Army Research Laboratory

Finding Attitude of a Spin Axis From Roll Angles

by Andrew A. Thompson

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Andrew A. Thompson
Weapons and Materials Research Directorate, ARL

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14. ABSTRACT This report describes a method for determining the attitude of a spin axis from rotational measurements between known locations. The described method goes beyond "finding down" to solve for the attitude. This solution was conceived as a method to enhance a GPS receiver on a spinning body. The method was made more general by only dealing with locations and directions and is potentially useful for a wider class of applications. The generalized problem has been solved and simulations have been run to verify the implementation of the solution. Pressure waves and electromagnetic energy could be used; systems could combine magnetic, visual, radar, IR, and acoustic information to determine attitude. The method described has received U.S. Patent 7,388,538 B1.					
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1. Introduction

This report describes a method for determining the attitude of a spin axis from rotational measurements between known locations. This solution was conceived as a method to enhance a GPS receiver on a spinning body. The method was made more general by only dealing with locations and directions and is potentially useful for a wider class of applications. The generalized problem has been solved and simulations have been run to verify the implementation of the solution.

The ability of a single sensor to determine both position and attitude is appealing from a tactical perspective. Software modification and the addition of appropriate antennas to a GPS receiver would realize a measurement that includes position and attitude. This package would provide all the information needed for input to control mechanisms for improving the precision of projectiles, indirectly increasing the lethality. The increase in lethality is a direct result of control algorithms reducing the delivery error. The accuracy of these algorithms is a function of the position and attitude accuracy. Obtaining an acceptable estimate of the spin angle has been a stumbling block in this area and is often referred to as the "finding down" problem. The described method goes beyond "finding down" to solve for the attitude.

Another example of a possible use of this method would be to find the attitude of a spacecraft. Using stars as known sources of energy and rotational information, the spin axis of the spacecraft could be found. Similarly, this method could be applied to estimate the earth's or any other planet's spin axis. It is also possible to imagine systems using information from many spectral bands to obtain the needed roll information. Both pressure waves and electromagnetic energy could be used; systems could combine magnetic, visual, radar, IR, and acoustic information to determine attitude. The method described has received U.S. Patent 7,388,538 B1.

2. Coordinate Estimation

This section discusses some of the issues with GPS and coordinate adjustment. Coordinate adjustment is used within GPS receivers to process pseudorange measurements.

Coordinate adjustment is central to precise estimation of location. Surveying and geodesy both have contributed to this body of literature. Wolf and Ghilani[1] describe the way coordinate estimation is used in surveying. Strang and Borre[2] discuss the way coordinate adjustment is used

[1]Wolf, P. R.; Ghilani, C. D. *Adjustment Computations: Statistics and Least Squares in Surveying and GIS*; John Wiley and Sons: New York, NY, 1997.

[2]Strang, G.; Borre, K. *Linear Algebra, Geodesy, and GPS*; Wellesly-Cambridge Press: Wellesly, MA, 1997.

in geodesy. Thompson[3] discusses how angles, recast as inner product measurements, can be used within the umbrella of coordinate adjustment. Finding equations for the measurements in terms of the unknown location establishes the necessary function. The gradient of each of these functions is used as a row in a least-squares observation matrix. Using an iterative process, the location can be estimated.

3. Development Background

GPS receivers have advanced in complexity and features. Of particular interest are the receivers containing hundreds of thousands of correlators. These correlators are used to search the frequency-time delay space and find the GPS signal associated with a given satellite. After satellites have been acquired, many of these correlators are idle.

A spinning round will create a Doppler effect on the received GPS signal for any radially oriented, non-circumferential antenna. Spinning rounds typically use ring antennas to bypass this situation. Consider the situation with a spinning round using both a ring antenna and a patch (or spin-axis-aligned antenna.) The GPS receiver will use the ring antenna to acquire and track the GPS signals; thus, the signal tracking information is available within the receiver. The patch antenna will have a frequency shift associated with its received signal for its rotation cycle, except when it is moving perpendicular to the satellite direction. By using the tracking information from the ring antenna and interrogating the patch antenna, the point of alignment can be found. The idle correlators can be used for this task. Using this information and timing information, roll angles between satellites can be found. With knowledge of position and direction to each satellite and three such measurements of roll angle, it is possible to find the orientation of the spin axis. Also note that a directional antenna could be used, removing the need for a Doppler shift. Signal parameters like signal-to-noise ratio could also be used to find roll angles between satellites or beacons.

The previous discussion gives a possible implementation of the method. The requirements are to get roll angles between known directions. To visualize roll angles, consider a half plane attached to a spin axis. As this plane rotates, points in the containing space enter and leave this plane; the roll angle between two points is the amount of roll between one point leaving this rotating plane and the other point entering. Figure 1 is a graphic representation of this idea. The green rectangle represents a plane of observation for the sensor. An object in this plane is observed. As the observing plane spins to the position of the yellow rectangle, another object is observed. The roll angle between these observations is recorded as the measurement. Due to the geometry,

[3]Thompson, A. *Advances in Applied and Computational Mathematics*; Nova Science Publishers: New York, NY, 2006; Chapter 18.

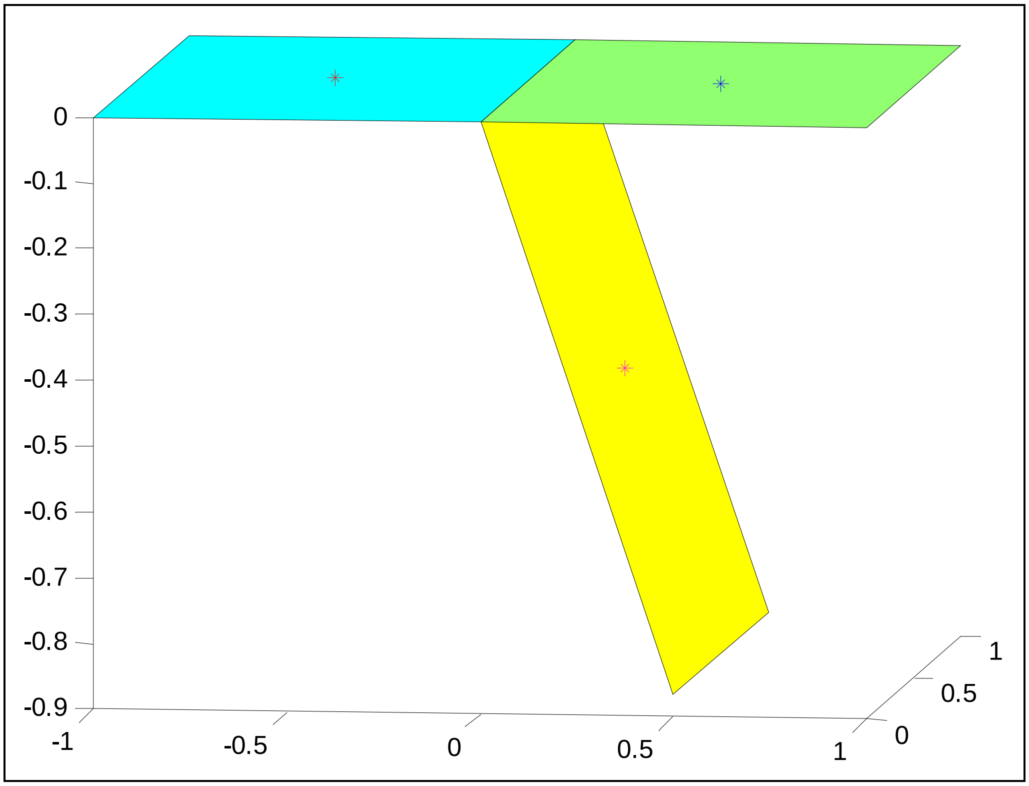

Figure 1. Roll angle.

the constraints imposed by the roll angles are sufficient to define the orientation or attitude of the spin axis. The following extreme situations are discussed in an attempt to clarify the nature of the information being used. Consider two directions or vectors and the plane formed by them. If the spin-axis vector is orthogonal to this plane, then the roll angle between the two directions is the same as the angle between the two directions. If the spin axis is in the same plane between the two directions, then the roll angle will be 180°, shown in figure 1 as the green and cyan rectangles. However, if the spin axis is located in the same plane so that both directions are to one side, the roll angle between the two will be 0°, as they will both be in the same half plane. The roll angle between two directions depends on the orientation of the spin axis. Three roll measurements will define the attitude of the spin axis; thus, four known directions are required. For GPS systems, four satellites are always in view.

It is a small extension to consider range systems with beacons at known locations being used as known directions. Lasers, infrared, and other portions of the electromagnetic spectrum can be used as energy sources; obviously, stars could make reliable energy sources. Sound waves and pressure waves of various types can be used as long as the direction to the energy source is known. The ability to monitor the precise attitude of a spinning shaft has uses in commercial enterprise as well as military and space applications.

4. Nonlinear Least Squares

A brief review of least squares will be given first. Start with the over-determined equation $FX = Y$, where *X* is the vector of unknown coordinates, *Y* represents measurements, and *F* is a matrix model of a known linear relationship. Then project both sides onto the column space of the matrix *F*. Multiply each side by the transpose of *F* and then, assuming *X* is multiplied by a square matrix of full rank, multiply each side by the inverse. The result will be

$$X = (F^t F)^{-1} F^t Y . \qquad (1)$$

This result, discussed in many textbooks, will be used as the core of an iterative procedure for nonlinear least squares. Both *F* and *Y* are known values, while *X* represents the unknown parameters.

Suppose the functional relationship, f, between the measurements and the coordinates (X) is known. This function is typically nonlinear. To start, the value of X needs to be guessed or approximated. Denote this value as X_0. The gradient matrix for the measurements with respect to the coordinates will now be denoted by F. If X_0 is reasonably close to X, then the following linearized approximation is valid:

$$f(X) = f(X_0) + F_{X_0}(X - X_0). \qquad (2)$$

The left side is the measurement vector, while the first term on the right is the evaluation of the functional relationship at the point X_0. Moving this term to the left side allows the interpretation of the left side as the deviation of the measurements from the approximated coordinates. The gradient evaluated at the point is multiplied by the unknown deviation from the approximation of the location. The previous equation can be written in the form $Y = FX$ as follows:

$$f(X) - f(X_0) = F_{X_0}(X - X_0). \qquad (3)$$

Linear least squares can be used to solve for the location deviation. After the solution is added to the current approximation (and becomes the new current approximation), the process can be repeated until the left side gets close to 0. If this is understood, what remains is to discuss realizations of f and F for different measurements in terms of coordinates. Generally, within the *F* matrix, different types of measurements can be used; however, only roll angles will be considered. Each measurement will define a row of the matrix.

5. Roll Angle Measurements

First note that all the quantities are vectors on the unit sphere. All directions are normalized. The unknown quantity is the vector representing the spin axis. Consider the ideal case where the sensor only observes energy in a half plane attached to the spin axis. Recall that the Gram-Schmidt process is a way to set up an orthogonal basis for a set of vectors. For two vectors, we can use this to create two orthogonal vectors. Consider two known directions and a spin axis. The roll angle is the angle between the orthogonal components of the known directions to the spin angle. First, each of these vectors will be described, and then their inner product will give the angle between them. Let *x* represent the unknown coordinates of the spin axis; let d_1 and d_2 be two known unit directions. The orthogonal or radial components of each direction are

$$r_1 = d_1 - \langle d_1 \ , x \ \rangle x, \tag{4}$$

and

$$r_2 = d_2 - \langle d_2 \ , x \ \rangle x, \tag{5}$$

where the brackets indicate the inner product function. Assume these new vectors are normalized using normalization factors n_1 and n_2. The inner product between these will yield the cosine of the angle between them.

$$\cos(\theta) = \left\langle \frac{d_1 - \langle d_1, x \ \rangle x}{n_1}, \frac{d_2 - \langle d_2, x \ \rangle x}{n_2} \right\rangle = \left\langle \frac{d_1 - p_1 x}{n_1}, \frac{d_2 - p_2 x}{n_2} \right\rangle, \tag{6}$$

where the inner products are defined by p_1 and p_2. In the sequel, the additional subscript indicates the component of the represented quantity. This can be written out as

$$= \frac{1}{n_1 n_2}\left((d_{11} - p_1 x_1)(d_{21} - p_2 x_1) + (d_{12} - p_1 x_2)(d_{22} - p_2 x_2) + (d_{13} - p_1 x_3)(d_{23} - p_2 x_3)\right). \tag{7}$$

Next, this expression will be expanded and then rearranged into a more concise group of symbols. Temporarily ignoring the normalization factors, multiply out each of the terms as follows:

$$\begin{aligned} &= d_{11}d_{21} - p_1 d_{21} x_1 - p_2 d_{11} x_1 + p_1 p_2 x_1^2 + d_{12} d_{22} - p_1 d_{22} x_2 - p_2 d_{12} x_2 + p_1 p_2 x_2^2 \\ &\quad + d_{13} d_{23} - p_1 d_{23} x_3 - p_2 d_{13} x_3 + p_1 p_2 x_3^2. \end{aligned} \tag{8}$$

Now the regrouping of terms results in

$$\begin{aligned} &= d_{11}d_{21} + d_{12}d_{22} + d_{13}d_{23} - p_1\left(d_{21}x_1 + d_{22}x_2 + d_{23}x_3\right) \\ &\quad - p_2\left(d_{11}x_1 + d_{12}x_2 + d_{13}x_3\right) + p_1 p_2\left(x_1^2 + x_2^2 + x_3^2\right). \end{aligned} \tag{9}$$

This result can be written in terms of inner products as follows:

$$= \langle d_1, d_2 \rangle - p_1 \langle d_2, x \rangle - p_2 \langle d_1, x \rangle + p_1 p_2 \|x\|^2 . \tag{10}$$

Recall all the original vectors, including the sensor spin-axis vector, were considered to be normalized. The preceding equation can be recast as

$$= \langle d_1, d_2 \rangle - p_1 p_2 - p_2 p_1 + p_1 p_2 . \tag{11}$$

Reintroducing the normalization factor,

$$= \frac{\langle d_1, d_2 \rangle - p_1 p_2}{n_1 n_2} = \frac{\langle d_1, d_2 \rangle - \langle d_1, x \rangle \langle d_2, x \rangle}{n_1 n_2} . \tag{12}$$

Now, the relationship between the cosine of the roll angle and the coordinates of the normalized spin axis has been established. Using this functional relationship, the gradient of the cosine of the roll angle with respect to the coordinates of the unknown spin axis can be determined. Next, an expression for the partial with respect to the ith component of the spin axis is needed as follows:

$$\frac{\partial \left(\dfrac{\langle d_1, d_2 \rangle - \langle d_1, x \rangle \langle d_2, x \rangle}{n_1 n_2} \right)}{\partial x_i} = - \frac{d_{1i} \langle d_2, x \rangle + d_{2i} \langle d_1, x \rangle}{n_1 n_2} . \tag{13}$$

This assumes the normalization factors make negligible contributions to the partial derivative. Assuming the normalization factors are constant simplifies each least-squares iteration, since the partials associated with the normalization factors are ignored. Since the exact partial is not being used, there will be a suboptimal convergence. The tradeoff between the number of steps saved per iteration and the additional number of iterations needed can be used to analyze this situation. The numerator of the roll angle expression contains the necessary information; thus, it seems the normalization factors do not contain information pertinent to the roll angle. In some situations, the extra complexity due to the partials of the normalization factors may be tolerated; however, as the simulations have shown, the normalization factors can be treated as constants in the partial. Although this simplification reduces implementation complexity, it results in a suboptimal convergence. At this point, it is possible to form a row in the previously described *F* matrix that corresponds to the cosine of the observed roll angle. To estimate the three coordinates of the spin axis, a minimum of three rows is required. By dealing directly with the cosine of the angle, the partial is simplified in comparison to the partial of the angle with respect to the coordinates. This perspective is no problem mathematically.

6. Simulation

A Matlab toolbox was designed to test these ideas and verify that the method would produce accurate results. After completing this validation, the set of GPS toolbox functions from Kai Borre was used to test the method under realistic conditions. In these simulations, the orientation of the spin axis was found. The method works; what remains is to work out details with specific applications. Matlab code for the simulation is included as the appendix.

7. Precise Derivative

In a previous discussion of using roll angles to determine attitude, it was stated that a reduced form of the derivative could be used in the linearization step of the iterative process. The simplification was justified through the use of a simulation. This simplification is investigated to give a more quantitative perspective to the former qualitative statement. The expression under investigation follows and represents a roll angle about a spin axis (x) between two known directions as follows:

$$\frac{\langle d_1, d_2 \rangle - \langle d_1, x \rangle \langle d_2, x \rangle}{n_1 n_2}. \tag{14}$$

Here the d terms are known directions of unit magnitude, and x is an unknown direction of unit magnitude. The terms in the denominator are both normalization factors and are as follows:

$$n_i = \left\| d_i - \langle d_i . x \rangle x \right\|. \tag{15}$$

Adding an additional subscript to identify the component, this can be written as

$$n_i = \begin{pmatrix} \left(d_{i1} - (d_{i1}x_1 + d_{i2}x_2 + d_{i3}x_3)x_1 \right)^2 + \left(d_{i2} - (d_{i1}x_1 + d_{i2}x_2 + d_{i3}x_3)x_2 \right)^2 + \\ \left(d_{i3} - (d_{i1}x_1 + d_{i2}x_2 + d_{i3}x_3)x_3 \right)^2 \end{pmatrix}^{\frac{1}{2}}, \tag{16}$$

or more concisely,

$$n_i = \left(\sum_{j=1}^{3} \left(d_{ij} - (d_{i1}x_1 + d_{i2}x_2 + d_{i3}x_3)x_j \right)^2 \right)^{\frac{1}{2}}. \tag{17}$$

The partial of equation 17 is described by the following equation:

$$\frac{\partial n_i}{\partial x_j}=\frac{1}{2}\frac{2\left(d_{ij}-(d_{i1}x_1+d_{i2}x_2+d_{i3}x_3)x_j\right)\left(-2d_{ij}x_j-\sum_{k\neq j}d_{ik}x_k\right)+2\sum_{k\neq j}\begin{pmatrix}d_{ik}-(d_{i1}x_1+d_{i2}x_2\\+d_{i3}x_3)d_{ij}x_k\end{pmatrix}}{n_i}. \quad (18)$$

The numerator of the first expression has the following derivative:

$$\frac{\partial\langle d_1,d_2\rangle-\langle d_1,x\rangle\langle d_2,x\rangle}{\partial x_i}=d_{1i}\langle d_2,x\rangle+d_{2i}\langle d_1,x\rangle. \quad (19)$$

Now that the individual pieces have been defined, they will be assembled using the following equation. First, let p represent the numerator of the first expression. The expression of interest can be written as

$$\frac{p}{n_1n_2}. \quad (20)$$

The partial derivative of it can be found using the product rule as follows:

$$\frac{\partial pn_1^{-1}n_2^{-1}}{\partial x_i}=\frac{\partial p}{\partial x_i}n_1^{-1}n_2^{-1}-pn_2^{-1}n_1^{-2}\frac{\partial n_1}{\partial x_i}-pn_1^{-1}n_2^{-2}\frac{\partial n_2}{\partial x_i}. \quad (21)$$

This can now be evaluated by substituting the former expressions in the previous equation. If the directions and spin axis are known, then each of the terms will indicate the magnitude of the particular partial's influence in relation to the total. If the second and third terms are relatively large, this can differ significantly from the abbreviated partial used in the simulation. The appendix includes code for evaluating the partial.

A simulation was designed to compare the partial of p with the partial of $\frac{p}{n_1n_2}$. The measure of agreement was the inner product of the normalized partials. If they both point in the same direction, the inner product is 1; if the inner product is 0, the partials are perpendicular, and negative values will result in divergence in the estimation process. Three cases were investigated. In the first case, the two known directions and the spin axis were chosen randomly. The results are shown in figure 2.

Although the majority of the cases are in the correct direction and thus lead to convergence, there are instances approaching complete divergence. From figure 2, it can be conjectured that using the partial of p will result in suboptimal convergence that will take a step in the wrong direction on ~10% of the iterations and move in the appropriate direction on ~70% of the steps, as indicated by positive inner products.

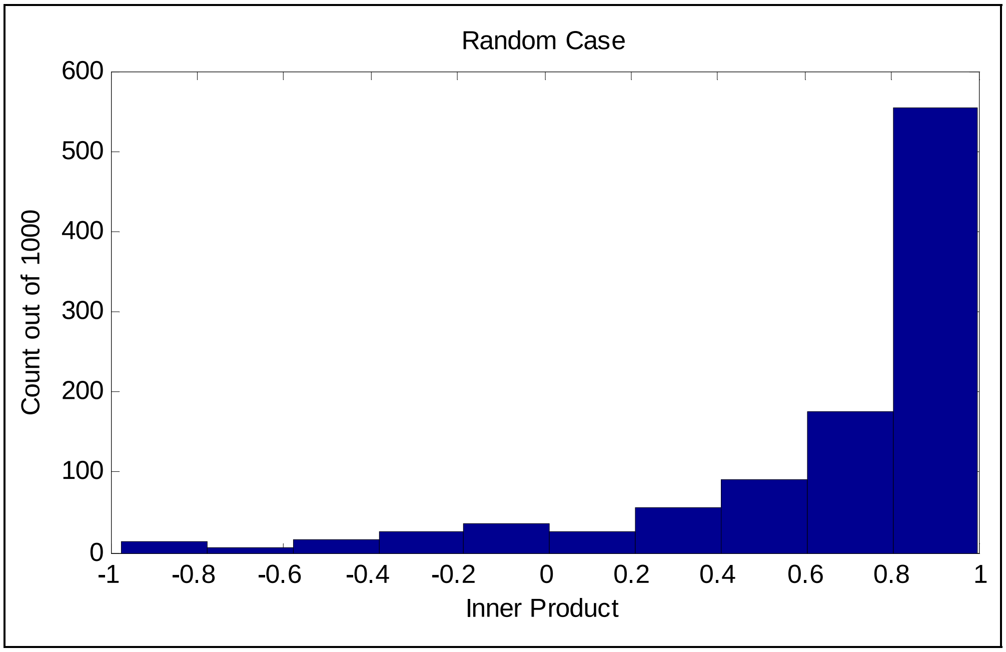

Figure 2. Agreement for randomly chosen directions and spin axis.

A second simulation was performed with the two known directions perpendicular to each other. Figure 3 illustrates the results of the simulation. Comparing this to figure 2, it can be seen that the number and magnitude of divergent instances have decreased. In this case, ignoring the partials of the normalization factors has a smaller effect on the convergence of the estimator, and less than 2% of the cases are divergent. Figure 4 shows that if the two known directions are 120° apart, 10% of the cases diverge.

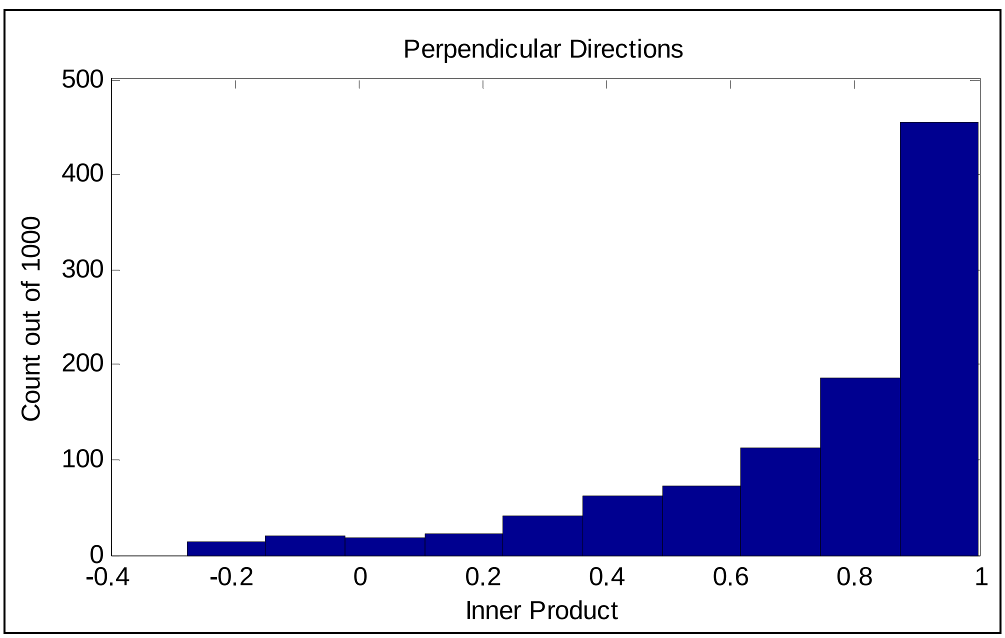

Figure 3. Inner products for perpendicular directions.

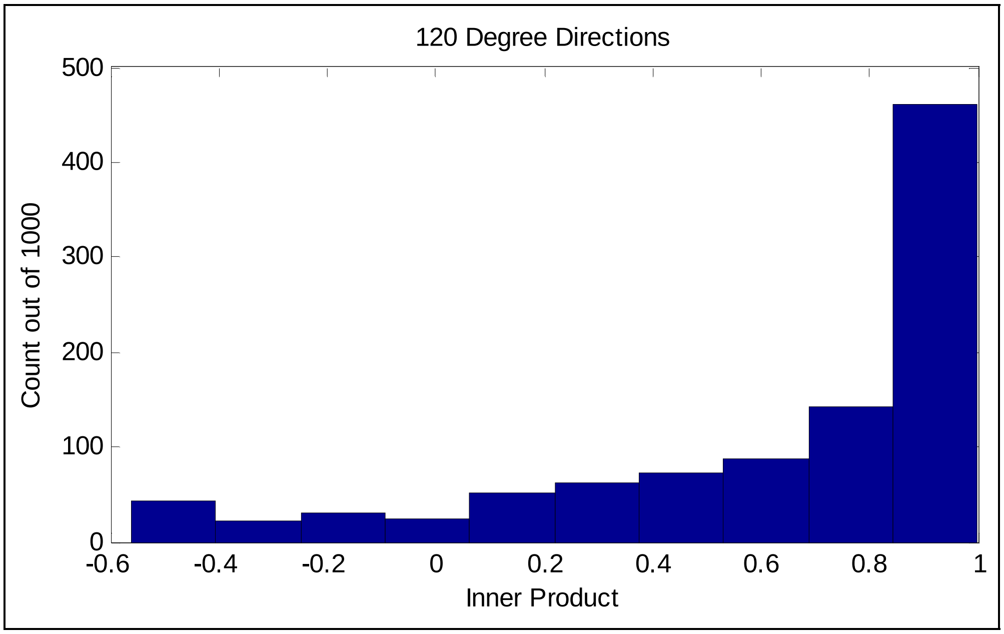

Figure 4. Inner products for 120° separation of directions.

Although the estimator converged while ignoring the partials of the normalization factors, the results of this section indicate it may be at the cost of extra iterations. An implementation of the method that does not include the partials of the normalization factors should present a rationale for their omission.

8. Closing Remarks

This method can have widespread application. Attitude estimation is a problem in many fields, from detecting small shifts in snow to predicting avalanches to aerospace applications. Many systems' sole purpose is to estimate the attitude of an object. A method that estimates attitude from known directions and roll angle measurements has been developed, and simulations show that if the requirements are met, accurate estimates of the axis orientation can be made. For artillery systems, this is a solution to the "find down" problem. For systems already equipped with GPS for position information, this will allow the receivers to be extended and also deliver attitude information. This method can be used on a spinning sensor where the directions to known sources of energy are known. It would also be possible to simulate a spin axis with a steerable antenna. A single sensor providing both location and attitude information would solve many tactical and test range problems.

Appendix. Matlab Code

This appendix appears in its original form, without editorial change.

```
function [dip, num]=ipiter(a,meas)

%for i=1:n a(:,i)=a(:,i)/norm(a(:,i)); end
aip=a'*a;  %inner products
[m n]=size(a);
%sweep the spin axis component from the other vectors
sa=a;
for i=2:n
   sa(:,i)=a(:,i)-aip(1,i)*a(:,1);
end
anorm=sqrt(sum(sa.^2));  %norm of the vectors orth to spin axis
%get the measurements c notes
num=anorm;
for i=2:n-1
   num(i)=(aip(i,i+1)-aip(1,i)*aip(1,i+1))/(anorm(i)*anorm(i+1));
end
%c the angular situation
%acos(num)*180/pi;

% find differential
dip=zeros(n-2,3);  %initialize
for j=2:n-1
   denom=anorm(j)*anorm(j+1);
   for i=1:3
      dip(j-1,i)=-(a(i,j)*aip(1,j+1)+a(i,j+1)*aip(1,j))/denom;
   end
end
```

routine ipconverge

```
tst=astart(:,1)';
%a(:,1)=[-.2;-.9;.0001];
it=0;
[r n_ipc]=size(meas);
n_ipc=n_ipc-1;
while (1-abs(tst*a(:,1))>.000001)
%for i=1:20
   it=it+1;
   [dip num]=ipiter(a,meas);
   y=(meas(2:n_ipc)-num(2:n_ipc))';
   xtx=inv(dip'*dip);
   yadj=xtx*dip'*y;
   a(:,1)=a(:,1)+yadj;
   a(:,1)=a(:,1)/norm(a(:,1));
   if it>100 break; end
end
```

```
%function y=gpsipsim()

close all;
clear all;

% ------------------------------------------------------------
% --- Read ephemeris (RINEX nav-file or YUMA almanac)      ---
% --- and delete double occurences of SVN's                ---
% ------------------------------------------------------------

eph = sortrows(rdyuma('yuma79.txt'));

% ----------------------------------------
% --- Station coordinates (WGS84 / XYZ) ---
% ----------------------------------------

station(1) = 3923625; % Delft (Geodesy-building)
station(2) =  298462;
station(3) = 5002803;
cutoff     = 10;      % Cutoff elevation in degrees

% ----------------
% --- Time/span ---
% ----------------

tsat = mktsat ('28-FEB-2001 12:00:00','28-FEB-2001 14:00:00',300);

% -----------------------------------------------------
% --- Compute satellite positions (XYZ WGS84 & AZ/EL) ---
% -----------------------------------------------------

[xsat,ysat,zsat,azim,elev,rotmat]    = cpaziele (tsat,eph,station);

%get local directions form the rotation matrix
local_vert=-rotmat(3,:)';
local_north=rotmat(1,:)';
%pick a random angle to rotate about the local vert
%make a quaternion rotation about the vert
%find the new axis by rotating local north about the vert
d_r=2*pi*rand;
q_d=q_aa2quat(local_vert,d_r);
axis=q_rotate(q_d,local_north);
%follow above procedure choose rotation
%make rotation quat to rotate about previous horizantal axis
%calculate new axis by rotaing the verticle axis
d_r=45*pi/180;
```

```
q_d=q_aa2quat(axis,d_r);
s_axis=q_rotate(q_d,local_vert);
a=zeros(3,29);
[r n_epochs]=size(tsat);
con_its=[];
est_err=[];
dxtx=[];
itlist=[];
i_noncon=0;
for i=1:n_epochs
    vis_sat =   find (elev(:,i) > cutoff);
    n_direct=size(vis_sat);
    for j=1:size(vis_sat)
        tmp=[xsat(vis_sat(j),i);ysat(vis_sat(j),i);zsat(vis_sat(j),i)];
        tmp=tmp-station';
        tmp=tmp/norm(tmp);
        a(:,j)=tmp;
    end
    a=[s_axis a];
    n=n_direct+1;
    a=a(:,1:n);
    aip=a'*a;
    sa=a;
    for i=2:n
      sa(:,i)=a(:,i)-aip(1,i)*a(:,1);
    end
    anorm=sqrt(sum(sa.^2));
    num=a(1,:);
    aip=a'*a;
    for i=2:n_direct
     num(i)=(aip(i,i+1)-aip(1,i)*aip(1,i+1))/(anorm(i)*anorm(i+1));
    end
    meas=num;
    astart=a;
    d_r=2*pi*rand;
    q_d=q_aa2quat(s_axis,d_r);
    axis3=q_rotate(q_d,axis);
    d_r=10/180*pi;
    q_d=q_aa2quat(axis3,d_r);
    axis_p=q_rotate(q_d,s_axis);
    a(:,1)=axis_p;
    ipconverge
    con_its=[con_its it];
    err=astart(:,1)-a(:,1);
    est_err=[est_err err];
    dxtx=[dxtx det(xtx)];
```

```
    if it>100
        i_noncon=i_noncon+1;
        noncon(i_noncon).a=astart;
        noncon(i_noncon).sa=axis_p;
        noncon(i_noncon).meas=meas;
        i
        xtx
        anorm
        aip
        a
    end
end
```

example of use

```
gpsipsim
ipinvest
a=noncon.a
ipinvest
plot_hs
a=noncon.a
r_set
plot_hs
con_its
gpsipsim
con_its
help find
find(con_its>100)
noncon
size(noncon)
nonconinv
plot_hs
pplot_hs
plot_hs
noncon
r_noncon=noncon
con_its
rotatenocon
rotatnocon
rotatnoncon
noncontemp=noncon
noncon=r_noncon;
nonconinv
plot_hs
```

```
%function y=ipadjust()

n=5; %n must b more than 3
a=randn(3,n);

for i=1:n a(:,i)=a(:,i)/norm(a(:,i)); end

aip=a'*a  %inner products

%sweep the spin axis component from the other vectors
sa=a;
for i=2:n
   sa(:,i)=a(:,i)-aip(1,i)*a(:,1);
end
anorm=sqrt(sum(sa.^2));  %norm of the vectors orth to spin axis
%get the measurements c notes
num=anorm;
for i=2:n-1
   num(i)=(aip(i,i+1)-aip(1,i)*aip(1,i+1))/(anorm(i)*anorm(i+1));
end
%c the angular situation
acos(num)*180/pi

% find differential
dip=zeros(n-2,3);  %initialize
for j=2:n-1
   denom=anorm(j)*anorm(j+1);
   for i=1:3
      dip(j-1,i)=-(a(i,j)*aip(1,j+1)+a(i,j+1)*aip(1,j))/denom;
   end
end

%function y=ipconverge_d(d,rotaxis)


d_r=d/180*pi
q_d=q_aa2quat(rotaxis,d_r)
axis=q_rotate(q_d,v)
a(:,1)=axis
ipconverge
y=it


%function y=ipinvest(a)

aip=a'*a  %inner products
```

```
%sweep the spin axis component from the other vectors
sa=a;
[r n]=size(a);
for i=2:n
    sa(:,i)=a(:,i)-aip(1,i)*a(:,1);
end
anorm=sqrt(sum(sa.^2));  %norm of the vectors orth to spin axis
%get the measurements c notes
num=anorm;
for i=2:n-1
    num(i)=(aip(i,i+1)-aip(1,i)*aip(1,i+1))/(anorm(i)*anorm(i+1));
end

astart=a;
meas=num;
tst=astart(:,1)';

a(:,1)=starta
%
it=0;
[r n_ipc]=size(meas);
n_ipc=n_ipc-1;
seq=[];
while (1-abs(tst*a(:,1))>.000001)
%for i=1:20
    it=it+1;
    [dip num]=ipiter(a,meas);
    y=(meas(2:n_ipc)-num(2:n_ipc))';
    xtx=inv(dip'*dip);
    yadj=xtx*dip'*y;
    %af=yadj-yadj'*a(:,1)*a(:,1);  this method oscilated toom uch
    %yadj_1=norm(yadj)*af/norm(af);
    temp=a(:,1)+yadj;  %method2
    temp=temp/norm(temp); %m2
    t_2=temp-a(:,1);   %m2
    t_2=norm(yadj)/norm(t_2)*t_2;  %m2
    a(:,1)=a(:,1)+1*t_2;   %m2
    a(:,1)=a(:,1)/norm(a(:,1)); %m2
    %a(:,1)=temp/norm(temp); %method 1
    seq=[seq a(:,1)];
    if it>100 break; end
end
figure;plot3(seq(1,:),seq(2,:),seq(3,:),'-+')
grid on;
hold on;plot3(astart(1,1),astart(2,1),astart(3,1),'ro')
plot3(seq(1,1),seq(2,1),seq(3,1),'go')
```

```
%this was to plot eigenvalues from [d v]=eig(xtx)
%x=astart(:,1);for iz=1:3 plot3([x(1) x(1)+.02*v(1,iz)],[x(2) x(2)+.02*v(2,iz)], [x(3)
x(3)+.02*v(3,iz)]); end

%function y=ipstart()

ipadjust
astart=a;
meas=num

v=astart(:,1)
vperp=[v(2);-v(1);0]
deg=2*pi*rand
pertaxis=q_aa2quat(v,deg)
rotaxis=q_rotate(pertaxis,vperp)

d5=5/180*pi
q5=q_aa2quat(rotaxis,d5)
axis_5=q_rotate(q5,v)
a(:,1)=axis_5
ipconverge

d10=10/180*pi
q10=q_aa2quat(rotaxis,d10)
axis_10=q_rotate(q10,v)
a(:,1)=axis_10
ipconverge

%function y=iptest()

n=3; %n must b more than 3
a=randn(3,n);

for i=1:n a(:,i)=a(:,i)/norm(a(:,i)); end
a;
a2=a.^2;

sum(a2)

aip=a'*a
ip1=aip(1,3);
ip2=aip(2,3);
sa1=a(:,1)-ip1*a(:,3);
sa1'*a(:,3)
sa2=a(:,2)-ip2*a(:,3);
s_ip=sa1'*sa2/(norm(sa1)*norm(sa2));
```

```
acos(s_ip)*180/pi
acos(aip(1,2))*180/pi
sa2'*a(:,3)
ns1=norm(sa1);
ns2=norm(sa2);

num=aip(1,2)-aip(1,3)*aip(2,3);
acos(num/(ns1*ns2))*180/pi

% find differential
dip=sa1;
for i=1:3
   dip(i)=-(a(i,1)*aip(2,3)+a(i,2)*aip(1,3));
end
dip=dip/(ns1*ns2)


%function y=nonconinv()

[r c]=size(noncon);

for i=1:c
   a=noncon(i).a
   starta=noncon(i).sa
   ipinvest
end

%function y=plot_hs()
n=20
theta = pi*(-n:2:n)/n;
phi = (pi/2)*(0:2:n)'/n;
X = cos(phi)*cos(theta);
Y = cos(phi)*sin(theta);
Z = sin(phi)*ones(size(theta));
%figure
plot3(X,Y,Z,'y')
[r,c]=size(Z);
hold on
for i=1:r
   plot3(X(i,:),Y(i,:),Z(i,:),'y')
end

%function y=r_set()

q1=q_aa2quat([1;1;0],pi/4)
```

```
ar=a;for j=1:9 ar(:,j)=q_rotate(q1,a(:,j)); end
a=ar
ipinvest

%function y=rotatnoncon()

r_noncon=noncon;

q1=q_aa2quat([1;1;0],pi/4); %rotational axis and angular rotation
[r c]=size(noncon);

for i=1:c
    ar=noncon(i).a;for j=1:9 ar(:,j)=q_rotate(q1,a(:,j)); end
    r_noncon(i).a=ar;
    r_noncon(i).sa=q_rotate(q1,noncon(i).sa);
end
```

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