determinants are fully articulated by a theoretical paradigm explicitly

designed to characterize risk. The second improvement which can be made is to allow for ex ante real exchange rate changes. This extension is presented in Levine (1986).

Estimation of equation (24') or (25') will consider each of these improvements. Using equation (24') has the advantage of being able to distinguish the relative explanatory powers of ex ante real return differences and ex ante real exchange rate movements. It has the disadvantage that real exchange rate movements would then have to be explicitly modeled. Estimation of equation (25') has the advantage that it can be estimated completely within the context of the IAPM. The next step is clear: estimate equation (25') using the empirical techniques outlined in Section V.

VII. Conclusions

This paper has applied techniques developed by Merton (1973) and Ross (1976) to the problem of pricing assets and deriving optimal portfolio shares in a model where investors' real returns differ internationally. I derive an intertemporal, international arbitrage pricing model incorporating both stochastic PPP deviations and stochastic inflation rates. Imposing (1) the no-arbitrage condition; and (2) the condition that the supply and demand for assets are always equal, this paper obtains equilibrium asset prices and optimal portfolio shares.

The IAPM relaxes more assumptions than previous international asset pricing models, e.g., stochastic PPP deviations, stochastic, imperfectly correlated inflation rates, and time-varying covariances are permitted without concomitant restrictions on agents utility functions. Furthermore, the IAPM is empirically tractable. Because of these features, the model can form the basis of applied theoretical and empirical research.

This paper applies the model to the forward exchange market's risk premium. The lack of an empirically tractable model of the time-varying risk premium has hindered empirical inquiry into the sources of intertemporal variation between forward and corresponding future spot exchange rates. Section VI derives an equation of the risk premium that in future work will allow investigators to discover the risk premium's economic determinants.

Appendix A

This Appendix provides additional steps in moving from equation (3') to equation (6).

Recall that equation (3'), the nominal return on asset i in country j (assuming i is its home country), is

$$(A1) \quad \frac{dQ_i}{Q_i} = \mu_{Q_i} dt + \sum_{s=1}^K b_{Q_i}^s \delta_s$$

The instantaneous real return on asset i in country j is $d[Q_i/P_i D_{ij}]/Q_i/P_i D_{ij}$. After using Ito's Lemma this becomes

$$(A2) \quad \frac{d[Q_i/P_i D_{ij}]}{Q_i/P_i D_{ij}} = \frac{dQ_i}{Q_i} - \frac{dP_i}{P_i} - \frac{dD_{ij}}{D_{ij}} - \frac{dQ_i dP_i}{Q_i P_i} \\ - \frac{dQ_i dD_{ij}}{Q_i D_{ij}} + \frac{dP_i dD_{ij}}{P_i D_{ij}} + \frac{dD_{ij}^2}{D_{ij}^2} + \frac{dP_i^2}{P_i^2}$$

Now use equations (2), (3) and (5) to obtain:

$$(A3) \quad \frac{d[Q_i/P_i D_{ij}]}{Q_i/P_i D_{ij}} = \left[\frac{\mu_{Q_i}}{Q_i} - \frac{\mu_{P_i}}{P_i} - \frac{\mu_{D_{ij}}}{D_{ij}} - \text{cov}(Q_i, P_i) \right. \\ \left. + \text{Cov}(P_i, D_{ij}) + \text{Var}(D_{ij}) + \text{Var}(P_i) \right] dt \\ + \left[\sum_{s=1}^K dZ_s \sigma_s (b_{Q_i}^s - b_{P_i}^s - b_{D_{ij}}^s) \right]$$

The following multiplication rules have been used:

$$(dt)^2 = 0$$

$$(dt)(dz_s) = 0$$

$$(dz_s)(dz_r) = \text{correlation between state variable } s \text{ and } r.$$

So that:

$$\text{Cov}(Q_i, P_i) = \left[\sum_{s=1}^k b_{Q_i}^s \sigma_s dZ_s \right] * \left[\sum_{s=1}^k b_{P_i}^s \sigma_s dZ_s \right],$$

and the remaining covariances and variances are similarly defined.

The term in the first pair of brackets in equation (A3) is the expected real return of asset i evaluated in country j . Let this term equal α_j^i .

$$\text{Let } B_s^{i,j} = b_{Q_i}^s - b_{P_i}^s - b_{D_{ij}}^s,$$

$$\text{and } \delta_s = \sigma_s dZ_s.$$

Equation (A3) then becomes

$$(A4) \quad \frac{d[Q_i/P_i D_{ij}]}{Q_i/P_i D_{ij}} = \alpha_j^i + \sum_{s=1}^K B_s^{i,j} \delta_s, \quad \text{Q.E.D.}$$

Appendix B

This Appendix more rigorously defines ψ_j , Ω_j , and ϕ_j .

Since capital gains are the only sources of income, the instantaneous change in real wealth of a country j resident is

$$(B1) \quad dW_j = \sum_{i=1}^N q_j^i (\alpha_j^i dt + \sum_{s=1}^K B_s^{i,j} \sigma_s dZ_s) - C_j dt.$$

Let

$$\psi_j = \begin{vmatrix} B_1^{i,j} \sigma_1 & \dots & B_k^{i,j} \sigma_k \\ \vdots & & \vdots \\ B_1^{N,j} \sigma_1 & \dots & B_k^{N,j} \sigma_k \end{vmatrix}.$$

(B1) then becomes

$$(B2) \quad dW_j = W_j q_j^i \alpha_j^i dt - C_j dt + W_j q_j^i \psi_j dZ$$

where ψ_j is the $N \times K$ matrix containing the sensitivity of each asset's real return to changes in the fundamental state variables according to a resident of country j .

After maximizing utility subject to the given constraints, the following matrices are constructed:

Ω_j is an $N \times N$ matrix of real asset return covariances according to residents of j .

Φ_j is an $N \times K$ matrix of covariances between real asset returns and the K state variables.

The $(\ell, m)^{th}$ element of Ω_j is the covariance between the real return on asset ℓ and the real return on asset m , and is equal to

$$\Omega_j(\ell, m) = \left[\sum_{s=1}^K B_s^{\ell, j} \sigma_s dZ_s \right] * \left[\sum_{s=1}^K B_s^{m, j} \sigma_s dZ_s \right].$$

The (a, x) element of Φ_j is the covariance between the real return on asset a and the state variable x , and is represented by

$$\Phi_j(a, x) = \left[\sum_{s=1}^K B_s^{a, j} \sigma_s dZ_s \right] * \sigma_x dZ_x.$$

Appendix C

This Appendix derives the factor risk premium for a representative individual from country j , equation (16), assuming that the state variables follow (1) independent diffusion processes, and (2) correlated diffusion paths.

Consider first the simple case when the diffusion processes are independent. Equation (15a) is rewritten:

$$(C1) \quad \alpha_j - \alpha_j^0 \cdot 1 = R_j \Omega_j q_j^* + \Phi_j A_j.$$

Now, form a (hypothetical) portfolio whose expected real rate of return, α_j^s , depends only on state variable s , and has unit systematic risk with state variable s . Multiply (C1) by q_j^s , which represents this hypothetical portfolio's vector of asset shares, and replace A , to obtain:

$$(C2) \quad \alpha_j^s - \alpha_j^o = \frac{J_{w\theta}}{J_w} \sigma_s^2,$$

where $\sigma_s^2 = q_j^s \phi_j$ because this portfolio only varies with the s state variable.

Using the implicit function theorem on equation (13), we know that

$$\begin{aligned} \frac{J_{w\theta}}{J_w} &= \frac{-\mu_{cc} \frac{dC}{d\theta_s}}{\mu_c} \\ &= \left(\frac{-\mu_{cc} \cdot C}{\mu_c} \right) \frac{dC}{d\theta_s(C)}. \end{aligned}$$

From Bellman's Principle of Optimality, we know that $\left(\frac{-\mu_{cc} \cdot C}{\mu_c} \right) = \left(\frac{-J_{ww} \cdot W}{J_w} \right) =$ investors' coefficient of relative risk aversion. Thus, equation (C2) can be rewritten as

$$(C3) \quad \alpha_j^s - \alpha_j^o = (R) \left(\frac{\partial C}{\partial \theta_s(C)} \right) \sigma_s^2 \quad \text{Q.E.D.}$$

The second part of this Appendix derives the factor risk premium in the case of covarying state variables. This section follows directly from Cox, Ingersoll, and Ross (1985), and is merely included here for completeness.

The hypothetical portfolio, q_j^s , is now more accurately interpreted as a portfolio whose excess expected real return has only the risk of factor s but, the s^{th} factor covaries with the other state variables. The factor risk premium then becomes

$$\alpha_j^s - \alpha_j^o = \left[\left(\frac{-J_{ww}}{J_w} \right) (\text{cov}(W^*, \theta_s)) + \sum_{x=1}^K \left(\frac{-J_{w\theta_x}}{J_w} \right) (\text{cov}(\theta_x, \theta_s)) \right],$$

where W^* is optimally invested wealth. Note that if the conditions of the first part of this Appendix are imposed, the first term in brackets disappears, and of the K following terms only $\frac{-J_{w\theta_s}}{J_w} \text{cov}(\theta_s, \theta_s)$ would remain. For a more detailed explanation see Cox, Ingersoll, and Ross (1985) and Roll and Ross (1980).

BIBLIOGRAPHY

- Adler, M., and B. Dumas, "International Portfolio Choice and Corporation Finance: A Synthesis," Journal of Finance 38 (June 1983), 925-84.
- Bilson, J.F.O., "The Speculative Efficiency Hypothesis," Journal of Business 54 (July 1981), 435-51.
- Breedon, D.T., "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," Journal of Financial Economics 7 (September 1979), 265-96.
- Brock, W.A., "Asset Prices in a Production Economy," in The Economies of Information and Uncertainty, ed. J.J. McCall. Chicago: University of Chicago Press, 1982.
- Chen, N., R. Roll, and S.A. Ross, "Economic Forces and the Stock Market," University of Chicago Working Paper (1985).
- Cho, D., "On Testing the Arbitrage Pricing Theory," Journal of Finance 39 (December 1984), 1485-1502.
- _____, C. Eun, and L.W. Senbet, "International Arbitrage Pricing Theory: An Empirical Investigation," Journal of Finance 41 (June 1986), 313-30.
- Cornell, B., "Spot Rates, Forward Rates and Exchange Market Efficiency," Journal of Financial Economics 5 (August 1977), 55-65.
- Cox, J.C., J. Ingersoll, and S. Ross, "An Intertemporal General Equilibrium Model of Asset Prices," Econometrica 53 (March 1985), 363-84.
- Cumby, R., and M. Obstfeld, "A Note on Exchange-Rate Expectations and Nominal Interest Differentials," Journal of Finance 36 (June 1981), 697-704.
- Darby, M.R., "Movements in Purchasing Power Parity: The Short and Long Runs," in M.R. Darby et al., The International Transmission of Inflation, NBER Monograph Series, Chicago: University of Chicago Press, 1983.

- Dhrymes, P., I. Friend, and B. Gultekin, "A Critical Reexamination of the Empirical Evidence on the Arbitrage Pricing Theory," Journal of Finance 39 (June 1984), 323-46.
- Domowitz, I., and C.S. Hakkio, "Conditional Variance Exchange Market," Journal of International Economics, 19 (1985), 47-66.
- Dybvig, P., and S. Ross, "Yes, the APT is Testable," Journal of Finance 15 (September 1985), 1173-188.
- Fama, E.F., "Forward and Spot Exchange Rates," Journal of Monetary Economics (November 1984), 319-38.
- _____, and A. Farber, "Money, Bonds, and Foreign Exchange," American Economic Review 67 (September 1979), 639-49.
- Fleming, W.H., and R.W. Rishel, Deterministic and Stochastic Optimal Control, New York: Springer-Verlag, 1975.
- Grauer, F., R. Litzberger, and R. Stehle, "Sharing Rules and Equilibrium in an International Capital Market under Uncertainty," Journal of Financial Economics 3 (June 1976), 233-56.
- Hansen, L.P., and R.J. Hodrick, "Risk Averse Speculation in the Forward Foreign Exchange Market: An Econometric Analysis of Linear Models," in Exchange Rates and International Macroeconomics, ed. J.A. Frenkel. Chicago: University of Chicago Press, 1983.
- _____, and R.J. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," Journal of Political Economy, 88 (October 1980), 829-53.
- Hodrick, R., "International Asset Pricing with Time-Varying Risk Premia," Journal of International Economics 11 (November 1981), 573-87.
- _____, and S. Srivastava, "An Investigation of Risk and Return in Forward Foreign Exchange," Journal of International Money and Finance 3 (April

- 1984), 5-29.
- Korajczyk, R.A., "The Pricing of Forward Contracts for Foreign Exchange," Journal of Political Economy 93 (April 1985), 346-68.
- Kouri, P., "International Investment and Interest Rate Linkages under Flexible Exchange Rates," in The Political Economy of Monetary Reform, ed. R. Aliber. London: Macmillan & Co., 1977.
- _____ and J.B. de Macedo, "Exchange Rates and the International Adjustment Process," Brookings Papers (1:1978), 111-50.
- Levine, R., "The Pricing of Forward Foreign Exchange Rates," unpublished ms., UCLA (October 1986).
- Los, L.A., "Econometrics of Models with Evolutionary Parameter Structures," unpublished Ph.D. dissertation, Columbia University (1984).
- _____, "Regression Analysis, and Diagnosis by an Evolutionary Parameter Estimator," Journal of the American Statistical Association (forthcoming 1986).
- _____ and C. McC. Kell, "Computer Programs for the Estimation of Economic Models with Evolutionary Parameter Structures," Federal Reserve Bank of New York Version 4.0 (December 1985).
- Lucas, R.E., "Interest Rates and Currency Prices in a Two-Country World," Journal of Monetary Economics 10 (1982), 335-60.
- Malliaris, A.G., and W.A. Brock, Stochastic Methods in Economics and Finance, New York: North-Holland Press, 1982.
- Merton, R.C., "An Intertemporal Capital Asset Pricing Model," Econometrica 41 (September 1973), 867-87.
- Richard, S.F., "A Generalized Capital Asset Pricing Model," TIMS Studies in the Management Sciences 11 (1979), 215-32.

Roll, R., "A Critique of the Asset Pricing Theory's Test; Part 1: On Past and Potential Testability of the Theory," Journal of Financial Economics 4 (March 1977), 129-76.

_____, "Violations of Purchasing Power Parity and Their Implications for Efficient International Commodity Markets," in M. Sarnat and G. Szego, eds., International Finance and Trade, vol. 1, Cambridge, MA: Ballinger, 1979.

_____ and S. Ross, "An Empirical Investigation of the Arbitrage Pricing Theory," Journal of Finance 35 (December 1980), 1073-103.

Ross, S.A., "The Arbitrage Theory of Capital Asset Pricing," Journal of Economic Theory 13 (December 1976), 341-60.

_____, "Return, Risk, and Arbitrage," in Risk and Return in Finance, eds. I. Friend and J. Bicksler. Cambridge, MA: Ballinger, 1976.

_____, and M.W. Walsh, "A Simple Approach to the Pricing of Risky Assets with Uncertain Exchange Rates," Research in International Business and Finance, 3 (1983), 39-54.

Sercu, P., "A Generalization of the International Asset Pricing Model," Reveu de l'Association Francaise de Finance 1 (June 1980), 91-135.

Shanken, J., "The Arbitrage Pricing Theory: Is It Testable," Journal of Finance 37 (December 1982), 1129-140.

Solnik, B., "An Equilibrium Model of the International Capital Market," Journal of Economic Theory 8 (August 1974), 500-24.

_____, "International Arbitrage Pricing Theory," Journal of Finance 38 (May 1983), 449-57.

Stockman, A.C., "Risk, Information, and Forward Exchange Rates," in The Economics of Exchange Rates, eds., J.A. Frenkel and H.G. Johnson. Reading, MA: Addison-Wesley, 1978).

Stulz, R.M., "A Model of International Asset Pricing," Journal of Financial Economics 9 (December 1981), 383-406.

_____, "On the Effects of Barriers to International Investment," Journal of Finance, 36 (September 1981), 923-34.

_____, "The Forward Exchange Rate and Macroeconomics," Journal of International Economics, 12 (May 1982), 285-99.

_____, "Currency Preferences, Purchasing Power Risks and the Determination of Exchange Rates in an Optimizing Model," Journal of Money, Credit and Banking, 16 (1984), 302-16.

International Finance Discussion Papers

<u>IFDP NUMBER</u>	<u>TITLES</u>	<u>AUTHOR(s)</u>
	<u>1986</u>	
294	An International Arbitrage Pricing Model with PPP Deviations	Ross Levine
293	The Structure and Properties of the FRB Multicountry Model	Hali J. Edison Jaime R. Marquez Ralph W. Tryon
292	Short-term and Long-term Expectations of the Yen/Dollar Exchange Rate: Evidence from Survey Data	Jeffrey A. Frankel Kenneth A. Froot
291	Anticipated Fiscal Contraction: The Economic Consequences of the Announcement of Gramm-Rudman-Hollings	Robert A. Johnson
290		
289	Deposit Risk Pooling, Irreversible Investment, and Financial Intermediation	Robert A. Johnson
288	The Yen-Dollar Relationship: A Recent Historical Perspective	Manuel H. Johnson Bonnie E. Loopesko
287	Should Fixed Coefficients be Reestimated Every Period for Extrapolation?	P.A.V.B. Swamy Garry J. Schinasi
286	An Empirical Analysis of Policy Coordination in the U.S., Japan and Europe	Hali J. Edison Ralph Tryon
285	Comovements in Aggregate and Relative Prices: Some Evidence on Neutrality	B. Dianne Pauls
284	Labor Market Rigidities and Unemployment: The Case of Severance Costs	Michael K. Gavin
283	A Framework for Analyzing the Process of Financial Innovation	Allen B. Frankel Catherine L. Mann

Please address requests for copies to International Finance Discussion Papers, Division of International Finance, Stop 24, Board of Governors of the Federal Reserve System, Washington, D.C. 20551.

International Finance Discussion Papers

<u>IFDP NUMBER</u>	<u>TITLES</u>	<u>AUTHOR(S)</u>
282	Is the ECU an Optimal Currency Basket?	Hali J. Edison
281	Are Foreign Exchange Forecasts Rational? New Evidence from Survey Data	Kathryn M. Dominguez
280	Taxation of Capital Gains on Foreign Exchange Transactions and the Non-neutrality of Changes in Anticipated Inflation	Garry J. Schinasi
279	The Prospect of a Depreciating Dollar and Possible Tension Inside the EMS	Jacques Melitz
278	The Stock Market and Exchange Rate Dynamics	Michael K. Gavin
277	Can Debtor Countries Service Their Debts? Income and Price Elasticities for Exports of Developing Countries	Jaime Marquez Caryl McNeilly
276	Post-simulation Analysis of Monte Carlo Experiments: Interpreting Pesaran's (1974) Study of Non-nested Hypothesis Test Statistics	Neil R. Ericsson
275	A Method for Solving Systems of First Order Linear Homogeneous Differential Equations When the Elements of the Forcing Vector are Modelled as Step Functions	Robert A. Johnson
274	International Comparisons of Fiscal Policy: The OECD and the IMF Measures of Fiscal Impulse	Garry Schinasi
273	An Analysis of the Welfare Implications of Alternative Exchange Rate Regimes: An Intertemporal Model with an Application	Andrew Feltenstein David Lebow Anne Sibert

1985

(partial listing)

272	Expected Fiscal Policy and the Recession of 1982	William H. Branson Arminio Fraga Robert A. Johnson
271	Elections and Macroeconomic Policy Cycles	Kenneth Rogoff Anne Sibert