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THE ALMOST IDEAL DEMAND SYSTEM AND ITS APPLICATION IN GENERAL EQUILIBRIUM CALCULATIONS

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Summary

Deaton and Muellbauer developed the Almost Ideal Demand System (AIDS) to help analyze consumer behavior. Since its introduction in 1980, AIDS has become a staple of demand theory and has been used in numerous empirical studies. The model facilitates the quantification of consumer demand, based on the well-established theoretical axioms of optimal consumer behavior. The word "ideal" expresses the intention to link actual data to pure theory, i.e., the maintained hypothesis of consumer behavior. "Almost" conveys the fact that results obtained by using the system may have only a limited validity. AIDS belongs to the general class of the flexible functional form approach to demand analysis, a methodology that has been in use since the advent of high-speed computers in the 1960s.

Sherman Robinson pioneered the application of AIDS in trade-related computable general equilibrium (CGE) models in the early 1990s. These models can be run either with AIDS or the constant elasticity of substitution (CES) system to determine the shares of domestically-sourced and foreign commodities under various scenarios. One of these models is the Western Hemisphere Free Trade Area (WHFTA) model. The investigation of the applications of AIDS in the WHFTA model shows that the methodology offers significant advantages in examining the consequences of trade liberalization between the United States and various Latin American countries.[1]

Introduction

The Office of Economics of the USITC uses and maintains a number of computable general equilibrium (CGE) models, and it monitors methodological developments in the field. An understanding of AIDS and its applications in CGE analysis is essential to improving existing models and depicting consumer demand more realistically in future models.

The paper describes AIDS and its place in CGE analysis. It introduces Sherman Robinson's Western Hemisphere Free Trade Area (WHFTA) model, which is equipped with the AIDS feature, and makes specific observations about its application. Background theoretical information is presented in the appendices. Appendix A describes those aspects of the linear expenditure system that are relevant in the derivation and CGE applications of AIDS; Appendix B shows some pertinent features of the Cobb-Douglas utility function; Appendix C treats the Cobb-Douglas utility function as a special case of the constant elasticity of substitution (CES) utility function; Appendix D describes the Cholesky factorization, a methodology that accompanies the application of AIDS in CGE models; and Appendix E provides a numerical illustration of the application of AIDS. A bibliography of cited works and other sources appears at the end of the paper. Bibliographical references in text are in parentheses

[1] The USITC Office of Economics is in the process of building its own Latin American Regional (LAR) model based on the WHFTA model.

The Almost Ideal Demand System (AIDS)

Angus Deaton and John Muellbauer first described AIDS in *The American Economic Review* (Deaton and Muellbauer, 1980). Their stated goal in devising the system was to improve upon the so-called "flexible functional form" approach to consumer demand analysis.

AIDS as a Flexible Functional Form

The modeling technique based on the flexible functional form approach, widely used in the sciences, may be characterized by the following equation:

$$F(x)=\sum_n C_n f_n(x) \tag{1}$$

where the left side stands for the estimated values of some system, and the C's are parameters, each of which is attached to a function "f" that has to be specified by the modeler. Each element in the sum, i.e., a given parameter coupled with its matching function, stands for one of the effects that the entire equation (1) is designed to capture. The symbol "n" represents the total number of effects the model considers. In demand analysis, "n" is typically 2, since analysts are interested in two kinds of effects; the income effect and the substitution effect. The whole approach is called flexible, because it allows a free choice in the number and combination of the effects to be considered and the types of functions to be used in describing them. Equation (1) is regarded as "generally linear," because it adds up equations and because it remains linear in the parameters even if the equations that make it up may not be linear. (For a detailed description of the flexible functional approach to economic modeling, see Chambers, 1988, pp. 158-202.)

To be able to ascertain the existence of solutions, the functions "f" need to be at least twice continuously differentiable. If approximations involve only the first differentials of equation (1), they are called "first-order" approximations, even if the functions selected are twice or more differentiable. If approximations involve the second differentials, they are called "second order approximations." Second order approximations subsume first order approximations. As it will be seen, AIDS is a first order approximation.

The fundamental principle of the flexible functional form in demand analysis is the neoclassical postulate that consumers optimize their choices. The mathematical conditions of optimal consumer behavior are the explicit basis of the functions that make up equation (1) and they are presumed to be implicitly present in the actual data matched with them. In terms of symbols, equation (1) contains all the usually considered parameters of consumer behavior, that is, intercepts (denoting initial levels of consumption), own- and cross-price elasticities, and income elasticities (depicting substitution and income effects, respectively). The fitting of actual data, representing some arbitrary demand system at a given level of commodity breakout, with equation (1) will result in the approximation of the unknown parameters.[2]

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[2] At any meaningful commodity detail, the number of parameters supplied is likely to be considerably fewer than the theoretical maximum. For example, in an 11-sector system, the

Approximations of an arbitrary equation system (such as a system of demand equations that underlie actual consumption data) through the flexible functions approach may have validity only for the data used. The flexible functional approach checks validity only in the "neighborhood" of the solution generated by a given set of data.[3] The flexible functional form approach cannot guarantee the validity of calibrated parameters once a different data set is used. Approximations gained through equation (1) are "locally flexible" approximations, i.e., the flexibility in determining the parameter vector C is restricted to the data used. If the observation vector "x" in equation (1) contains k elements, the number of parameters that may be estimated by the system will be at least (k+1)(k+2)/2. [4]

The advantage of approximations gained through other methods than the flexible functional form is that they may have "global" validity. For example, equations that are versions of the geometric mean, such as the CES equations, including their well-known subcategory the Cobb-Douglas-type equations, all have reliable "global" properties. They may be matched with any number of data sets, their mathematical properties guarantee that once their parameters are calculated, these will remain valid for any number of data sets. Their disadvantage vis-a-vis the flexible functional form is that they are restrictive in the number of parameters they use. The CES method would estimate as many parameters as the number of observations plus 2 constants, i.e. (k+2) parameters. The Cobb-Douglas-type approach would estimate as many parameters as the number of observations plus 1 constant, i.e. (k+1) parameters. To summarize, the flexible functional form is superior in the number of parameters it can compute, but it may be restrictive in accommodating changes in the data that generated the parameters. The CES and Cobb-Douglas approaches are superior in their readiness to handle variety in data used, but they are relatively restrictive in the number parameters they can compute.

The flexible functional form approach to demand analysis began to emerge in the 1960s with the increased availability of high-speed computers and detailed consumption data. Prior to that epoch, economists had to be content with a comparatively restricted number of parameters and, in general, less detailed data, embodied in shorter times series. Before AIDS, the so-called "Rotterdam" model and the transcendental logarithmic (translog)[5] model were the best known

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number of theoretically possible cross-price elasticities (the most populous category among the parameters) is 11 x 10 x.9!/2! x 9!= 55.

[3] For the flexible functional form estimate to be valid at the given data set, the associated Wronksian matrix must be non-singular. For details, see (Chambers, 1989, p. 163).

[4] The formula provides for the number of parameters in the case the estimate is based on the cross-effects of one single variable and one type of constant. For example, in an 11-sector system, the formula would yield 78, representing the sum of 55 cross-effects, 11 "own"-effects, 11 sector constants, and 1 overall function constant.

[5] The so-called translogarithmic, or shortly translog, functions represent abbreviated Taylor series approximations of logarithmic functions at the domain value of "1." The translog form of estimation is sometimes referred to as a "second-order Taylor approximation," because it

flexible functional form approaches to demand analysis (Deaton and Muellbauer, 1986, pp. 67-73; Chistensen, Jorgensen, and Lau, 1975. For a succinct survey of the flexible functional form approach to demand analysis, see Chalfant, 1987.)

Deaton and Muellbauer claim (Deaton and Muellbauer, 1980) that AIDS has major advantages over other flexible functional form models in testing the maintained hypotheses of demand theory, such as the zero homogeneity of demand functions.[6] Moreover, according to the inventors, parameters obtained through AIDS for the national level correspond better to observed household demand behavior than parameters obtained through other flexible functional form models.

Derivation of AIDS

The Cobb-Douglas utility function, reflecting additive preferences between subsistence and above-subsistence level consumption, is at the root of AIDS. Following the notations of Deaton and Muellbauer, if consumption is divided between subsistence level "a" and above-subsistence or "bliss" level "b" for a given commodity, the Cobb-Douglas direct utility function takes the following form:

(2) $$V(q) = a^{1-u}\, b^{u}$$

where "1 - u" indicates the proportion of subsistence and "u" the proportion of "bliss" level consumption. The indirect cost function, containing the utility levels derived from consumption may be written as follows:

(3) $$C(U,p) = (p.a)^{1-u}\,(p.b)^{u}$$

where "p" stands for the price vector. In estimating expenditure levels, "p.a" and "p.b" may be replaced with general functions to be specified later:

(4) $$C(U,p) = a\,(p)^{1-u}\, b(p)^{u}$$

If the newly-specified cost function remains linearly homogenous in the functions "a" and "b," which, in turn, remain linearly homogenous in prices, the demand equations derived from it will be homogeneous to a degree of zero in prices. Since the Cobb-Douglas-type utility function is closely tied to the linear expenditure system (Appendix A), cost functions based on it are called "general linear cost functions." Since in this type of cost function, expenditure shares are independent of prices (Appendix B), it may also be called "price-independent general linear cost

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involves the first and the second differentials of the estimated equation. For details, see (Apostolakis, B.E., 1988).

[6] Much of the theoretical achievements of demand analysis are based on the assumption that demand functions are homogeneous to a degree of zero in prices and total expenditures. In other words, if all prices and expenditures change in the same proportion, the quantity demanded would remain unchanged.

(5) $$\ln C(U,p)=(1-u)\ln a(p)+u\ln b(p)$$

functions" (PIGL). Monotonic transforms do not change the order of utility levels attributed to the various commodities in a utility function. Hence, a PIGL cost function may take a logarithmic form as shown in equation (5).

Such functions are called "price-independent, general linear log functions" or "PIGLOG" functions for short. Deaton and Muellbauer began the derivation of the AIDS by specifying a(p) and b(p) for this skeleton version of the PIGLOG class cost function (Deaton and Muellbauer, 1980.) They specified the following two functions with the Greek letters indicating parameters:

(6) $$\ln a(p)=a_0+\sum_k \alpha_k \ln p_k+\frac{1}{2}\sum_k \sum_j \gamma_{kj}\ln p_k \ln p_j$$

(7) $$\ln b(p)=\ln a(p)+\beta_0\prod_k p_k^{\beta_k}$$

These equations were selected, because they lead to a system of demand functions with desirable properties. Equation (6) is the so-called translog price index. Combining (6) and (7) and substituting them into the PIGLOG function (5) yields the AIDS cost function:
Equation (8) corresponds to equation (1), describing the general flexible functional format. The

(8) $$\ln C(U,p)=a_0+\sum_k \alpha_k \ln p_k+\frac{1}{2}\sum_k \sum_j \gamma_{kj}\ln p_k \ln p_j+u\beta_0\prod_k p_k^{\beta_k}$$

demand functions can be derived from this cost function. By Shepard's lemma, the first derivative of the cost function with regard to a commodity's price yields an equation that describes demand for the same commodity (Diewert, 1971).

(9) $$\frac{\partial C(U,p)}{\partial p_i}=q_i$$

Notice that the demand function approximations are "first order" since they are based on first derivatives. AIDS demand functions are defined in terms of budget shares. Multiplying both sides of (9) by $p_i/C(U,p)$ results in

(10) $$\frac{\partial C(U,p)}{\partial p_i}\frac{p_i}{C(U,p)}=\frac{q_i p_i}{C(U,p)}=w_i$$

Since the left side of equation (10) is an elasticity type expression, it may be written as

(11) $$\frac{\partial \ln C(U,p)}{\partial \ln p_i}$$

Thus, the logarithmic differentiation of the cost function (8) gives market shares:

(12) $$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i U \beta_0 \prod p_k^{\beta_k}$$

Since utility maximization means spending at the level of the budget constraint, U may be expressed. Setting equation (8) equal to total expenditure (X), expressing total utility (U), and substituting the result into equation (12) will be the AIDS demand function in share form:

(13) $$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(\frac{X}{P})$$

where P is the translog price index, equation (6). The interpretation of AIDS is simple: Market shares change with relative prices and real expenditures. The expression (X/P) in the last term of equation (13), denoting real expenditures, is often treated as a constant. The restrictions on the parameters are as follows:

(14) $$\sum_i \alpha_i = 1 \quad \sum_i \gamma_{ij} = 0 \quad \sum_i \beta_i = 0 \quad \sum_j \gamma_{ij} = 0 \quad \gamma_{ij} = \gamma_{ji}$$

These restrictions ensure the four properties of a consistent demand system, described in Appendix A. The particularity of AIDS is that it allows a limited departure from the restrictions involving cross-price effects, i.e., the gammas, for the sake of realism in estimating demand. This, of course, is a benefit with some risk. (See more on the subject in the section entitled "A Word of Caution.")

Equation (13) gives rise to so-called quasi-homothetic consumer preferences, because it contains an intercept. Even at zero expenditures, there is demand for a commodity in question. [7] In this it contrasts with the implied homothetic preferences derived from demand share

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[7] A function e(x) is homothetic if it can be written as f((g(x)), where "f" is a monotonically increasing, and "g" is a linearly homogenous function. Since monotonicity is generally subsumed in utility analysis (i.e., the analysis usually shows the consequences of a constantly increasing budget constraint as, for example, in mapping indifference curves), linear homogeneity remains the crucial criterion. Thus, in general, a community has homothetic preferences if increases in expenditures bring constant returns to the value of its utility function. Homothetic preferences imply straight line Engel curves crossing the origin, that is unitary expenditure elasticities. Once a straight line Engel curve has an intercept, it no longer has a unitary expenditure elasticity. Nevertheless, elasticity along such lines approaches unity as total expenditures increase. Therefore, linear Engel curves with an intercept are said to stand for quasi-homothetic preferences.

estimations through the constant elasticity of substitution (CES) approach. (See more on the contrasts between AIDS and CES in the section entitled "The Place of AIDS in CGE Models" and in Appendix C.) In CGE models, where prices and expenditures are endogenous, AIDS-generated market shares no longer correspond to any preference system associated with Engel curve estimations.

Substitution of the Stone Price Index

The prices of a national economy are perhaps the best examples of high level but less than complete collinearity. They tend to move together with the business cycle and in response to economic policies. (See, for example, Evans, 1969, p. 598.) The presence of collinearity in regression analysis is known to create severe difficulties when the aim is to separate the explanatory role of specific independent variables. However, collinearity actually helps when the sole purpose is to estimate the level of the endogenous variable. A significant level of stable collinearity in data means that different sets of coefficient estimators may serve as equally good substitutes for one another.

Deaton and Muellbauer suggested that the Stone price index may be used in lieu of the translog price index. Since exponential weights in the Stone index correspond to the market shares of the commodities or commodity groups weighed (Appendix B), its substitution into equation 13 simplifies calculations, allowing for a straightforward application of OLS. In CGE models the choice between the two versions requires careful consideration. The use of linear regression to estimate budget share equations that contain the translog price index could pose statistical problems (Hanson, Robinson, Tokarick, 1993). On the other hand, the translog version tends to inject more precision into the details than the Stone version, pointing the users of a CGE model to particular problems relevant to the analysis at hand. (For more on this subject, see "Some Observations on Running WHFTA with Various Functional Forms," later in this paper.)

Uses of AIDS

Since its invention in 1980, AIDS has been used in demand studies for the following seven, often closely interrelated purposes: (1) Approximation of unknown parameters; (2) test of the symmetry of cross-price elasticities; (3) test on the zero price homogeneity of a demand system; (4) test of separability of products (i.e., if cross-price elasticities are indeed zero between two products, belonging to two presumably separate commodity groups); (5) test of homotheticity (i.e., test if expenditure elasticities are indeed unitary); (6) test of demand theory itself in light of practical results; and (7) calculations of market shares in general equilibrium calculations.

A Word of Caution

Deaton and Muellbauer qualified their system with the adjective "almost" for good reasons. Although they never specified the gap between the absolutely and almost ideals, two groups of potential problems with AIDS stand out: first, the AIDS does not guarantee a global optimum. Solution of the demand system with AIDS means that the calculations confirmed the existence of an at least strictly quasi-concave cost surface segment in the neighborhood of the optimum solution.[8] Second, exogenous expenditure and substitution elasticities in an AIDS model, as in other flexible functional form models, are known to cause departures from the restrictions listed in equation (14). These problems are not necessarily shortcomings in the flexible functional form. They may be regarded as simplifications imposed by the maintained hypotheses of general demand theory. During the past decades, some empirical work based on flexible functional forms produced such convincingly good statistical results even while violating theoretical restrictions that a number of influential theoreticians began to doubt the validity of some fundamental aspects of maintained demand theory. (See, for example, Christensen, et al. 1975.) Equally influential scholars refuted these attacks and the debate over the merits of the existing demand theory continues (Deaton and Muellbauer, 1986, p. 74).

Nevertheless, departures from theoretical restrictions could cause problems in general equilibrium calculations where consumer demand is only a subsystem in the overall model. If the AIDS-based consumer demand calculations turned up one single negative market share, the entire general equilibrium calibration would come to a halt. The negative market share describes an upward sloping demand curve, representing irrational consumer behavior. (The following footnote shows the derivation of the positive slope from a negative market share.)[9] It means that the second derivative of the cost function with regard to the price of the given commodity is

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[8] "Strictly quasi concave" represents a minimum requirement. It means that the curve is expected to have other than a perfectly concave surface. "Weakly quasi concave" would allow for the possibility of a perfectly concave surface. The determinant of the matrix of the second derivatives (Hessian) obtained from a strictly quasi concave surface is expected to be a negative semidefinite. (I.e., all eigenvalues are nonpositive.) In contrast, the determinant of the Hessian of a strictly concave surface is expected to be a negative definite. (I.e., all eigenvalues are negative.)

[9]

$$\frac{p_i q_i}{X} = -w_i \qquad p_i q_i = -w_i X = -C \qquad q_i = -\frac{C}{p_i} \qquad q_i = -C p_i^{-1} \qquad \frac{\partial q_i}{\partial p_i} = C p_i^{-2} > 0$$

positive,[10] i.e., the consumer is buying more at higher prices, becoming worse off than doing nothing in the face of a price rise. The curvilinear cost curve, which should be concave in each price, allowing for some straight line segments on a two-dimensional illustrative chart, is positive instead and it accelerates in the place examined. That is the reason for the positive sign of the second derivative (the slope of the demand curve). The passive behavior would mean a zero second derivative, since the no-action cost curve is an upward sloping straight line. (For details, see Varian, 1992, p. 73.)

This single symptom of nonconcavity in the cost curve would disqualify the determinant of the compensated cross price derivatives (Hessian) from being a negative semidefinite. One of the requirements for the Hessian to be a negative semidefinite is that its diagonal elements, i.e., the "own" compensated price responses, be negative or zero. One restriction of the optimization process is that it stops when it finds a positive sign in the main diagonal. (The other standard requirement that the matrix be symmetric is imposed through restrictions built into the model.) The process is fair, because even a single positive sign in the main diagonal signals general problems with the system. Since market shares must add up to one, the presence of a negative share would imply the presence of least one other share that exceeds one. For example, in a Cobb-Douglas type cost function, the second derivative of the cost curve with regard to a price that has an exponent greater than one is positive, indicating the presence of another convex segment on the cost curve.

Forcing solvability on an AIDS model restricts its flexibility. Although these restrictions ensure compliance with economic theory, they might generate economically nonsensical solutions. Thus, the AIDS approach may be inappropriate to analyze shocks that tend to move market shares significantly from their initial values. (See more on the subject in the section entitled "Problems that May Emerge and Their Correction" and in Appendix D.)

AIDS in General Equilibrium Calculations

The Place of AIDS in CGE Models

Current CGE models incorporate a two-stage decision approach in determining commodity market shares. In the first stage, the models use the LES to determine demand for composite commodity groups. In the second stage, the models divide each group into domestic output and imports. The choice between the CES and AIDS emerges only in this second stage.

Thus far, in CGE models the CES has been the prevalent functional form used to disaggregate demand for a composite commodity into domestically-sourced and foreign-sourced components. Two simplifications inherent in this approach motivated research to replace it. First, the CES approach is based on a single elasticity of substitution per commodity group. Yet developed economies have a demonstrably greater ability to substitute imports for domestic

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[10] The positive sign of the own-compensated price elasticity in the Slutsky matrix implies that the good in question is "inferior," when, owing to the simplifications of the maintained hypotheses, such goods must be explicitly ignored when analysis hinges upon the concavity of the cost curve.

products than developing economies. Experience shows that in most commodity groups, the actual elasticities of substitution in developed countries are greater than in developing countries. Second, the CES approach brings with it uniformly unitary income elasticities for all commodities and all countries. The unitary income elasticity in the CES approach follows from the fact that CES aggregation represents homothetic Engel curves, i.e., functions linking demand for a commodity to total expenditures are upward sloping straight lines crossing the origin. (At zero expenditure, there is no (effective) demand for either domestically-sourced or foreign goods in the commodity group.) Elasticities along such lines are always unitary. However, statistical evidence indicates that the income elasticities of imports tend to be larger than unity. During the postwar era, the growth of world trade outpaced the growth of aggregate world output. As a result of increased international specialization, and consequent economic interdependence, producers and consumers spent an increasing share of their income on imported goods. Evidence also shows that the income elasticity of import demand is generally larger for developed than for developing countries.

The AIDS approach in CGE models enlarges the number of behavioral parameters by reducing the restrictions of models not based on the flexible functional approach. It introduces as parameters different elasticities of substitution for the commodity shipped from different sources, and different income elasticities for different countries or regions in the model. By allowing variety in both cross-country substitution patterns and import demand, the AIDS approach can incorporate the basic Armington assumption about the imperfect substitutability between imported and domestic goods.

Allowing differences in substitution and income elasticities can have a significant influence on the assessment of the welfare effects of trade liberalization. In particular, analysis of trade flows in newly-formed trading blocs, using differentiated substitution and income elasticities is likely to reveal a less balanced flow of trade within the bloc than an analysis using the customary CES specifications. Moreover, the imbalance in trade within a newly-formed bloc is bound to have implications for trade outside the bloc. (For the first application of CGE analysis to evaluate the consequences of regional trade agreements, see Benjamin, 1995.)

AIDS Market Share Equations in an Armington Trade Model

In an Armington-type model, where commodities imported from different countries are considered different commodities that are not perfect substitutes for one another, the AIDS demand share equation (13) takes the following form:

$$w_{i,k,c1} = \alpha_{i,k,c1} + \sum_{c2} \gamma_{i,k,c1,c2} \ln P^{m}_{i,k,c2} + \beta_{i,k,c1} \ln[\frac{X_{i,k}}{P_{i,k}}] \tag{15}$$

where "i" stands for the commodity in question, "k" for the reference country, "c_1" for the country whose market share the equation estimates, and "c2" stands for the other countries in the model. Whereas in equation (13), the gammas capture the impact of substitutability between commodity "i" and other commodities "j" upon the budget share of "i" in only one country, the

gammas in equation (15) capture the impact of substitutability between different sources of imports in commodity "i" in different countries, indexed over "k". Whereas in equation (13), the betas capture the influence of the income elasticity in the "i-th" commodity, the betas in equation (15) reflect the income elasticity in commodity "i" in country "k" from an import source.

Equation (16) shows the relationship between income elasticities (denoted with epsilons) and betas, and equations (17) and (18) show the relationship between substitution elasticities (denoted with sigmas) and gammas, in case the Stone price index is used:

$$\beta_{i,k,c1} = (\varepsilon_{i,k,c1} - 1) * w_{i,k,c1} \tag{16}$$

$$\gamma_{i,k,c1,c2} = (\sigma_{i,k,c1,c2} - 1) * w_{i,k,c1} w_{i,k,c2} \tag{17}$$

$$\gamma_{i,k,c1,c1} = (\sigma_{i,k,c1,c1} - 1 + \frac{1}{w_{i,k,c1}}) * w_{i,k,c1}^2 \tag{18}$$

These equations show that, in addition to the respective elasticities, only market shares play a role in determining the value of the two parameters.[11]

In a CGE model, the AIDS market shares are endogenous. AIDS equations are linked to the rest of the model through shared variables. They represent a subsystem of the entire model. Nevertheless, in addition to calculating market shares, running the model with AIDS allows for the completion and systematization of behavioral parameters. Completion of the behavioral parameters occurs when the number of equations exceeding the number of unknowns allows for the calculation of unknown betas or gammas, and by extension for the missing parameters. Entering some market shares as fixed is a way to make room for parameters among the unknowns to be calculated. Systematization occurs through scaling of the parameters to make them conform to the algebraic requirements of maintained demand theory. (Appendix E provides a numerical illustration.)

Counterfactual simulations use the complete set of behavioral parameters initially entered or subsequently generated by the model's base solution. AIDS market shares may be different from actual ones, because they represent shares under the assumptions of utility maximization and equilibrium. The theoretical expectation is that as a result of optimizing behavior encompassing an entire system of linked economies, actual market shares will converge to AIDS market shares. Such flexible-function-generated "equilibrium" elasticities are often used outside the context of the model that originally produced them. A model equipped with the AIDS feature can be switched to the CES approach, by suppressing cross-country elasticities of substitution in equation (17), and by setting income elasticities to one in equation (16). Substituting "1" as

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[11] The derivations are too long to be presented in this paper. The interested reader may obtain them in the form of hand-written documentation from the USITC Office of Economics. The USITC documentation is based entirely on the work of Sherman Robinson. For the formula describing the relationship between substitution elasticities and gamma parameters in the translog version, see (Robinson, S, Soule, M.J., and Weyerbrock, S., 1992, p. 10).

income elasticity in equation (16) causes the betas to vanish.

Problems that May Emerge

Policy experiments using AIDS may shock a model beyond its capacity to accommodate the changes being tested. The inconsistencies created may be mathematical and/or economic. Mathematical inconsistency in this context means that the parameters cannot accommodate the experiment-induced change in the data set which served as the basis for their calculation. Economic inconsistency means that the experiment did not produce economically meaningful results. The two may occur together or separately. In 1986, Kostas Despotakis published a study (Despotakis, 1986) on classifying and analyzing the two types of inconsistencies found in 2- and 3-input flexible production function estimates. Despotakis, following already established mathematical terminology, called the range of mathematically well-behaved results the "outer domain," and the subset of economically consistent results within it, the "inner domain." The pioneering study, which is still one of the most recommended published works on the subject, drew attention to the fact that the analysis of inconsistencies in the context of general equilibrium analysis is much more complex than in the limited framework of a 2- or 3-input production function. It also called for more investigation in this area. The growing practical application of CGE models appears to necessitate more thought on accommodating the inconsistencies of flexible functional form estimates, an inevitable concomitant of their many advantages over the nonflexible functional form estimates.

In a CGE model using AIDS, the departure of one single market share from the 0-to-1 interval is sufficient to create a mathematical inconsistency. Given the economic constraint that all market shares must add up to one, this means that at least one market share becomes negative. As indicated before, the negative market share breaks the model, because it implies an upward-sloping demand curve, i.e., the presence of a convex segment on the cost curve.

The axioms of choice demand that the utility function be an upward sloping, concave surface, allowing for some occasional "flat" spots on it. This ensures that the upper contour set, the set lying above any slice into the utility surface, is convex. The convexity of choices is, of course, a fundamental requirement for optimization. In case of a three-dimensional utility surface, representing the utility derived from the consumption of two commodities, the convexity of the upper contour sets guarantees that the horizontal slices (or level sets) of the utility function, i.e., the indifference curves, are convex to the origin. The convexity of these indifference curves, again with some flat spots allowed, reflects the diminishing marginal rates of substitution, an indispensable mathematical condition of optimum choice. The irregularity detected in the cost-curve dual of the utility function would be manifest in a concave bend in the otherwise convex-to-the origin indifference curves.

In the Robinson models, this problem is corrected by using the principle of the so-called Cholesky factorization, which guarantees the convexity of choices (Appendix D). The imposition of the mathematically conforming approach on the choices rectifies at once all the interrelated mathematical inconsistencies caused by any market share straying out of the 0-to-1 interval. Nevertheless, this intervention causes the system to lose an undetermined degree of flexibility and it may or may not produce economically consistent results. Thus, in a CGE model equipped

with the AIDS feature, the outer domain can be expanded easily but not without the danger of narrowing the inner domain.

Recommendations for Analyzing Model Results

As Despotakis pointed out, the inner domain of flexible functional form estimates is unquantifiable. In other words, an a priori limitation of the mathematically feasible solutions to a subset of economically meaningful solutions is not possible at this point. The analyst is left to use a different combination of methods to determine the economic usefulness of results in each instance. Later, these methods might become systematized, but in the current state of the art, a great deal is left to the analyst.

Policy simulations may fall into the small, medium, and large categories, according to the intensity of the shock caused to the model. The realm of changes may be classified into macro and micro levels. The analysts may expect policy simulation changes in macro-aggregates to be commensurate with the intensity of shocks, providing a basis for judging the economic consistency of simulation results. On the sectoral level, the correspondence between the intensity of shocks and the consequences may be less predictable. A small shock could cause major perturbations in some sectors as the model restores micro-balances. Micro-balances mean that sectoral output levels are consistent with sectoral input levels, sectoral demand equals sectoral supply, and the economic agents do not violate their budget constraints.

Using these basic principles as a guide, some additional calculations may be performed to determine if a counterfactual policy simulation produces economically meaningful results, i.e., if a newly-generated solution falls within the inner domain. One group of such calculations might be designed to investigate how some variables critical to the experiment vary around the benchmark point. Pushing the model outside the outer domain intentionally may be useful. Another group of calculations may involve econometric work using post-benchmark data. Such calculations could be performed relatively easily, if the parameters used by the model were available independently of the model's base data. In evaluating the consequences of counterfactual experiments, the analyst should also use information obtained from sources other than the CGE model.

Despite its shortcomings, the CES approach does not violate theoretical restrictions. CES functions are self-duals. A CES utility function is concave in the quantities of commodities consumed, as the corresponding CES cost function is concave in the prices of those commodities. Changes in the ratios of commodities consumed (market shares) cannot break this correspondence. Any sector-specific set of substitution elasticities could be used to evaluate the consequences of any policy experiment.

Therefore, it may be a good practice to run policy simulations using both the AIDS and CES approaches, and to look at both results before drawing conclusions. However, it should be pointed out that the choice of functional form has a strong influence on the results from a policy experiment at both the macro and micro levels (McKitrick, 1995). Each approach creates its own universe of scales. Numerically identical behavioral parameters may have different implications under the two systems, and there are no standards by which to judge which approach produced the better results in a given policy experiment.

The WHFTA Regional CGE Model

The Western Hemisphere Free Trade Area (WHFTA) model was conceived and is being maintained by Sherman Robinson. It includes the United States, Mexico, Argentina, Brazil, Chile, and the rest of the world (ROW) as a unit. All units except ROW are described by CGE equations. The model incorporates the following 11 sectors: corn, other program crops, fruits and vegetables, other agriculture, food processing, light manufacturing, oil, intermediary goods, consumer durables, capital goods, and services.

The Social Accounting Matrix (SAM) contains the following entries for each unit: activity, commodity, land, urban skilled labor, urban unskilled labor, rural labor, professional labor, households, enterprises, government, $country_1$, $country_2$, $country_3$, $country_4$, rest of the world, and capital account.

The model can be run either by using CES aggregation or the AIDS. To run the model with CES aggregation, it is sufficient to restrict substitution elasticities among different countries to be identical in the same product category and to set expenditure elasticities of demand to 1 The model can also be run with mixed CES aggregation and AIDS, i.e., only some trade relations are subjected to a more detailed determination inherent in the AIDS process. In running AIDS, assumptions concerning the substitution elasticities and expenditure elasticities can be varied to correspond to various hypotheses. Policy experiments may be specified to use earlier calculated parameters or to generate new ones.

The parameters for running the model with AIDS equations incorporate the theoretical expectations that substitution elasticities are relatively high for developed countries and relatively low for developing countries. The WHFTA model classifies the United States and ROW as developed, and the rest of the countries in the model, i.e., Mexico, Argentina, Brazil, and Chile, as developing countries. AIDS also allows a variation in the substitution elasticities of commodities. The theoretical expectation is that substitution elasticities must be low for capital goods and high for homogenous agricultural products. WHFTA assigns the following substitution elasticities to the commodity sectors:

Capital goods: very low (0.6)
Intermediary goods: medium low (0.8)
Oil: unitary (1.0)
Food, other agricultural products, light manufacturing, and services: slightly elevated (1.4)
Fruits and vegetables, and consumer durables: medium high (2.0)
Corn and other program crops: high (4.0)

The expenditure elasticities of imports were chosen by Robinson et al. using econometric estimations. These elasticities vary between 0.9 and 3.0, with an average value close to 2.0. The developed countries have higher expenditure elasticities than the developing ones.

Some Observations on Running WHFTA with Various Functional Forms

The following functional forms were used in running the model: (1) CES; (2) partial AIDS with varying substitution elasticities, but expenditure elasticities set uniformly at 1;[12] (3) full AIDS i.e., with both substitution and expenditure elasticities varying, using the Stone price index; and (4) full AIDS using the translog price index.

Four counterfactual trade liberalization experiments served as the basis for the observations: (1) Elimination of trade barriers between the United States and Chile; (2) elimination of trade barriers between Argentina and Brazil, combined with the effects of NAFTA in the U.S.-Mexico subregion; (3) elimination of trade barriers between the U.S.-Mexico NAFTA subregion and Brazil; and (4) elimination of trade barriers between the U.S.-Mexico subregion of NAFTA and Argentina. The base run and the experiments contained a fixed level of trade deficit for the group of countries included in the model.

Changes in the following variables were observed as a result of varying the functional forms: Real GDP, real absorption (i.e., real GDP minus intermediary consumption), trade volumes, and international terms of trade (i.e., world prices of exports divided by world prices of imports.) The experimental runs fell into two categories: runs with behavioral parameters provided with the model and runs with altered parameters.

Observations on Using Model Parameters

The four functional forms produced only minor variations in GDP levels. Nonetheless, by a slight margin, the CES version produced the largest real GDP figures in all experiments. No clear sequence emerged among the three AIDS versions. In all experiments, the AIDS/translog version produced the largest real absorption.

The CES version produced the largest export levels for all the experiments, and the AIDS/translog version was an uncontested second. No clear pattern emerged between the other two functional forms. Since the trade deficit was fixed for the group of countries as a whole, the CES version also produced the largest import figures in the four experiments.

The international terms of trade were set as 1 for all functional forms. With the exception of Mexico, which showed a minor decline in its international terms of trade in experiments 3 and 4, this measure remained unchanged within the three decimal points range for all the countries and in all the experiments. Percentage changes varied from one functional form to a another, but to a statistically insignificant extent. Furthermore, the algebraic signs of the changes were largely consistent across specifications, i.e., regardless of the model, the changes tended to point in the

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[12] Income elasticities were made equal to 1 by setting the beats uniformly to zero in equation (16). When the beats are zero, the relationship between substitution elasticities (sigmas) and the price parameters (gammas) remains the same regardless of whether the Stone or the translog price index is used. See equations (17) and (18) for the relationship between gammas and sigmas in case the Stone price index is used.

same direction.[13]

In growth experiments the CES approach should be expected to produce larger terms-of-trade effects than either of the AIDS/translog or the AIDS/Stone version. This was the result of experiments performed on models similar in concept. (For details, see Robinson, S., Soule, M.J., and Weyerbrock, S., 1992).

Observations on Using Altered Parameters

Alteration of a single elasticity of substitution results in a rescaling of substitution elasticities in the given trade link. The rescaled values reflect the percentage increase of alteration. The following tabulation shows substitution elasticities in Mexico's corn trade with the model parameters, based on the translog version:[14]

	U.S.	Mexico
Mexico.U.S.	-21.386	4.000
Mexico.Mexico	4.000	-0.748

That is, $\sigma_{k,c1,c2} = 4.000$, where k = Mexico, c_1 = U.S., and c_2 = Mexico. Increasing the elasticity of substitution between the United States and Mexico to 5 in corn trade results in the following:

	U.S.	Mexico
Mexico.U.S.	-26.732	5.000
Mexico.Mexico	5.000	-0.935

The algebraically imposed adding up characteristic, shown in equation (14), results in a proportional increase in the negative entries, i.e., in the "own" elasticities. Each entry in the second tabulation is 25 percent higher in absolute value than in the first one. Consequently, the net result of increasing a cross-country elasticity is to increase price responsiveness in the entire

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[13] Some models using the CES functional form showed large terms-of-trade effects in trade liberalization experiments (Shiells, Roland-Holst, and Reinert, 1993). The models in question used substitution elasticities in the neighborhood of 1. This implies small import supply elasticities and large optimal tariffs--the reciprocal of the first is the measure of the second. Since the terms-of-trade changes indicate unexploited opportunities to impose optimal tariffs, any model that generates large changes in optimal tariffs will also show large swings in terms-of-trade. For details on the relationship between changes in terms-of-trade and optimal tariffs in CGE policy simulations, see (Benjamin, 1995).

[14] The "5.000" in the tabulation represents the substitution elasticity in Mexico between Mexican and U.S. corn. The main diagonal shows the "own" elasticity of locally produced and U.S. corn, respectively. In the model, the United States is the only foreign supplier of corn to Mexico.

trade of the given commodity.

The recalculation of parameters with the altered substitution elasticity in Mexico's corn sector left a trace, however infinitesimal, on the GDP and real absorption of all the countries in the model. Reasonably, the effects were the largest for Mexico. The effect on U.S. variables was infinitesimal. The effects on Mexico were the largest in experiments 3 and 4, obviously because these experiments involved major extensions of the free trade area. The increase in the substitution elasticity increased trade or left it unchanged. Thus the net effect was an increase in overall trade.

The translog version accommodated the change the most reasonably. It slightly raised Mexico's corn purchases from the United States in the base run and in experiments 1 through 3, leaving it unchanged in experiment 4. The Stone version did not react to the change in the base run and in experiments 1 and 2, but reacted much more forcefully than the translog version in experiments 3 and 4. It is interesting to note that the actual market share of U.S. corn in Mexican domestic consumption approximated the AIDS market share in 1995. This is a reminder that the AIDS market share is also a behavioral parameter, showing the responsiveness of total costs to changes in prices (equation 10). The change in AIDS market shares was infinitesimal as a result of the change in the substitution elasticity, an observation that remained valid for the rest of the experiments reported here.

The alteration of a single elasticity of substitution affects price responsiveness only in the particular commodity group and particular trade link. The entered elasticity will remain intact in the translog version only. In the above example, there were only two suppliers in the market examined. However, in commodity sectors where there are several foreign suppliers, the alteration of substitution elasticity between two sources leaves substitution elasticities between each pair of other sources unaffected. For example, the following row shows substitution elasticities in the capital goods sector of Mexico, with regard to U.S. imports:

	U.S.	Mexico	Argentina	Brazil	Chile
Mexico.U.S.	-1.313	0.586	0.586	0.586	1.000

Changing the entry under Mexico from 0.586 to 2.000 will result in the following substitution elasticities:

	U.S.	Mexico	Argentina	Brazil	Chile
Mexico.U.S.	-3.727	2.000	0.586	0.586	1.000

Thus, the new elasticity "2.000" remained unchanged and only the corresponding trade link, i.e., U.S.-Mexico was recalculated. The following tabulation shows changes in the Stone version of substitution elasticity in the same experiment. The first line shows substitution elasticities before, the second line shows them as a result of the experiment:

	U.S.	Mexico	Argentina	Brazil	Chile
Mexico.U.S.	-1.888	1.102	0.011	0.011	0.425
Mexico.U.S.	-4.302	2.516	0.011	0.011	0.425

In this version also only cross elasticities of the given trade link were affected, but as a result of recalculating the substitution elasticities with the Stone equation, the "2.000" became "2.516."

The alteration of a single elasticity of income affects all income elasticities in the particular commodity sector of the given reference country.--Entering a new income elasticity of demand gives a signal to the model that the ratio of income elasticities in a given commodity group and in a given country has changed. The recalculated values will reflect the ratio that the newly entered income elasticity has created. The following tabulation shows income elasticities of demand in the capital sector of the United States:

	U.S.	Mexico	Argentina	Brazil	Chile
U.S.	0.723	2.008	2.008	2.008	2.008

Changing the original 2.5 for Mexican products (which led to 2.008 after rescaling above) to 3.0 results in the following:

	U.S.	Mexico	Argentina	Brazil	Chile
U.S.	0.721	2.402	2.402	2.402	2.402

The number entered (2.5) was altered along with all the items in the row. The following tabulation shows income elasticities in corn in Mexico, as entered and recalculated as a result of running the model:

	U.S.	Mexico
Entered	1.400	0.960
Recalculated	1.360	0.933
Entered	1.400	1.000
Recalculated	1.317	0.941
Entered	1.400	2.000
Recalculated	0.735	1.050

Manipulation (i.e., increase or decrease) of elasticities results in nonmonotonic transformations of macro variables.[15] The increase in a given substitution or income elasticity

[15] This observation does not intend to introduce a new theorem, it only confirms the model's regular, expected behavior. As it is well-known from the literature on the foundations of

causes an increase (or decrease) in any macro variable only to a certain extent, beyond which a decrease (increase) sets in. For example, in the unaltered version of the model (with the income elasticity of corn in Mexico equaling 0.960), experiment 1 slightly reduced Mexico's GDP from the base run level, and experiment 2 left it unchanged. Experiments 3 and 4 each increased it slightly. Switching income elasticities from 0.96 to 1 slightly amplified both movements. However, switching the income elasticity from 1 to 2 amplified the movement in experiments 1 and 2, but not in experiments 3 and 4. Mexican GDP with income elasticity in corn equaling 2 settled between the results produced with the base run and with income elasticity equaling 1. This is the result regardless of the choice between the translog and the Stone price elasticity. A similar phenomenon of reaching a maximum at income elasticity equaling 1 could be observed in the variable real absorption.

Total exports and imports for the countries in the model grew from experiment 1 through experiment 4. However, the extent of growth varies according to the choice in the functional form. The following table shows the increase in the level of exports and imports from the respective levels in experiment 1:

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general equilibrium theory, a system (economic or mechanical) can be in an equilibrium state only if the forces composing it are subject to constraints. Otherwise, the stress among the forces could not be in balance, i.e., the resultant of the forces acting on the system could not reach zero. Since in a CGE model, variables and parameters represent economic forces (i.e., individual drives and preferences straining against the constraints of production possibilities), functional relationships exist between each pair of them. Returning to the observation in the text, the parameter considered independent variable (income elasticity) may assume any reasonable continuum of values, forming a domain that is a closed and bounded set. Since the investigated dependent variable (GDP, real absorption, and export volumes) react to changes in the chosen parameter between and including an upper and a lower bound, the range of the function will also be a closed and bounded set. These topological conditions guarantee that the mapping will be meaningful, but not that it will be a continuous, differentiable function. The range may contain gaps and inaccessible regions. Nevertheless, the range will contain distinct values, allowing for the identification of the minimum and maximum values on a given domain. For details on the mathematical background of general equilibrium, see (McKenzie, L., 1981; and Arrow, K.J. and Debreu, G., 1954).

Functional form	Growth in imports (%)	Growth in exports (%)
CES	10.04	12.85
AIDS, with betas set to zero	6.71	8.59
Translog price index		
income elasticity = 0.96	8.64	11.05
income elasticity = 1	9.13	11.68
income elasticity = 2	9.10	11.65
Stone price index		
income elasticity = 0.96	7.99	12.22
income elasticity = 1	8.50	10.87
income elasticity = 2	8.48	10.85

Under both the translog and Stone versions, the growth in trade peaks when the income elasticity equals one. This table also shows that the lack of variation in cross-country substitution elasticities, as under the CES, yields a larger growth in trade than with variation. (As mentioned before, the income elasticities are unitary under both functional forms.)

Appendices

Appendix A

The Linear Expenditure System (LES)

The LES reflects additive preferences as expressed by the Cobb-Douglas utility function, i.e., the geometric weighing of commodities. The LES in CGE models uses the following basic formulation:

$$p_i q_i = p_i \gamma_i + \beta_i (X - \sum_k p_k \gamma_k) \tag{19}$$

where the gammas and betas are parameters. The gammas represent the subsistence or required minimum level consumption in commodity "I," whereas the betas stand for the marginal propensity to consume the same commodity above the subsistence or required minimum level. The theoretical cost function that corresponds to (1) is as follows:

$$C(U, P) = \sum_k p_k \gamma_k + U \prod p_k^{\beta_k} \tag{20}$$

where the second term represents the marginal cost of a unit of utility (util) in the form of the Stone price index, since the betas add up to one.[16] Setting the cost equal to the available budget limit, X, the indirect utility function may be expressed:

$$\psi(X, P) = U = (X - \sum_k p_k \gamma_k) / \prod p_k^{\beta_k} \tag{21}$$

The direct utility function, containing only physical quantities, follows from the indirect utility function:

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[16] The utility cost function is the inverse of the indirect utility function, which uses expenditures and prices as arguments. The demand curves derived from minimizing costs subject to a given level of utility--in practice total expenditures-- are called "Hicksian demand curves," after J.R. Hicks. The demand curves derived from maximizing utility--in practice the consumption of goods--subject to a budget constraint are called "Marshallian demand curves," after A. Marshall. Since the problems of finding the Hicksian and Marshallian demand functions are duals, the two solutions yield identical results. Whereas the Hicksian approach is based on minimizing total costs, considering the utility fixed; the Marshallian approach is based on maximizing utility, considering total costs fixed. The mutual determination between total costs and levels of utility, and the prevalence of optimizing consumer behavior are the underlying assumptions that allow for the duality between the two approaches (Deaton and Muellbauer, 1980, p. 41.)

(22) $$V(Q)=\prod (q_k-\gamma_k)^{\beta_k}$$

The mathematical derivation of (22) from (21) may be spared by recognizing that (22) repeats in real terms what (21) states in value terms, since the Stone price index in (21) functions as a deflator. The Stone price index determines a geometrical weighing of utility contributions by various commodities. In (21), geometric averaging is applied to prices and in (22), to quantities. Equation (22) tacitly assumes that as a result of optimum choice, the ordering of commodities by their contribution to total utility is identical to their ordering in value, as expressed by the Stone price index. This utility function is of the Cobb-Douglas type, which is linearly homogenous. Increases in the level of consumption generate constant returns to the level of utility, pushing the indifference curves in a North-Eastern direction by the same ratio as the quantities consumed increase.

The LES conforms to the four properties required for a consistent and complete demand system. The property of zero homogeneity in prices (the "absence of money illusion requirement)" follows from the formulation of the equations. Multiplication of each price term by the same constant will leave the system unperturbed both in terms of level and allocation of expenditures. The adding-up property (i.e., expenditures on individual commodities add up to the budget constraint) and the property of symmetry (i.e., cross-partial price derivatives are equal) are algebraically imposed. The negativity property (i.e., the demand curves generated by the system are either downward sloping or horizontal) follows from actual calculations. Compliance with the fourth property is fulfilled when the matrix of compensated cross-price elasticities (also called the substitution or Slutsky matrix) forms a negative semidefinite. The latter means that the own-price derivatives are non-positive, i.e., zero or negative. The terms of the substitution matrix may be expressed in elasticity terms. The algebraic manipulations that lead from the own- and cross- derivatives to the respective elasticities do not change the signs of the terms.

An LES with non-origin intercepts, as presented here and used in contemporary CGE models, implies income elasticities that vary according to the commodity group. This can be seen easily by remembering that the betas in equation (19) represent marginal propensities to consume and that income elasticity for a particular commodity group is the marginal propensity to consume divided by the average propensity to consume in the commodity group. In face of varying marginal propensities to consume, income elasticities would be the same across commodity groups if, and only if, average propensities differed between any two groups by exactly the same ratio as their marginal propensities. To illustrate the problem, let the linear consumption function for two commodity groups be as follows:

(23) $$C_1=b_1Y+d \qquad C_2=b_2Y+e$$

where the marginal propensities to consume are denoted by the "bs" and consumption at subsistence levels, by "d" and "e," respectively. To have identical income elasticities the following condition would need to hold:

(24) $$\frac{b_1}{\int C_1 dY/Y} = \frac{b_2}{\int C_2 dY/Y}$$

However, this cannot hold, since

(25) $$\frac{b_1}{b_1 Y^2/2 + dY} \neq \frac{b_2}{b_2 Y^2/2 + eY}$$

Both the numerators and denominators differ.

The LES with non-origin intercepts represents strong separability or block additivity among various commodity groups. Since such LES functions contain different income elasticities for different commodity groups, these do not expand at the same rate as a result of expansion in total outlays. That is, the expansion of purchases in one commodity group as a result of a unit increase in total outlays could not be predicted based on the expansion of purchases in another commodity group. The relationship between expenditures and demand within given commodity groups is quasi-homothetic. (U.S. International Trade Commission, September 1991, p. 15)

Appendix B

The Cobb-Douglas utility function

Optimization involving the Cobb-Douglas utility function subject to constraints expressed as LES leads to market shares that will equal the utility function's exponents. If a two-commodity ordinal utility function is

(26) $$U = q_1^{a} q_2^{1-a}$$

and the corresponding cost function is

(27) $$X = p_1 q_1 + p_2 q_2$$

then the Langrangian equation may be written as

(28) $$\max L = a \ln q_1 + (1-a) \ln q_2 - \lambda (p_1 q_1 + p_2 q_2)$$

since taking the logarithm of the utility function is a monotonic transformation, which, by definition, does not change the order of commodity ranking. The first condition equations will be

(29) $$\frac{\partial L}{\partial q_1}=\frac{a}{q_1}-\lambda p_1=0$$

(30) $$\frac{\partial L}{\partial q_2}=\frac{(1-a)}{q_2}-\lambda p_2=0$$

These equations yield

(31) $$\frac{a}{q_1 p_1}=\frac{1-a}{q_2 p_2}$$

(32) $$a(q_2 p_2)=q_1 p_1-q_1 p_1 a$$

(33) $$q_1 p_1=a(q_1 p_1+q_2 p_2)$$

Since the term in the parenthesis in equation (32) is total expenditure, X, spending on commodity 1 is the "a-th"proportion of X. Similarly, spending on commodity 2 will be the residual "(1 - a)-th" proportion of X.

The following simple transformation retains the identity between the amount spent on a commodity as expressed by the product of price and quantity and by the proportion of total spending:

(35) $$p_1 q_1=aX=b+a(X-c)$$

(34) $$p_2 q_2=(1-a)X=d+(1-a)(X-c)$$

provided that b + d = c. With some difference in notation, this is exactly the LES for two commodities, as shown in (19):

(36) $$p_1 q_1=a(X-\gamma_1 p_1-\gamma_2 p_2)$$

(37) $$p_2 q_2=\gamma_2 p_2+(1-a)(X-\gamma_1 p_1-\gamma_2 p_2)$$

Generalization from the two-commodity case does not alter the validity of this illustration.

Models of consumer behavior that are based on an underlying Cobb-Douglas type utility

are sometimes called generalized linear (sometimes, linear approximate) models.

Appendix C

Cobb-Douglas as a special case of the CES utility function

The derivation closely follows P.S. Armington's 1969 IMF staff paper article (Armington, 1969, pp. 167 and 168.) After defining demand for commodity "i" with the constant-elasticity-of-substitution (CES) functions as

(38)
$$X_i=[\sum_i b_i X_i^{-\rho_i}]^{-\frac{1}{\rho_i}}$$

demand for commodity "i" from country "j" may be expressed as follows:

(39)
$$x_{ij} = b_{ij}^{\sigma_i} X_i (\frac{p_{ij}}{p_i})^{-\sigma_i}$$

Multiplying both sides by p_{ij} yields

(40)
$$p_{ij}x_{ij} = b_{ij}^{\sigma_i} (p_i x_i)(\frac{p_{ij}}{p_i})^{1-\sigma_i}$$

Finally, expressing this sum as a fraction of total spending on commodity "i" gives

(41)
$$\frac{p_{ij}x_{ij}}{P_i X_i} = b_{ij}^{\sigma_i} (\frac{P_{ij}}{P_i})^{1-\sigma_i}$$

Since substitution elasticities are uniformly "1" in the Cobb-Douglas case, the second term in equation (41) reduces to the fixed proportion b_{ij}. Thus, neither the absolute level of prices, nor the relative level of the given commodity's price plays a role in determining the budget share. Hence, the market share derived from Cobb-Douglas utility function is called "price independent." Note also that expenditure shares are independent of total expenditure.

Consumer demand based on a Cobb-Douglas-type utility function generates fixed expenditure shares, reflecting strong separability of preferences or block additivity. This creates the same type of rigidity on the consumption side of a CGE model as the "Leontief" fixed input coefficient system on the production side. However, this rigidity is defendable at higher levels of disaggregation where it is actually used. The relative constancy in the proportions of household budgetary allocations to broad groups of commodities and services is statistically evident.

Although the derivation of AIDS market shares at the most detailed level of disaggreagtion begins with the Cobb-Douglas utility function, the application of the flexible functional form detaches the results of AIDS from its limited starting point. Setting the parameter gammas and betas at zero in the AIDS market share equation (text equation 15), leads back to the Cobb-Douglas case of constant market shares, unresponsive to changes in prices and expenditures.

Appendix D

Cholesky factorization

A definite (positive or negative) matrix "S" can always be written in the form $S = A\,D\,A^T$, where "A" is a lower triangular matrix with 1s on its main diagonal, and "D" is a diagonal matrix containing only positive or negative numbers, depending on whether "S" is a positive or a negative definite. The method of finding "A" and "D" is called triangular factorization. The following shows the first three rows and columns of "A" and "D," regardless of the size of "S:"

$$(42) \qquad A = \begin{bmatrix} 1 & 0 & 0 \\ s_{21}s_{11}^{-1} & 1 & 0 \\ s_{31}s_{11}^{-1} & h_{32}h_{22}^{-1} & 1 \end{bmatrix} \qquad D = \begin{bmatrix} s_{11} & 0 & 0 \\ 0 & h_{22} & 0 \\ 0 & 0 & h_{33} - h_{32}h_{22}^{-1}h_{23} \end{bmatrix}$$

where $h_{22} = (s_{22} - s_{21}s_{11}^{-1}s_{12})$, $h_{33} = (s_{33} - s_{31}s_{11}^{-1}s_{13})$, and $h_{23} = h_{32} = s_{23} - s_{21}s_{11}^{-1}s_{13})$. (For a detailed description of finding "A" and "D" for the general, "n x n" case, see, Hamilton, 1994, pp. 87-91). Further, "S" may be written as $S = A\,D^{0.5}\,D^{0.5}\,A^T$, where $D^{0.5}$ indicates the square root of each element in the diagonal matrix. Since the transpose of a diagonal matrix is itself, "S" may also be written as $S = (A\,D^{0.5})\,(A\,D^{0.5})^T$. Defining $P = A\,D^{0.5}$, "S" may be represented as $S = PP^T$. This is called the Cholesky factorization of "S." The result is unique (Papules, 1991, pp. 207 and 506). Multiplication shows that, indeed, $S = A\,D\,A^T$. For example, if:

$$(43) \qquad A = \begin{bmatrix} 1 & 0 & 0 \\ 4/3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 4/3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

An obvious condition of the procedure is that the elements under the square roots in $D^{0.5}$ must all be positive or zero. If all the elements are positive in the matrix, "S" is a positive definite. If all the elements are negative after finding the square roots, "S" is a negative definite. In this

example, S is a positive definite.

(44)
$$S = \begin{bmatrix} 9 & 12 & 0 \\ 12 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Further, since $D^{0.5}$ is obtained by taking the square root of the diagonal elements,

(45)
$$P = AD^{0.5} = \begin{bmatrix} 1 & 0 & 0 \\ 4/3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then, the Cholesky factorization will mean the following:

(46)
$$PP^T = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 12 & 0 \\ 12 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix} = S$$

If the matrix "S" is a negative definite, all the diagonal elements have negative signs. In this case, the procedure is the same, except that a negative sign needs to be put in front of the square roots when calculating $D^{0.5}$.

The optimization procedure in the Robinson CGE models invokes this principle when the

(47)
$$S = \begin{bmatrix} -9 & 12 & 0 \\ 12 & -16 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Slutsky matrix is not a negative semidefinite. For example, consider the matrix "S" in (47). "S" is a negative semidefinite since it is singular, symmetric, and its eigenvalues are 0, -1, and -25. (At least one of the eigenvalues needs to be zero for a matrix to be a negative semidefinite.) If the above were a substitution matrix, its elements would stand for the second derivatives of the cost function that the AIDS approach estimated at the point determined by actual consumption data. The negative elements in the main diagonal show the convexity of the choices, or equivalently, that the demand curves are downward sloping for all the commodities considered, or equivalently, that the system does not contain negative market shares. Some zeros in the main diagonal would be permissible in complying with these requirements. The symmetry and positive sign of the off-diagonal elements show compliance with required regularity conditions, i.e., only

substitutes are considered and the degree of substitutablility between the two products is equal. In terms of the WHFTA model, this could mean, for example, that in Mexico, capital goods from Argentina are equally substitutable for capital goods from Chile.

As indicated in the text, one problem with flexible functional estimates in a CGE model is that a drastic counterfactual policy experiment could jolt a market share out of the 0-to-1 range. If this occurs, the calculation could not be completed since the Slutsky matrix would not be a negative semidefinite. For example, it might look like this:

$$S = \begin{bmatrix} -9 & 12 & 0 \\ 12 & -16 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{48}$$

This matrix is no longer a negative semidefinite. Its eigenvalues are 0, +1, and -25. In terms of the AIDS approach, the positive number in the main diagonal would indicate that for the commodity in question the demand curve is upward sloping, or equivalently, that the share of the commodity is negative, or equivalently, that the convexity of choices is not universal at the given point on the total cost curve.

At first glance, the solution to the problem appears trivial. In terms of algebra, the correction of the 1 in the main diagonal to a small negative number or zero could take care of the problem at the cost of some estimable loss in the validity of calculations. However, the correction of the problem in a CGE context must take into account the algebraically imposed requirements of demand theory.

The WHFTA model uses the following approach to solve this problem: (1) It puts the number of bilateral trade links in a given reference country (k) and commodity (I) in the main diagonal with a negative sign; i.e., each trade link contributes "-1" to the size of the "own elasticity;" (2) it rescales cross-country substitution elasticities based on the algebraic requirement that the price parameters (gammas) should add up to zero by row and by column (equation 14). The result guarantees that the Slutsky matrix will be at least a negative semidefinite, ensuring solvability.

Appendix E

A numerical illustration of the application of AIDS

If the market shares are exogenous, the AIDS may be used to determine unknown parameters. For example, assume that in reference country "k" supplies in a given commodity group "i" come from two import sources and from domestic production. The data bank for the analysis contains the market shares for the three sources, prices, total expenditures on the commodity group, the price index for the commodity group, and some behavioral parameters found in the literature. The task is to calculate the missing parameters, using the Stone price index. Applying equation (15), dropping "k" and "i" for convenience, equation (24) serves as the basis for the calculations:

(49) $$w_{c1} = \alpha_{c1} + \sum_{c2} \gamma_{c1,c2} + \beta_{c1} \ln [\frac{X}{P}]$$

where both "c1" and "c2" vary to stand for the 3 sources of "c1," "c2," and "k." Let the market shares be as follows: $w_{c1} = 0.3$; $w_{c2} = 0.4$; and $w_{ck} = 0.3$. Let 4 be the price of source country 1 (p_1), 3 the price of source country 2 (p_2), and 4 the domestic price (p_k). Let 713 be expenditures on the commodity group (X), and 1 the commodity price index (P). The prices and expenditures are arbitrary numbers, but 1 was chosen as the commodity price index to simplify calculations.

Assume that research identified the following "own" elasticities: $\sigma_{c1,c1} = -4.333$; $\sigma_{c2,c2} = -3.5$, and $\sigma_{ck,ck} = -4.333$. (These are the so-called Allen elasticities, i.e., the compensated own price elasticities divided by their respective market shares.) Assume that expenditure elasticities found in the literature were as follows: $\varepsilon_{c1} = 1.33$, $\varepsilon_{c2} = 1.000$, $\varepsilon_{ck} = 0.67$. (The numbers were chosen so that the market-share-weighted expenditure elasticities add up to 1.0, avoiding the need for scaling.) Then, based on equation (24) the following equation system will serve as the basis for the calculations:

$$w_{c1} = \alpha_{c1} + \gamma_{c1,c1} \ln p_1 + \gamma_{c1,c2} \ln p_2 + \gamma_{c1,ck} \ln p_k + \beta_{c1} \ln X$$
$$w_{c2} = \alpha_{c2} + \gamma_{c2,c1} \ln p_1 + \gamma_{c2,c2} \ln p_2 + \gamma_{c2,ck} \ln p_k + \beta_{c2} \ln X$$
$$w_{ck} = \alpha_{ck} + \gamma_{ck,c1} \ln p_1 + \gamma_{ck,c2} \ln p_2 + \gamma_{ck,ck} \ln p_k + \beta_{ck} \ln X$$

Of course, $w_{c1} + w_{c2} + w_{ck} = 1$. Moreover, based on equation (14),

$$\gamma_{c1,c1} + \gamma_{c1,c2} + \gamma_{c1,ck} = 0$$
$$\gamma_{c2,c1} + \gamma_{c2,c2} + \gamma_{c2,ck} = 0$$
$$\gamma_{ck,c1} + \gamma_{ck,c2} + \gamma_{ck,ck} = 0$$
$$\alpha_{c1} + \alpha_{c2} + \alpha_{ck} = 1$$
$$\gamma_{c1,c2} = \gamma_{c2,c1}$$
$$\gamma_{c1,ck} = \gamma_{ck,c1}$$
$$\gamma_{c2,ck} = \gamma_{ck,c2}$$
$$\beta_{c1} + \beta_{c2} + \beta_{ck} = 0$$

Using equation (16) for expenditure elasticities and equations (17) and (18) for substitution elasticities, the following parameters can be determined: $\beta_{c1} = 0.1$; $\beta_{c2} = 0$; $\beta_{ck} = -0.1$; $\gamma_{c1,c1} = -018$; $\gamma_{c2,c2} = -0.32$; $\gamma_{ck,ck} = -0.18$; Substituting the actual market shares, the predetermined parameters, and the natural logarithms of prices and total costs into the market share equations yields the following:

$$0.3 = \alpha_{c1} - 0.18 \times 1.3863 + \gamma_{c1,c2} \times 1.0986 + \gamma_{c1,ck} \times 1.3863 + 0.1 \times 6.5695$$
$$0.4 = \alpha_{c2} + \gamma_{c2,c1} \times 1.3863 - 0.32 \times 1.0986 + \gamma_{c2,ck} \times 1.3863$$
$$0.3 = \alpha_{ck} + \gamma_{ck,c1} \times 1.3863 + \gamma_{ck,c2} \times 1.0986 - 0.18 \times 1.3863 - 0.1 \times 6.5695$$

Since the betas are predetermined, the system provides 10 equations. However, one of

the gamma cross identities is uniquely determined by the other two identities. Thus, the 9 variables, representing unknown parameters, are matched with 9 equations, allowing for the derivation of the unknown parameters. The solution of the equation system, which can be conveniently solved with GAMS, yields the following values for the unknown parameters: $\alpha_{c1} = 0.280$; $\alpha_{c2} = 0.308$; $\alpha_{ck} = 0.412$; $\gamma_{c1,c2} = \gamma_{c2,c1} = 0.160$; $\gamma_{c1,ck} = \gamma_{ck,c1} = 0.02$; $\gamma_{c2,ck} = \gamma_{ck,c2} = 0.160$. The missing substitution elasticities may be recovered by applying equation (18): $\sigma_{c1,c2} = \sigma_{c2,c1} = 2.33$; $\sigma_{c1,ck} = \sigma_{ck,c1} = 1.22$; and $\sigma_{c2,ck} = \sigma_{ck,c2} = 2.33$. The complete equation system looks as follows:

$$w_{c1} = 0.280 - 0.18 \times \ln p_1 + 0.16 \times \ln p_2 + 0.02 \times \ln p_k + 0.1 \times \ln X$$
$$w_{c2} = 0.308 + 0.16 \times \ln p_1 - 0.32 \times \ln p_2 + 0.16 \times \ln p_k$$
$$w_{ck} = 0.412 + 0.02 \times \ln p_1 + 0.16 \times \ln p_2 - 0.18 \times \ln p_k - 0.1 \times \ln X$$

In a CGE model, the market shares are endogenously generated as part of the model's overall optimization process. If the above equation system had been part of a CGE model, its parameters would serve as the basis for calculating the consequences of policy simulations on model shares. For example, if a policy experiment generates prices such as $p_1 = 3$, $p_2 = 3$, and $p_k = 4$, then the system will calculate market shares as $w_{c1} = 0.351$, $w_{c1} = 0.354$, and $w_{c1} = 0.295$. The system shows a remarkable resilience against producing negative market shares. It took a combination of $p_1 = p_2 = 5.47$, $p_k = 40.45$, and $X = 992.27$ to produce $w_{c1} = 0.389$, $w_{c2} = 0.628$, and $w_{ck} = -0.017$. This bodes well for using AIDS in CGE models.

BIBLIOGRAPHY

Agrawal, N. and Powell, A.A., "MAIDS Under Additive Preferences: Some Early Estimates," Impact Research Center, University of Melbourne, Preliminary Working Papet, number OP-67, March 1989.

Alston, J.M., Carter, C.A., and Green, R., "Whither Armington Trade Models?," American Journal of Agricultural Economics, May 1990, pp. 455-467.

Apostolakis, B.E., "Translogarithmic Production and Cost Functions: A Synopsis," The Economic Studies Quarterly, vol. 39, no. 1, March 1988.

Armington, P.S., "A Theory of Demand for Products Distinguished by Place of Production," International Monetary Fund Staff Papers, International Monetary Fund (IMF), Washington, DC, 1969, pp. 159-176.

Arrow, K.J. and Debreu, G., "Existence of an Equilibrium for a Competitive Economy," Econometrica, July 1954, 22(1954), pp. 265-290.

Benjamin, N., "Notes on CGE Model Simulations of Regional Trade Agreements," paper presented at the ASSA conference, January, 1995.

Berndt, E.R., The Practice of Econometrics: Classic and Contemporary, Addison-Wesley Publishing Co., Reading, MA, 1990.

Blanciforti, L. and Green, R., "An Almost Ideal Demand System Incorporating Habits: An Analysis of Expenditures on Food and Aggregate Commodity Groups," The Review of Economics and Statistics," 1983, pp. 511-515.

Chalfant, A.J., "Globally Flexible, Almost Ideal Demand System," Journal of Business & Economic Statistics, April 1987, pp. 233-241.

Chambers, R.S., Applied Production Analysis, Cambridge University Press, Cambridge, U.K., 1989.

Christensen, L.R., Jorgenson, and D.W., and Lau, L.J., "Transcendental Logarithmic Utility Functions," The American Economic Review, June 1975, pp. 367-383.

Deaton, A., Understanding Consumption, Clarendon Press, Oxford, U.K., 1992.

Deaton, A. and Muellbauer, J., Economics and Consumer Behavior, Cambridge University Press, Cambridge, U.K., 1986.

Deaton, A. and Muellbauer, J., "An Almost Ideal Demand System," The American Economic Review, June 1980.

Dervis, K., de Melo, J., and Robinson, S., General Equilibrium Models for Development Policy, Cambridge University Press, Cambridge, U.K., 1982.

Despotakis, R.A., "Economic Performance of Flexible Functional Forms," European Economic Review, 30 (1986) 1107-1143, North Holland.

Diewert, W.E., "An Application of the Shepard Duality Theorem: A Generalized Leontief Production Function," Journal of Political Economy, vol. 79, 1971, pp. 481-507.

Diewert, W.E., "Intertemporal Consumer Theory and the Demand for Durables," Econometrica, vol. 42, 1974, pp. 497-516.

Diewert, W.E., "Symmetry Conditions for Market Demand Functions," Review of Economic Studies, vol. 47, pp. 595-601.

Eales, J.S. and Unnevehr, L.J., "Demand for Beef and Chicken Products: Separability and Structural Change, American Journal of Agricultural Economics, vol. 70, August 1988, pp. 521-532.

Evans, K. M., Macroeconomic Activity, Harper & Row, Publishers, New York, NY, 1969.

Green, R. and Alston, J.M., "Elasticities in AIDS Models," American Journal of Agricultural Economics, May 1990, vol. 72, pp. 442-445.

Hamilton, J.D., Time Series Analysis, Princeton University Press, Princeton, NJ, 1994.

Hanson, K., Robinson, S., and Tokarick, S., "U.S. Adjustment in the 1990s: A CGE Analysis of Alternative Trade Strategies," International Economic Journal, vol. 7, no. 2, Summer 1993, pp. 1-23.

McKenzie, L.W., "The Classical Theorem on Existence of Competitive Equilibrium, Econometrica, 49(1981), pp. 819-841, July 1981.

McKitrick, R.R., "The Econometric Critique of Computable General Equilibrium Modeling: The Role of Functional Form Choice," paper presented at the Sixth International CGE Modeling Conference, Waterloo, Ont., October 1995.

Papoulis, A., Probability, Random Variables, and Stochastic Processes, McGraw-Hill, Inc., New York, NY, 1991.

Perroni, C. and Rutherford, T.F., "Regular Flexibility of Nested CES Functions," European Economic Review, 39 (1995) 335-343, North Holland.

Rimmer, T.M. and Powell, A.A., "An Implicitly Directly Additive Demand System: Estimates for Australia," paper presented to the Third General Equilibrium Modeling Conference, Wilfred Laurier University, Waterloo, Ontario, Oct. 24-25, 1992.

Robinson, S. et al., Structure of the GNAFTA CGE Model, International Food Policy Institute, Washington, DC, February, 1995.

Robinson, S., Soule, M.J., and Weyerbrock, S., "Import Demand Functions, Trade Volumes, and Terms of Trade Effects in Multi-Country Trade Models," Working Paper, Department of Agricultural and Resource Economics, University of California at Berkeley, January, 1992.

Shiells, C.R., Roland-Holst, D.W., and Reinert, K.A., "Modeling A North American Free Trade Area: Estimation of Flexible Functional Forms," Review of World Economics (Weltwirtschaftliches Archiv), J.C.B. Mohr (Paul Siebeck) Tubingen, ISSN 0043-2636, vol. 129, no. 1, 1993, pp. 55-77.

Stone, J.R.N., "Linear Expenditure Systems and Demand Analysis: An Application to the Patterns of British Demand," Economics Journal, vol. 64, pp. 511-527, 1954.

U.S. International Trade Commission: An Introduction to the ITC Computable General Equilibrium Model, USITC publication 2423, Washington, D.C., September 1991.

Varian, H.R., Microeconomic Analysis, W.W. Norton & Company, New York, NY, 1992.