A Guide for Evaluating

Reforestation

and

Stand Improvement

Projects

In Timber Management Planning on the

National Forests

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FOREWORD

This handbook describes a procedure for economic evaluation of reforestation and timber stand improvement projects. It indicates a method for scheduling available funds to projects that will yield the highest rates of financial return.

Usually there are values in addition to timber that must be considered on National Forest projects of this kind. These values, and the cost of developing them to accomplish multiple-use objectives on a project area, should also be appraised to arrive at an overall project evaluation. Economic evaluation for timber production alone, however, provides a benchmark for rating project priorities.

Economic evaluation of projects shows the significance of inefficiencies and resultant higher costs, which reduce the economic justification for projects. The analysis of costs and expected returns should thus guide efforts to improve treatment procedures and encourage better project administration to reduce costs and increase returns from expenditure of public funds.

In private enterprise, the profit motive provides the incentive to produce with maximum efficiency. To a large extent, economic principles that apply to investment of funds for reforestation and timber stand improvement on private land should apply to the setting of priorities for such work on National Forest land.

This handbook was initially designed to develop economic criteria for the guidance of reforestation and timber stand improvement work on National Forests in the Appalachian Mountains, and has been generalized to apply to this work on all National Forests. Continuing research and field experience in the economic evaluation of projects should lead to better local guides and more and better accomplishment per dollar of public funds invested.

Edward P. Cliff

Chief, Forest Service

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INTRODUCTION

Timber stand improvement and reforestation projects are an important and growing part of the total timber management program on the National Forests. These projects are undertaken to increase timber growth and quality, to reduce the risk of fire, weather, and pest losses, to transfer growth from low to high value trees, to return denuded areas to production quickly, and to enhance other forest uses and values as well. Economic appraisals of the various opportunities for stand improvement and reforestation are helpful in deciding how much effort to devote to each aspect of timber management, and what kinds of projects to favor. This handbook outlines procedures for rating and comparing proposed stand improvement and reforestation projects.

BASIC CONCEPTS

Economic desirability, in terms of costs and returns, is a principal consideration in establishing priorities for reforestation and stand improvement projects on the National Forests. This section summarizes basic economic concepts used to estimate economic desirability, compare competing projects, and to allocate funds among projects.

MEASURING ECONOMIC DESIRABILITY

The economic desirability of a project is measured by how much it adds to the value of the timber stand, in relation to its cost and the length of time required to bring the stand to harvest. Value is used, rather than volume or some other physical quantity, because it is the only measure of output that reduces all changes in timber yield to a common denominator. For example, a 20-percent increase in the yield of select-grade lumber cannot be compared directly with a 10,000 board-foot increase in per acre volume, but the dollar values of these yield changes can be compared.

Both costs and values have to be considered because omitting either can lead to erroneous priorities. Where differences in the value added by various projects are ignored, the least costly projects are likely to be favored, even though slightly more expensive projects may add several times as much value. When costs are ignored, projects adding the most value tend to be favored without regard to their cost.

Time is the third important element that must be considered. When time is ignored and both cost and value are taken into account, projects with equal returns per dollar of cost are likely to be given equal priority. Yet some of these projects might accomplish their value increase in only a few years, whereas others require a longer period of time.

Cost, value added, and the time between investment and return vary from one project to another, and each influences economic desirability. All three, however, are taken into account by the value growth rate of a project. The usual way of expressing value growth rate is the annually compounded percentage increase—a compound interest rate.

Compound interest is the preferred measure of the rate of value growth not because it is intrinsically better than any other measure, but because it is used so commonly to describe investments that it is universally understood and accepted. Economists call it the *internal rate of return* or the *rate of return to investment.*

To summarize, the internal rate of return is a widely understood and accepted measure of an investment's economic desirability. It measures the rate at which an investment grows toward the return it eventually generates, and takes into account the amount and timing of both costs and returns. Methods of computing rates of return, and some comparisons with other measures of economic desirability, are discussed later.

COMPARING PROJECTS AND ALLOCATING FUNDS

Each year funds available for reforestation and stand improvement must be allocated among a large number of management units. There is opportunity to allocate these funds to areas and projects that will produce the highest internal rates of return. How can these opportunities be identified?

Capital budgeting is an analytical system developed by economists to answer just this sort of question. It is so called because it budgets capital funds among competing investment projects. There are several steps in applying the capital budgeting system. For budgeting stand improvement and reforestation projects within a Region it might work as follows:

1. In the course of periodic compartment examinations, areas would be found where reforestation or cultural treatments would improve timber output. Type of treatment needed, acreage to be treated, project cost, internal rate of return, and important nontimber effects would be estimated and noted for each project area.

2. Compartment examination records would be reviewed once each year by Forest staff. A listing would be prepared, for each working circle, of the reforestation and stand improvement projects proposed for the ensuing fiscal year. Separate lists might be made for work to be financed with appropriated funds and with K-V[1] funds. Projects would be listed in descending order of internal rate of return, along with their cost and other relevant data. Working circle lists would be forwarded to Regional staff. Table 1 shows a hypothetical listing.

TABLE 1.—*Hypothetical working circle list of scheduled reforestation and stand improvement projects to be financed from appropriated funds*

Compartment		Type	Treatment	Project area	Return rate	Project cost
				Acres	*Percent*	*Dollars*
Sampson	A36	Cove hdwd.	Thinning	870	12.3	6,090
Logan	F14	North. hdwd.	Thinning	1,310	11.2	9,170
Logan	F16	North. hdwd.	Weed, release	2,100	9.7	10,500
Logan	A24	Oak-hickory	Thinning	1,745	9.7	12,210
Joshua	B4	Cove hdwd.	Thinning	315	9.3	2,200
Sampson	D7	Conifer	Weed, release	720	9.0	3,600
Sampson	A14	Conifer	Thinning	975	8.5	6,820
Joshua	C10	North. hdwd.	Thinning	150	8.1	1,050
Joshua	C11	North. hdwd.	Weed, release	1,380	8.0	6,900
Joshua	C12	Cove hdwd.	Thinning	865	8.0	6,050
Evans	E4	Conifer	Weed, release	2,400	2.1	12,000
Logan	A28	Conifer	Planting	310	1.9	6,200
Evans	C12	Oak-hickory	Weed, release	950	1.4	4,750
Snyder	A17	Oak-hickory	Thinning	705	1.4	4,930
Snyder	A24	Oak-hickory	Weed, release	1,430	1.0	7,150
Snyder	D12	Conifer	Weed, release	480	0.8	2,400

[1] Knutson-Vandenberg Act of June 9, 1930 (46 Stat. 527; 16 U.S.C. 576).

3. Region staff would arrange all projects in order of their return rates on a composite list (table 2). A cumulative cost column would be added to indicate how far down the project listing the funds available for the next fiscal year would reach.

With this system each project is evaluated according to its economic productivity as measured by an internal rate of return. Projects compete for funds on the basis of their expected productivity. Those with the highest internal rates of return are financed first; progressively less productive ones are financed until available funds are exhausted. Exceptions could be made where nontimber benefits are important.

TABLE 2.—*Hypothetical composite list of scheduled reforestation and stand improvement projects to be financed from appropriated funds*

Working circle	Compartment	Return rate	Cost	Cumulative cost
		Percent	*Dollars*	*Dollars*
Sullivan	Sampson A36	12.3	6,090	6,090
Franklin	Windham F23	12.1	14,220	20,310
Franklin	Windham F18	12.1	7,400	27,710
Franklin	Webster B2	12.0	21,080	48,790
Webster	Elk A22	11.7	18,010	66,800
Pendleton	Erie D12	11.6	8,430	75,230
Sullivan	Logan F14	11.2	9,170	84,400
Pendleton	Erie A28	6.2	3,440	1,336,480
Pendleton	Tioga F4	6.2	13,270	1,349,750
Pendleton	Tioga F14	6.2	24,000	1,373,750
Franklin	Tucker D22	6.1	17,360	1,391,110
Pendleton	Erie B6	6.0	8,880	[1] 1,399,990
Pendleton	Wayne B18	5.8	15,500	
Webster	Wyoming A11	5.8	29,370	
Webster	Roane C17	5.7	37,480	
Pendleton	Wayne C31	5.5	7,630	

[1] Appropriated funds limit is $1,400,000, and Pendleton–Erie–B6 is the last fundable project.

The capital budgeting approach is only as good as the basic evaluations of the economic desirability of proposed projects. In addition to being as accurate as possible, evaluations should be consistent to enable comparison from one project to others. Furthermore, it is useful to have a standard way of evaluating projects—a standard measure of economic desirability, a standard method for determining which costs and returns to include, and a standard way of computing required statistics—to enhance the validity of comparisons. It is such a standard method of evaluation that is presented in the remainder of this handbook.

CHOOSING AND RATING PROJECTS

The steps in project evaluation are choosing the project, estimating added yield, determining project cost, valuing yield additions, and computing rate of return.

CHOOSING MODAL PROJECTS FOR EVALUATION

Sometimes it is desirable to evaluate individual projects. An individual evaluation may be particularly useful, for example, when a contemplated project is large or unusual. In many cases, however, a single evaluation can serve for a range of similar projects. This may be done by subdividing projects into a limited number of fairly homogeneous groups, and by choosing for evaluation the one project from each group that is most representative of that group. The projects chosen are termed "modal" projects because they will often be the modal or most frequently occurring project of the group.

Many factors may ultimately play a part in defining project groups, but it often helps to begin by determining the range and frequency of site and stand conditions making up the timber type, Forest, or other management unit chosen for study. National Forest compartment examination and timber inventory records are usually good sources of this information. Age, size class, species composition, density of stocking, and site quality are among the more important site and stand condition variables. Other characteristics that influence project cost or yield may also be important in particular types.

The analysis begins by subdividing the type on the basis of site and stand characteristics. Plot data are then sorted on this classification to gain some idea of the acreage contained in each site-stand class. This initial classification may be modified, by either combining classes or subdividing them. It is typical for the bulk of the acreage in a timber type to be concentrated in a relatively small proportion of these site-stand condition classes. The inventory data, thus, can give the analyst a good idea of the most important site-stand condition categories.

Each site-stand condition class will, of course, contain some variation in actual conditions. The next task is to identify the modal site quality and stand condition that will be used to represent the class. Plot data for the more important stand classes will indicate the most frequently occurring site index and stand condition for the class. Sometimes a class will be proposed initially that turns out to include two conditions that are different with respect to some characteristic. The analyst may wish to subdivide such classes so that he can deal with more homogeneous groups. Where the data are inadequate, class midpoints can be used or additional sampling can establish the particular set of site and stand conditions that should represent the class.

Additional field sampling may be needed for other reasons as well. For example, a characteristic with an important influence on the type of reforestation or stand improvement practice needed, its cost or response, and that was not measured by the inventory, may require a supplementary survey.

The next task is to prescribe a treatment for each site-stand condition class, based on common practice on public and private forests, available management guides, and research information. At the end of this procedure, there should be a set of site-stand condition classes, an indication of the extent of each class, a modal site index and stand condition reflecting the most frequent situation within each class, and a treatment prescription tailored to this modal condition. The modal site index and stand condition for each class, together with its related treatment, define the modal project that will be evaluated as representative of the class.

An evaluation study carried out for the George Washington National Forest illustrates this process of defining modal projects. A primary classification or grouping of forest conditions was used that involved four factors: Forest type, site class, main stand size class, and main stand age class. Three forest types were included in the study: The conifer, the oak, and the cove hardwood. Three levels of site quality were used to subdivide each type. The four main stand size classes used were seedling and sapling stands, pole stands, small sawtimber stands, and large sawtimber stands. Ten 20-year age classes formed the final classification.

Five hundred and seventy-two survey plots were sorted on this primary, four-way classification. These plots fell into 136 of the 360 primary classes. The reason that so few of the primary classes actually exist on the Forest is that several of the classification variables are correlated with one another. For example, cove hardwoods are seldom found on poor sites, so there are really only two site classes that this type occupies. Age class and stand size class are also correlated in this way.

This primary grouping of forest conditions was not sufficient to provide groups homogeneous enough for analysis. Further subdivision was needed in many primary classes. For example, one of the 136 primary classes was "Oak type—medium site—poletimber—41–60 yr. old." This class contained 50 of the 572 survey plots, or about 8.7 percent of the total sample. For the forest as a whole, this primary class encompasses about 65 thousand acres, on the basis of its representation in the sample.

This class was further subdivided on the basis of basal area. Stands in the class with less than 40 sq. ft. of basal area per acre in growing-stock trees were considered candidates for a type conversion treatment. Those with a growing-stock basal area above 40 sq. ft. per acre but a total basal area of less than 80 sq. ft. per acre were to be retained, but were considered to need no immediate treatment. Stands with growing-stock basal areas above 40 sq. ft. per acre and total basal areas above 80 sq. ft. per acre were considered to be candidates for a thinning treatment.

A subsample of the 50 survey plots that fell in the primary class showed that about one-half of the area in this class had a stocking density that suggested thinning, about 35 percent needed no immediate treatment, and about 15 percent might benefit by a conversion treatment. The sample plots that fell in the thinning subclass were much alike. The average d.b.h. for the main stand was 8.4 in. in these sample plots; total basal area averaged 96 sq. ft., and growing stock basal area averaged 73 sq. ft. Optimal residual basal area was determined to be 63 sq. ft. per acre, indicating a thinning treatment removing 33 sq. ft. of basal area per acre on the average.

The listing below illustrates the definition of a modal project:

CLASS TITLE: Well-stocked oak poletimber on medium sites

CLASS DEFINITION:

Type.—50 percent or more of net cubic-foot volume in white, black, scarlet and chestnut oaks, hickory and related species.

Site class.—51–70 ft. at 50 yr. for oak.

Size class.—5.0–10.9 in. main stand average d.b.h.

Age class.—Age of main stand trees averages 41–60 yr.

Basal area class.—Total basal area exceeds 80 sq. ft. per acre, of which at least 40 sq. ft. per acre is in growing-stock trees.

CLASS ACREAGE: 32,500 acres on the George Washington National Forest

MODAL CONDITIONS:

Site quality.—60 ft. at 50 yr. for oak.
D.b.h.—8.4 in. average for main stand trees.
Age.—50 yr.
Basal area.—96 sq. ft. per acre total; 73 sq. ft. per acre in growing-stock trees.
Species composition.—37 percent red, black, and white oaks
34 percent chestnut oak
11 percent hard pines
4 percent yellow poplar
15 percent scarlet oak and other species

TREATMENT:

Type.—Thinning and cull tree deadening.
Intensity.—33 sq. ft. of basal area removed per acre.
Residual stand.—63 sq. ft. of residual basal area.
—Average age and diameter not significantly changed.
—Residual species composition:
55 percent red, black, and white oaks
25 percent chestnut oak
15 percent hard pines
5 percent yellow poplar and other species

ESTIMATING ADDED YIELD

Reforestation and stand improvement projects are undertaken to increase timber yield. The term "yield" as used here refers not just to volume, as it does in most yield tables, but to any stand or tree characteristic that influences value and changes with time or stand management. Usually the timber yield for a single species must be defined in three ways: A quantity measure like volume or weight, a quality measure like tree grade or quality index, and a measure of average unit size like average d.b.h., because of the influence of size on logging and milling costs.

Sometimes practices exert most of their influence on a single yield factor. For example, pruning primarily influences tree and stand quality. Most practices, however, have multiple effects. A hardwood improvement cut, for example, may increase volume, quality, and average d.b.h. at harvest by retaining the larger, more vigorous, and better quality trees.

The analyst must adopt or develop a procedure for projecting timber yields that is uniform for all projects in a given forest type. This procedure should specify how yield factors are to be projected from a wide range of initial stand conditions. It should take into account the possibility of different rotation ages and alternate programs of subsequent silvicultural treatment. Yield projection is usually the most complex analytical task in preparing evaluations of reforestation and stand improvement projects.

Yield tables are a good starting point for relatively pure, even-aged stand conditions. Yield tables alone seldom suffice, however, and additional information will often be required. The stand table projection approach is often best for mixed and uneven-aged stand conditions. Projections should not ignore available research results. Studies that give estimates of yield, growth, or treatment response can be used directly in stand table projections, and indirectly in adapting yield tables to various conditions. Available research data, then, can serve as checkpoints in developing or extending available yield table and stand table projections.

Projection of quality may sometimes be difficult because information about the effect of time and treatment on quality is less abundant than information on volume and size. Yet it is necessary to make these projections, even when the basis is at best doubtful. An objective estimate, even though based on limited data, is better than a completely subjective one. Estimates can and should be revised as better data become available.

Yield must be measured as of some future date and with a particular intensity of future management in mind, since both subsequent management and harvest date influence yield. And it may be necessary to assume different management programs depending on whether or not the project is undertaken. In types where commercial thinning is assumed as a future practice, thinning yields as well as the final harvest yield must be included in the projections. Timber management guides and timber management plans will indicate typical rotation ages and subsequent management plans for the type. Both are frequently related to site quality. Select the most likely rotation age and future management regime for each modal project, and the most likely rotation age and future management regime that would apply on the same area without stand treatment.

Having developed a yield projection technique and made estimates of future management, it is now possible to estimate added yield. First, estimate yield without treatment for each modal project. Then, depending on the projection procedure used, either estimate directly the yield added by the project, or estimate total yield with treatment and let the difference between the yield estimates with and without treatment define yield added.

In summary, a standard yield projection technique is developed and applied to all modal projects in a single timber type, insofar as possible, to avoid the inconsistencies of using different methods for different projects. Future management is set for each project, with and without treatment, according to silvicultural guides and management plans. Yield without treatment is estimated for all modal projects, as well as yield with treatment or yield added by treatment.

DETERMINING PROJECT COST

Project cost includes all the costs incurred because the project was undertaken, just as project yield includes all the yield added because the project was undertaken.

The analyst will be interested primarily in estimating direct costs. Direct charges include those for the labor, equipment hours, materials used to complete the treatment, and other costs that can be directly assigned to the project and are not included in overhead, such as transportation to and from the project area, professional planning, and supervision. The indirect or overhead costs must also be included as a part of project cost. This is most easily done by adding to direct cost a fixed percentage that reflects the average relation between direct expenditure and overhead for the Forest or Region.

Direct costs can be estimated by determining the physical inputs needed to accomplish the required treatment and by multiplying each input by its current unit cost. These data can come from cost studies or records of the man-hours, equipment, and materials used on recent reforestation or stand improvement projects. Accounting records of dollar cost may require careful scrutiny before use, because they often group heterogeneous projects and may include charges for activities not directly related to stand treatment.

Project costs may be spread over several years. There are projects that must be carried out in two or more steps spaced several years apart. Two-stage pruning, for example, means pruning perhaps 10 feet of bole initially and the remaining 7 or 8 feet some years hence. Such projects are undertaken in the expectation that subsequent treatments will be carried out in later years. Compute costs for each treatment separately and estimate the date of each treatment. Use the present prices of labor, equipment, and materials to determine the cost of all treatments, unless there is sufficient evidence indicating price change.

Costs, even for identical treatments under identical stand conditions, can vary from project to project because of accessibility, crew efficiency, and other factors that the analyst cannot take into account. Those who use modal project evaluations as a guide to the rates of return that can be anticipated on actual projects may wish to recompute a return rate to reflect a cost level for an actual project different from that assumed in the modal project evaluation.

APPRAISING ADDED YIELD

Added yield ordinarily increases the stumpage income from the treated compartment. Needed now is an estimate of what this income increase will be for each modal project. Yield additions can be valued by applying current sales data or, where these are not available, the standard Forest Service stumpage appraisal procedures.

Whatever unit price estimates are used, begin by computing the value of the yield assortments expected if the modal project is not undertaken. Prices, costs, and profit margins can be adjusted to reflect average conditions where necessary. Next, value the yield assortment expected with treatment and compute the difference between the two appraisals. The second appraisal need not be made when the only significant effect of a project is to increase volume (e.g., some regeneration projects). Here the percentage increase in volume can be applied to the base appraisal to give value added.

There may be significant changes in product price and conversion cost levels by the time added yields are harvested. Changes in the general level of pulp or lumber prices and harvesting and processing costs—influencing all timber types and treatments to a similar degree—usually do not influence comparisons among projects importantly. Differential price and cost changes do. If conversion efficiency or product demand rise more rapidly for some kinds of timber than for others, then projects yielding these kinds of timber produce more value relative to other projects than is apparent when yield additions are appraised at present prices. However, current prices should be used in valuing yield until more reliable projections of future prices are available.

COMPUTING RATES OF RETURN

The costs and added values for each modal project are used to compute its internal rate of return. The procedures for making these calculations are simple variations of the familiar methods of present worth and future value computation used in forest finance.

One common problem in forest finance is to determine the present worth of a future value discounted at a given rate of compound interest. For example, consider a modal project with an added yield valued at \$73 per acre, and due 40 years after treatment. The present worth (i.e., at the time of treatment) of this added value depends on the interest rate chosen. For a 5 percent discount rate,

$$\begin{aligned} PW &= \$73/(1.05)^{40} \\ &= \$10.37 \text{ per acre} \end{aligned}$$

where PW = present worth. In this problem the interest rate and future value are given; present worth is the unknown.

The problem at hand is a variation of the above: to determine the interest rate that makes present worth zero, given an initial cost and a future added value. All the elements of the problem are given except the balancing interest rate, which is the unknown. Using the previous example and introducing a project cost of \$6.40 per acre, the equation becomes:

$$PW = 73/(1 + i)^{40} - 6.40$$

Substituting zero for PW and rearranging terms:

$$\begin{aligned} 6.40\,(1 + i)^{40} &= 73 \\ (1 + i)^{40} &= 73/6.40 \\ (1 + i)^{40} &= 11.41 \end{aligned}$$

and by consulting the 40 year line in a $(1 + i)^n$ interest table (Marty and Neebe 1966):

$$i = 6.3 \text{ percent}$$

Rephrased, the problem is to determine the rate at which project cost grows toward the additional stumpage income it produces. In the example above the problem can be stated: If \$6.40 were invested at compound interest, what would the rate of interest have to be to realize exactly \$73 40 years hence? When the project cost is incurred during a single year and the added return accrues at harvest—the simplest case—then the balancing rate of interest or internal rate of return can be computed in two simple steps: by first calculating the value of $(1 + i)^n$ and then by finding the i that has a corresponding value for n years in a $(1 + i)^n$ compound interest table. The formula for finding the value of $(1 + i)^n$ is simply:

$$(1 + i)^n = R/C \tag{1}$$

where R = per acre value added
C = per acre project cost

Some projects have more than one cost or more than one income or value added. Indeed, a project may call for a whole series of costs and generate a whole series of returns. Here is a general formula for any project.

$$C_1(1+i)^n + C_2(1+i)^{n-1} + C_3(1+i)^{n-2} + \dots + C_{n-1}(1+i)^2 + C_n(1+i) = R_1(1+i)^{n-1} + R_2(1+i)^{n-2} \dots + R_{n-1}(1+i) + R_n. \quad (2)$$

There is provision for a cost and return item for every year between initial treatment and final harvest in this equation. The unknown is the interest rate, i, that will equate the future value of costs with the future value of returns.

By convention, all costs occurring within a given year are considered to have been incurred at the beginning of the year, and all returns occurring within a single year are assumed to have accrued at the end of the year.

In most practical applications many of the yearly cost and return items will be zero, for few projects have costs and returns in every year. Consider a pruning project that calls for two treatments 10 years apart, with a single return in 50 years. For this project the equation above simplifies to:

$$C_1(1+i)^{50} + C_{11}(1+i)^{40} = R$$

If each pruning costs \$17 per acre and the added value at harvest is estimated to be \$128 per acre, then substituting these values we have:

$$17(1+i)^{50} + 17(1+i)^{40} = 128$$

or

$$(1+i)^{50} + (1+i)^{40} = 128/17 = 7.53$$

Since more than one cost is involved, there is more than one value of exponent n in the expression $(1+i)^n$. Thus i cannot be solved for by the direct, two-step method. Instead it must be found by a process of successive approximation, in which various values for i are substituted in the formula until one is found that satisfies the equation.

The process can be begun with any interest rate. If 6 percent is substituted in the pruning example, the equation becomes:

$$18.420 + 10.268 = 7.529$$
$$28.706 \neq 7.529$$

Since the cost side is far too high, the internal rate must be lower than 6 percent. Three percent gives:

$$4.384 + 3.262 = 7.529$$
$$7.646 \neq 7.529$$

This is still a little high. 2.9 percent gives:

$$4.176 + 3.138 = 7.529$$
$$7.314 \neq 7.529$$

Since 2.9 percent is too low and 3.0 percent is too high, the balancing rate must be somewhere between the two, and is probably closer to 3.0 percent than to 2.9. In this way the analyst can determine the internal rate of return within narrow limits. This basic method of successive approximation must be used whenever the cost-return equation contains more than one value for the exponent n.

The appendix provides additional compound interest formulas for simplified handling of various regular series of costs and returns. Also included in the appendix is a description of a computer program for interest rate computation (Row 1963). Where many interest rate analyses have to be made by the method of successive approximation, this program can save much time and expense.

TWO EXAMPLES OF MODAL PROJECT ANALYSIS

Examples may help the reader to a better understanding of project evaluation. The first example [2] involves precommercial thinning in ponderosa pine, for which initial site and stand conditions can be summarized as follows:

Type.—Pure, even-aged ponderosa pine
Site index.—120 ft. at 100 yr.
Size class.—Sapling stand
Stand age.—12 yr. after regeneration cut
Density.—2,500 stems per acre

This specific set of conditions might be considered representative of densely stocked stands of young ponderosa pine reproduction on medium sites.

A precommercial thinning is to be applied to the stand immediately, and is expected to accelerate the growth rate enough to attain the product objective 20 years sooner than would otherwise be so.

Frequently, the project influences future management as well as rotation length. If the stand receives a precommercial thinning now, at age 12, future management will include a thinning for pulpwood at age 25, another pulpwood thinning at age 40, saw log thinnings at ages 55, 70, and 85, a shelterwood harvest at age 100, and shelterwood removal at age 110.

If the stand is not thinned now, its future management will involve pulpwood thinnings at ages 40 and 55, saw log thinnings at ages 70, 85, and 100, and shelterwood reproduction cuts at ages 120 and 130. Tables 3 and 4 show these two management schedules, with project cost and output volumes and values. A $50-per-acre cost for reproduction treatment is deducted from the value of the final harvest in both programs. Unit values for both pulpwood and sawtimber increase with stand age, reflecting increases in average size and quality of timber.

Table 5 repeats the income and expense schedules for each program and adds a schedule ("Marginal") that indicates the difference between the two programs. Incomes in this schedule are those received in the precommercial thinning program beyond those available in the alternate program, or expenses without precommercial thinning that are avoided in the precommercial thinning program. Similarly, expenses in the marginal schedule are either those occasioned by precommercial thinning or incomes forgone.

The rotation age assumed, if precommercial thinning is employed, is 110 years; without this treatment rotation age is assumed to be 130 years. Table 5 shows the schedules of both treatment alternatives for 130 years, and so the schedule for the precommercial thinning program is extended 20 years into the second rotation, having incurred a second thinning expense

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[2] Harmon, Wendell H. Does precommercial thinning pay? U.S. Forest Service, Region 6. Portland, Oreg. 2 pp. mimeographed. 1963.

TABLE 3.—*Management program for ponderosa pine on medium sites, without precommercial thinning*

Stand age	Management activity	Per acre yields		Unit value	Per acre values	
		Pulpwood	Sawtimber		Expense	Income
Years		*Cords*	*M bd. ft.*	*Dollars*	*Dollars*	*Dollars*
40	Pulpwood thinning	10		4		40
55	Pulpwood thinning	27		6		162
70	Saw-log thinning		13	15		195
85	Saw-log thinning		14	25		350
100	Saw-log thinning		12	28		336
120	Shelterwood cut		22	32		704
130	Shelterwood removal		13	35		455
130	Reproduction treatment				50	

TABLE 4.—*Management program for ponderosa pine on medium sites, with precommercial thinning*

Stand age	Management activity	Per acre yields		Unit value	Per acre values	
		Pulpwood	Sawtimber		Expense	Income
Years		*Cords*	*M bd. ft.*	*Dollars*	*Dollars*	*Dollars*
12	Precommercial thinning				30	
25	Pulpwood thinning	8		4		32
40	Pulpwood thinning	27		6		162
55	Saw-log thinning		13	15		195
70	Saw-log thinning		14	25		350
85	Saw-log thinning		12	28		336
100	Shelterwood cut		22	32		704
110	Shelterwood removal		13	35		455
110	Reproduction treatment				50	

TABLE 5.—*Individual and marginal schedules of program expenses and incomes*

[Dollars]

Stand age	Individual schedules				Marginal schedule	
	Without early thinning		With early thinning			
	Expense	Income	Expense	Income	Expense	Income
Years						
12			30		30	
25				32		32
40		40		162		122
55		162		195		33
70		195		350		155
85		350		336	14	
100		336		704		368
110			50	455		405
120		704			704	
122			30		30	
130	50	455		[1] 84	321	

[1] The present value of incomes and expenses during the remainder of the second rotation, when they are discounted at 7.9 percent.

but no second-rotation incomes as yet. A terminal value of $84 per acre is shown here and is the discounted value, at stand age 20, of subsequent second-rotation incomes.

Table 6 shows the calculations that demonstrate that the rate of return to precommercial thinning, in this case, is very close to 7.9 percent. This is shown by discounting the expenses and incomes of the marginal schedule to the present and comparing them. Various discount rates were tried until one was found that balanced costs and returns. In the actual computations procedure, the terminal value for the precommercial thinning program was recomputed each time a new interest rate was tried on the marginal schedule.

The second example presents the evaluation of a pruning project in eastern white pine, adapted from Marty.[3] Pruning is a frequently recommended treatment for white pine since it improves timber quality markedly. The site and stand conditions assumed for this project are as follows:

Type.—Pure, even-aged eastern white pine
Site index.—60 ft. at 50 yr.
Size class.—Poletimber stand
Stand age.—30 yr. from seed
Density.—Fully stocked

The diameter distribution and total height by diameter class for this stand, based on yield table data, are shown in table 7.

[3] Marty, Robert. Timber investment decisions. 1961. (Ph.D. dissertation, Yale University, New Haven, Conn.)

TABLE 6.—*Computation of present worth for incomes and expenses related to precommercial thinning in ponderosa pine*

[Dollars]

Time hence	Discount factor [1]	Marginal schedule		Present values [2]	
		Expense	Income	Expense	Income
Years					
0	1.0000	30		30.00	
13	2.6871		32		11.91
28	8.4062		122		14.51
43	26.298		33		1.25
58	82.270		155		1.88
73	257.37	14		0.05	
88	805.16		368		0.46
98	1722.3		405		0.24
108	3683.9	704		0.19	
110	4289.0	30		0.01	
118	7880.0	321		0.04	
Totals				30.29	30.25

[1] Given by 1.079^n when n is years hence.
[2] Given by the expense or income divided by the discount factor indicated.

TABLE 7.—*Stand table for a fully stocked, pure white pine stand of 30 years on a medium site*

D.b.h. class	Number of trees [1]	Range in total height [2]
Inches		*Feet*
1.0–2.9	407	18–24
3.0–4.9	1463	25–30
5.0–6.9	280	31–34

[1] Based on average diameter and number of trees above 1 inch d.b.h. (table 19) and a general diameter distribution for fully stocked white pine stands (table 24) reported by Gevorkiantz and Zon (1930).

[2] Based on a percentage diameter-height distribution (table 178) reported by Brown and Gevorkiantz (1934).

The treatment chosen for analysis involved pruning 150 well-distributed pine per acre, all from the 6-inch d.b.h. class. Pruning was to be carried to 17½ ft. above the ground and accomplished in a single operation. A rotation age of 60 yr. was assumed for the stand. No other intermediate treatments were contemplated. Approximately 250 merchantable trees per acre were expected at harvest.

The response to pruning over a given period is closely related to the rate of tree growth. The expected diameter distribution 30 years hence at harvest age 60, for trees now in the 6-in. class, is shown in table 8. This estimate again is based on normal yield data (Gevorkiantz and Zon 1930).

The second stage in predicting the response to pruning was to estimate the expected increase in tree value for trees of various harvest diameters. Three steps were involved: Predicting the change in butt log quality index (QI), predicting the volume of butt logs at harvest, and predicting the value of the increase in QI for butt logs of the predicted volume.

TABLE 8.—*Diameter distribution 30 years hence for trees now in the 6-inch d.b.h. class* [1]

D.b.h. class	Expected number of trees
Inches	
Less than 9 or mortality	30
9.0–10.9	158
11.0–12.9	74
13.0–14.9	17
15.0–16.9	1
Total	280

[1] Based on average diameter and number of trees above 1 inch d.b.h. (table 19) and a general diameter distribution for fully stocked stands (table 24) reported by Gevorkiantz and Zon (1930).

Table 9 shows the derivation of the prediction of change in QI for trees of different harvest diameters. The second column shows the percent of total butt log volume outside the knotty core. These estimates assume that (1) pruning occurs when the tree is 6 inches d.b.h., (2) that 2 inches d.b.h. growth are required to enclose branch stubs and pruning wounds, and (3) that the scaling diameters of both the log and its knotty core are 2 inches less than their diameters at breast height.

The third column shows the QI of unpruned butt logs. It is based on an average of figures given for lumber grade recovery of second-growth New England white pine (Davis 1940; Holland 1960). For unpruned trees, the differences in butt log QI among the several diameter classes are assumed. As pine increases in size more and more butt log branches die, causing the formation of "loose" rather than "green" knots and thus a reduction in lumber grade recovery. Further size increases tend to reverse this downward trend as dead limbs drop and a clear shell develops. Because of the persistence of dead limbs, however, little or no clear shell develops until pine exceeds 16 inches d.b.h.

TABLE 9.—*Change in butt log quality index (QI) due to pruning* [1]

Harvest d.b.h. class	Clear volume with pruning	QI without pruning	QI with pruning	Change in QI due to pruning
Inches	*Percent*			
10	25	74	90	16
12	40	72	98	26
14	54	71	107	36
16	63	72	114	43

[1] See text for sources, assumptions, and derivations.

Column four shows the QI of logs that have been pruned. These estimates were derived by taking the weighted (by volume) average of the QI's of the knotty core and the clear shell. It was assumed that the knotty core would produce lumber with a QI like that for an unpruned log of the same size, and that the clear shell would produce select lumber with a QI of 138.

Column five shows the increase in QI due to pruning the butt log, given by the difference between columns three and four.

Table 10 shows the derivation of butt log volume. Volumes are according to the International log rule with separate allowance for expected cull due to weevil injury and other loss factors.

TABLE 10.—*Butt log volumes for white pine*

Harvest d.b.h.	Girard form class	But log volume [1]
Inches	*Percent*	*Board-feet*
10	83	23
12	83	38
14	82	56
16	82	78

[1] Based on Bickford (1951) and represents International one-fourth-in. volume corrected for average reductions due to weevil injury and other defect factors.

The increase in value for pruned logs stems from the improvement in lumber grade yield. The cost of conversion is assumed to be unchanged by pruning. Lumber price for the base lumber grade (No. 1 and 2 Common) is predicted to be $215 per M bd.-ft. in 1960 dollars 30 years hence. The estimate is based on a published price projection for white pine lumber (Holland 1960) and indicates that each board foot that is raised one point on the QI will increase tree value by $0.00215. This multiplied by the product of increase in QI due to pruning (table 9, column 5) and butt-log volume (table 10, column 3) gives the value added per tree by pruning. The results of these calculations are shown in the second column of table 11.

The average return per pruned tree is calculated in table 12. This computation, being a weighted average of value added of all trees in the 6-inch d.b.h. class at time of treatment, assumes that pruned trees will have the same diameter distribution at harvest as all trees in the 6-inch d.b.h. class. On a per-acre basis the 150 trees yield an added value at harvest of $193.50.

TABLE 11.—*Value response to pruning*

Harvest d.b.h.	Value added per tree [1]
Inches	*Dollars*
10	0.79
12	2.12
14	4.33
16	7.21

[1] Based on tables 9 and 10, and Holland (1960).

TABLE 12.—*Computation of average value added per pruned tree when pruned trees have the same diameter distribution at harvest as all trees*

D.b.h. class	Relative frequency	Value added per tree	Value added per class
Inches	*Percent*	*Dollars*	*Dollars*
Less than 9 or mortality	10.7	0	0
9.0–10.9	56.4	.79	.45
11.0–12.9	26.4	2.12	.55
13.0–14.9	6.1	4.33	.26
15.0–16.9	.4	7.21	.03
Average value per pruned tree			1.29

Two studies that report white pine pruning costs (Meyer 1940; Ralston and Lemmien 1956) for particular operations give physical performance rates of 176 and 130 bole-feet per hour. A range in performance rates from 50 to 200 bole-feet per hour has also been reported (Cline and Fletcher 1928). A performance rate of six trees per man-hour (approximately 105 bole-feet per man-hour) was assumed in the example.

If the average cost per man-hour is $1.75 for the pruning crew, then the direct labor charge amounts to $43.75 per acre. Supervision, transportation, equipment maintenance, and other overhead expenses might bring the per-acre cost to $55 or $60 for a compartment of average accessibility.

The return/cost ratio for this project, at a cost of $60 per acre, amounts to 3.225. This indicates an internal rate of return of about 4 percent for the 30-year period. A $50 cost per acre would produce a return rate of about 4.6 percent, whereas a $70 per acre cost would yield a return of about 3.4 percent.

LIMITATIONS OF RETURN RATE ESTIMATES

Project evaluations are neither perfectly reliable nor unaffected by other management activities and thus should be interpreted and used with care. This section contains a brief discussion of some of the limitations of rate of return estimates.

THE RELIABILITY OF RATE OF RETURN ESTIMATES

It has been stressed in several places that the data used to determine rates of return for modal projects are not completely reliable. Physical cost data, yield response data, and dollar prices are estimates that ordinarily will not always be exactly correct; thus, the rate of return estimates for modal projects may not always be exactly correct.

If all the information used were accompanied by statistical variance statements, the analyst could compute a variance estimate for each rate of return, and in this way present some information on the reliability of each estimate. Relative reliability can be an important consideration in ranking projects. Which should be ranked highest: a project that is almost sure to have a 4-percent rate of return, or one that might fall between 3 and 6 percent? There are differences of this sort within most groups of projects.

Unfortunately, the analyst seldom has the statistical information with which to objectively estimate these differences in reliability. However, it may be worthwhile sometimes to see how the return rate fluctuates under various assumptions about uncertain factors. A description of this analytical approach can be found in Marty (1964).

A second source of error in rate of return estimates comes in the application of guides. Rate estimates for modal projects will often be applied in situations somewhat different from that assumed for the modal project. So even if the modal project rate were exactly correct, when applied to non-modal conditions it may overestimate or underestimate the rate of return somewhat. This second type of error can be partly reduced by increasing the number of modal projects analyzed. The more stand conditions and treatment variations investigated by the analyst, the more probable will be a modal project that closely approximates each actual project.

RATE OF RETURN SENSITIVITY

The analyst should be aware that the sensitivity of a return rate, as a measure of economic desirability, decreases as the investment period lengthens. It requires only a 10-percent increase in returns, or decrease in costs, to cause a 1-percent increase in rate of return when the investment period is 10 years. When the investment period is 50 years, it requires a

60-percent increase in the return/cost ratio to cause a 1-percent increase in internal rate of return. Seventy-five-year investment periods, typical of many projects, require doubling of the return/cost ratio to increase a return rate 1 percent. What this means is that there may not always be large differences in the rates of return among modal projects, so that even small differences between rates are significant when investment periods are 30 years or more.

INTANGIBLE FACTORS

Rate of return estimates are based on dollar costs and returns of timber production. But there are other factors that influence the desirability of projects. These are often the intangible or unmeasurable aspects of a project. Frequently they relate to other products or services of the compartments being treated. What will the influence of a project be on wildlife habitat, on water yields, on scenic values? For most projects these effects may be negligible. Some projects, however, do cause marked changes in the flow of other products, sometimes increasing them, sometimes decreasing them. The analyst could consider these changes as additional costs or returns to the project if they could be evaluated in dollar terms. When, as often happens, they cannot be so measured, they cannot be included in calculations to determine internal rates of return. For the present, consideration of these intangible effects is left to those who apply the modal project evaluations. The guides described in this handbook do not take these effects into consideration.

INTERACTION AMONG MANAGEMENT ACTIVITIES

A regeneration or stand improvement project can significantly influence the value of future management activities. For example, pruning in a young pine stand increases the quality and value of volume growth. Therefore, subsequent cultural treatments that save or increase this growth produce more value than they would in unpruned stands. Improvement cuts in young hardwood stands also make subsequent management practices more profitable for similar reasons.

The analysis of single regeneration or stand improvement treatments does not take these interactions into account. Most projects probably do increase the value and profitability of subsequent treatments, but this is not counted as one of the returns or added values of the initial project. Thus, return rates for modal projects in young stands may underestimate true economic benefits

Strictly speaking, individual projects should not be the unit of management evaluation. Rather, series of treatments—entire management regimens—should be compared. This would allow assessment of the combined economic benefit of a series of interdependent treatments, which could be summarized by the net or average rate of return to the entire series of practices scheduled.

To summarize, return rate estimates for modal projects are not perfectly reliable, and there is an additional source of error in applying them under nonmodal conditions. The important things for the analyst to remember are that users should be made aware of any marked differences in the reliability of various modal project return estimates, and that application error can be reduced by increasing the number of modal projects so that more treatment variations and stand conditions are evaluated. Also, return rates, although not the only valid measure of economic desirability, are easily used and understood, but become less sensitive measures as investment period increases. Small changes in return rates are significant where the investment period is 50 years or more.

Rates of return measure only the net effect of tangible costs and returns to projects. Intangible effects, which may sometimes be important and even overriding, are not taken into account. Users should be made aware of this, and they must judge the effect of intangibles. Finally, guides that consider only single treatments ignore the relationships and interdependence among sequences of treatments. It may sometimes be necessary to undertake subsidiary analyses of treatment series when individual analysis may significantly misrepresent the real contribution of a treatment.

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APPENDIX

COMPOUND INTEREST FORMULAS

Formulas Dealing With Single Values

1. Future value of a current cost or income:

$$V_n = V_o(1 + i)^n$$

2. Present value of a future cost or income:

$$V_o = V_n/(1 + i)^n$$

Formulas Dealing With Series of Equal and Annual Values, Beginning 1 Year Hence

3. Present value of a perpetual series:

$$V_o = V/i$$

4. Future value of a terminable series:

$$V_n = V[(1 + i)^n - 1]/i$$

5. Present value of a terminable series:

$$V_o = V[(1 + i)^n - 1]/i(1 + i)^n$$

Formulas Dealing With Series of Equal, Periodic Values, Beginning 1 Year Hence

6. Present value of a perpetual series:

$$V_o = V/(1 + i)^t - 1$$

7. Future value of a terminable series:

$$V_n = V[(1 + i)^n - 1]/(1 + i)^t - 1$$

8. Present value of a terminable series:

$$V_o = V[(1 + i)^n - 1]/[(1 + i)^t - 1](1 + i)^n$$

where

i = the annual rate of interest expressed as a decimal
n = the number of years over which a value is discounted or compounded, or the number of years in a terminable series
V = an annually or periodically recurring value
t = the number of years between periodic recurrences of V
V_o = the present or initial value at year zero
V_n = the future or end value after n years

A COMPUTER PROGRAM FOR RATE OF RETURN COMPUTATION

The program published by Row (1963) was written to compute, simultaneously, rates of return for up to six investment alternatives, each with separate physical output data, under a common set of price and cost assumptions. Internal rate of return is given by the interest rate that most nearly equates future costs and returns with the capital investment necessary to obtain them. Copies of Row's paper are available.

The program is suitable for computing return rates for many stand improvement and regeneration projects. However, it has been altered to allow analysis of alternatives which extend beyond 99 years and which include more numerous costs and returns. Another change permits the analyst to choose the range and interval of interest rates to be used in computing present worths. It is possible, for example, to use rates lower than 3 percent or greater than 30 percent and to set the interest rate increment as small as 0.1 percent.

A printout of the revised program and instructions for preparing data input cards follow. The program is written in Fortran IV language and can be used with any IBM[4] 700/7000 series computer.

[4] Use of commercial names is for identification only and does not imply endorsement.

```
*      LABEL
CMARTY
C      INVESTMENT ANALYSIS PROGRAM SOUTHERN FOREST EXPERIMENT STATION
C     1CLARK ROW 62
       DIMENSION ANC(10),CANC(10),NC(6,200),PECO(6,200),N1(6,200),YLD1(6,
      1200),QUAL1(6,200),N2(6,200),QUAL2(6,200),N3(6,200),YLD3(6,200),QUA
      2L3(6,200),PR(3,20),CPR(3,20),FVAL(25),RATE(200),RTLOG(200),VALIN(
      36,200),LY(6),KCX(6),K1X(6),K2X(6),K3X(6),A(26),LI(6),YLD2(6,200)
       DIMENSION RINT(3)
       DISC(X)= RTLOI**X
142    READ(5,17000)(RINT(I),I=1,3)
17000 FORMAT (3F4.3)
      RATE(1)=RINT(1)
      DO 17001 I=2,200
      IF(RATE(I-1)-RINT(3)) 17002,17003,17003
17002  RATE(I)=RATE(I-1)+RINT(2)
      GO TO 17001
17003 LENGTH=I-1
      IF(MOD(LENGTH,2)) 17004,17005,17004
17004  LLNGTH=(LENGTH+1)/2
      GO TO 137
17005  LLNGTH=LENGTH/2
      GO TO 137
17001  CONTINUE
137   DO 138  I=1,LENGTH
138    RTLOG(I)=1.+RATE(I)
      READ(5,11)NO,LZ,LX,(LI(L),L=1,6),(LY(L),L=1,6),KX,KCXX,
     1(KCX(L),L=1,6),K1XX,(K1X(L),L=1,6),K2XX,(K2X(L),L=
     21,6),K3XX,(K3X(L),L=1,6),  JX,MX,NZ,NX
11 .  FORMAT (3I2,12I3,I2/21I3/7I3,4I2)
      READ(5,15)(A(I),I=1,14)
   15 FORMAT (14A5)
      READ(5,15)(A(I),I=15,26)
      WRITE(6,20 )
   20 FORMAT (91H1                                 INVESTMENT ANALYSIS PROGRAM
     1- SOUTHERN FOREST EXPERIMENT STATION)
      WRITE(6,21)NO,(A(I),I=1,14)
   21 FORMAT (14H0  PROBLEM NO , I4, 1H , 16A5)
  144 IF(LZ-2)145,146,147
  145 WRITE(6,23 )
   23 FORMAT (73H0                                                 ROTATI
     1ON LENGTH IN YEARS)
      GO TO 149
  146 WRITE(6,24 )
   24 FORMAT (70H0                                                    SIT
     1E INDEX IN FEET)
      GO TO 149
  147 WRITE(6,25 )
   25 FORMAT (69H0                                                    PRO
     1DUCTION SYSTEM)
  149 WRITE(6, 26) (LI(L), L=1,LX)
   26 FORMAT (I11,5I20)
      IF (KCXX) 160,160,152
  152 WRITE(6,27 )
   27 FORMAT (17H0  PERIODIC COSTS,/115H   YEAR    COST          YEAR    C
     1OST         YEAR    COST          YEAR    COST          YEAR    COST
     2     YEAR    COST)
      DO 155 KC=1,KCXX
      READ(5,13)(NC(L,KC),PECO(L,KC),L=1,6)
13     FORMAT(6(I3,F9.2)  )
  155 WRITE(6,29)(NC(L,KC), PECO(L,KC),L=1,LX)
```

```
   29 FORMAT (I6,F9.2,5(I11,F9.2))
  160 IF (K1XX) 170,170,162
  162 WRITE(6, 30)(A(I),I= 15,18)
   30 FORMAT (25H0  PERIODIC RETURNS FROM , 4A5)
      WRITE(6,31 )
   31 FORMAT (118H   YEAR YIELD QUAL      YEAR YIELD QUAL      YEAR YIELD
     1QUAL      YEAR YIELD QUAL      YEAR YIELD QUAL      YEAR YIELD QUAL)
      DO 165 K1= 1,K1XX
      READ(5,14)(N1(L,K1),YLD1(L,K1),QUAL1(L,K1),L=1,6)
14    FORMAT (6(I3,F5.1,F4.2))
  165 WRITE(6, 32) (N1(L,K1),YLD1(L,K1),QUAL1(L,K1),
     1L=1,LX)
   32 FORMAT (I6, F6.1, F6.2, 5(I8, F6.1, F6.2))
  170 IF(K2XX) 180,180,172
  172 WRITE(6,30)(A(I),I=19,22)
      WRITE(6,31 )
      DO 175 K2=1,K2XX
      READ(5,14)(N2(L,K2),YLD2(L,K2),QUAL2(L,K2),L=1,6)
  175 WRITE(6,32)(N2(L,K2),YLD2(L,K2),QUAL2(L,K2),L=1,LX)
  180 IF (K3XX) 190,190,182
  182 WRITE(6,30)(A(I), I=23,26)
      WRITE(6,31 )
      DO 185 K3=1,K3XX
      READ(5,14)(N3(L,K3),YLD3(L,K3),QUAL3(L,K3),L=1,6)
  185 WRITE(6,32)(N3(L,K3),YLD3(L,K3),QUAL3(L,K3),L=1,LX)
12     FORMAT (8F9.3)
  190 READ(5,12)(ANC(J), CANC(J),J=1,JX)
      DO 191 M=1,MX
1200  FORMAT(6F8.3)
191   READ(5,1200)(PR(K,M),CPR(K,M),K=1,KX)
  192 IF(NZ-1)195,195,193
  193 READ(5,12) (FVAL(N),N=1,NX)
  195 DO 390 J=1,JX
      DO 390 M=1,MX
  198 DO 390 N=1,NX
      WRITE(6, 34)ANC(J),CANC(J)
   34 FORMAT (37H0  ANNUAL COST                          $, F7.2,  46H
     1 CHANGE IN ANNUAL COST                        ,2PF11.2, 17H PERCENT PER
     2YEAR)
      WRITE(6,35)(A(4*K+11),A(4*K+12),
     1A(4*K+13),A(4*K+14),PR(K,M),A(4*K+11),A(4*K+12),
     2A(4*K+13),A(4*K+14),CPR(K,M),K=1,KX)
   35 FORMAT (12H   PRICE OF , 4A5, 5H    $,0PF7.2, 26H        CHANGE IN
     1PRICE OF , 4A5,2PF11.2, 17H PERCENT PER YEAR)
      IF(NZ-1)204,204,205
  204 WRITE(6,36 )
   36 FORMAT (30H   PERPETUAL INVESTMENT SERIES)
      GO TO 210
  205 WRITE(6,37) FVAL(N)
   37 FORMAT (37H   VALUE AT END OF ONE INVESTMENT   $, F7.2)
  210 DO 345 L=1,LX
      KCXA=KCX(L)
      K1XA=K1X(L)
      K2XA=K2X(L)
      K3XA=K3X(L)
      XLY=FLOAT(LY(L))
      ASSIGN 220 TO NZERO
       DO 340  I=1,LENGTH
      GO TO NZERO,(220,330)
220   DANC=0.0
      DCANC=0.0
```

```
      DKC=0.0
      DK1=0.0
      DK2=0.0
      DK3=0.0
      DFVAL=0.0
      RTLOI=RTLOG(I)
      DISCO = RTLOG(I)**XLY
      IF(ANC(J)) 225,230,225
  225 DANC=(ANC(J)*(DISCO-1.))/(RATE(I)*DISCO)
  230 IF (CANC(J)) 235,240,235
  235 DCANC=(CANC(J)*ANC(J)*(DISCO-XLY*RATE(I)-1.))/(RATE(I)**2*DISCO)
  240 IF(KCXA) 250,250,241
  241 DO 245 KC=1,KCXA
      XNC=FLOAT(NC(L,KC))
  245 DKC=DKC+PECO(L,KC)/DISC(XNC)
  250 IF (K1XA) 260,260,251
  251 DO 255 K1=1,K1XA
      XN1=FLOAT(N1(L,K1))
  255 DK1=DK1+(YLD1(L,K1)*PR(1,M)*QUAL1(L,K1)*(1.+CPR(1,M)
     1*XN1))/DISC(XN1)
  260 IF (K2XA)270,270,261
  261 DO 265 K2=1,K2XA
      XN2=FLOAT(N2(L,K2))
  265 DK2=DK2+(YLD2(L,K2)*PR(2,M)*QUAL2(L,K2)*(1.+CPR(2,M
     1)*XN2))/DISC(XN2)
  270 IF(K3XA)280,280,271
  271 DO 275 K3=1,K3XA
      XN3=FLOAT(N3(L,K3))
  275 DK3=DK3+(YLD3(L,K3)*PR(3,M)*QUAL3(L,K3)*(1.+CPR(3,M)*XN3))
     1/DISC(XN3)
  280 TDVAL=DK1+DK2+DK3-DANC-DCANC-DKC
      IF(NZ-1)290,290,295
  290 VALIN(L,I)=TDVAL *(1.+1./(DISCO-1.))
      GO TO 340
  295 DFVAL=FVAL(N)/DISCO
      VALIN(L,I)=TDVAL+DFVAL
      IF(VALIN(L,I))330,340,340
330   VALIN(L,I)=0.0
      ASSIGN 330 TO NZERO
  340 CONTINUE
  345 CONTINUE
      WRITE(6,38 )
   38 FORMAT (70H   PRESENT DISCOUNTED NET WORTH AT GIVEN ALTERNATIVE RA
     1TES OF INTEREST)
      WRITE(6,40)(LY(L),L=1,6),(LY(L),L=1,6)
   40 FORMAT (7H   RATE, I9, 5I8, 12H        RATE, I9, 5I8)
      DO 365  I=1,LLNGTH
      I13=I+LLNGTH
  365 WRITE(6,41) RATE(I),(VALIN(L,I),
     1L=1,6),RATE(I13),(VALIN(L,I13),L=1,6)
   41 FORMAT(2PF7.1,0PF11.2,0P5F8.2,2PF9.1,0PF11.2,0P5F8.2)
  390 CONTINUE
      READ(5,11) IEND
      IF(IEND-98)400,142,410
  400 WRITE(6,42 )
   42 FORMAT(23H0  ERROR IN INPUT CARDS)
410   CALL EXIT
      END
```

Data Input Instructions—Row Interest Rate Program as Altered

Card	*Columns*	*Item*	*Field*
Control—1	1–4	Minimum rate of interest	.XXX
	5–8	Interest rate increment	.XXX
	9–12	Maximum rate of interest	.XXX
Control—2	1–2	Problem number	XX
	3–4	Alternative type 01, 02, or 03	XX
	5–6	No. alternatives 01–06	XX
	7–24	3-digit no. identifying each alternative	XXX
	25–42	Length in years of each alternative (1–999)	XXX
	43–44	Max. no. different products (0–3)	XX

Card	*Columns*	*Item*	*Field*
Control—3	1–3	Max. no. periodic costs	XXX
	4–21	No. periodic costs, each alternative	XXX
	22–24	Max. no. returns for product 1	XXX
	25–42	No. returns for prod. 1 each alternative	XXX
	43–45	Max. no. returns prod. 2	XXX
	46–63	No. returns for prod. 2, each alternative	XXX
Control—4	1–3	Max. no. returns prod. 3 (0–50)	XXX
	4–21	No. returns for prod. 3, each alternative	XXX
	22–23	No. sets of annual costs (0–10)	XX
	24–25	No. sets of product prices (0–20)	XX
	26–27	Type of terminal calculation 01 or 02	XX
	28–29	No. of final values (0–25)	XX
Problem Name	1–72	Name of problem	
Product Names	1–20	Name of product 1	
	21–40	Name of product 2	
	41–60	Name of product 3	
Periodic Cost—*i*	1–3	Year of *i*th cost for alt. 1	XXX
	4–12	*i*th cost for alt. 1	XXXXXX.XX
	13–15	Year of *i*th cost of alt. 2	XXX
	16–25	*i*th cost for alt. 2	XXXXXX.XX
	26–28	Year of *i*th cost for alt. 3	XXX
	29–37	*i*th cost for alt. 3	XXXXXX.XX
	38–40	Year of *i*th cost for alt. 4	XXX
	41–48	*i*th cost for alt. 4	XXXXXX.XX
	49–51	Year of *i*th cost for alt. 5	XXX
	52–60	*i*th cost for alt. 5	XXXXXX.XX
	61–63	Year of *i*th cost for alt. 6	XXX
	64–72	*i*th cost for alt. 6	XXXXXX.XX
Product returns—*jk*	1–3	Year of the *j*th return for the *k*th product for alt. 1	XXX
	4–8	Volume of the *j*th yield for the *k*th product for alt. 1	XXX.X
	9–12	Quality index of the *j*th return for the *k*th product for alt. 1	X.XX
	—	and so forth for all six alternatives	
Annual costs—*i*	1–9	*i*th annual cost assumption	XXXXX.XXX
	10–18	*i*th change in annual cost assumption	XXXXX.XXX
	—	and so forth for up to 10 sets	—
Prices—*i*	1–8	*i*th unit price assumption for product 1	XXXX.XXX
	9–16	*i*th annual change in unit price assumed for prod. 1	XXXX.XXX
	—	and so forth for all 3 products—(additional cards of other sets of price assumptions to a maximum of 20)	—
Final Values—*i*	1–9	*i*th final value assumptions	XXXXX.XXX
	—	and so forth for a maximum of 25.	—
Terminal Card	1–2	Terminal code (98 or 99)	XX

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