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Use of Instantaneous Streamflow Measurements to Improve Regression Estimates of Index Flow for the Summer Month of Lowest Streamflow in Michigan

Scientific Investigations Report 2010–5236

U.S. Department of the Interior

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By David J. Holtschlag

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Conversion Factors

Multiply	By	To obtain
	Length	
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
	Area	
square mile (mi^2)	259.0	hectare (ha)
square mile (mi^2)	2.590	square kilometer (km^2)
	Volume	
cubic foot (ft^3)	0.02832	cubic meter (m^3)
	Flow rate	
foot per second (ft/s)	0.3048	meter per second (m/s)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
cubic foot per second per square mile [$(ft^3/s)/mi^2$]	0.01093	cubic meter per second per square kilometer [$(m^3/s)/km^2$]
inch per year (in/yr)	25.4	millimeter per year (mm/yr)
mile per hour (mi/h)	1.609	kilometer per hour (km/h)

Vertical coordinate information is referenced to the North American Vertical Datum of 1988 (NAVD 88).

Horizontal coordinate information is referenced to the North American Datum of 1983 (NAD 83).

Altitude, as used in this report, refers to distance above the vertical datum.

Water year: The 12-month period from October 1 through September 30. The water year is designated by the calendar year in which it ends.

Symbols Used in this Report

Symbol	Name
$\boldsymbol{Q}50$	The index flow defined as the minimum of median flows at a stream site for July, August, and September, in cubic feet per second.
$\boldsymbol{rY}50$	The index water yield defined as the square root of water yield corresponding to the square root of the quantity $\boldsymbol{Q}50$ divided by the upstream drainage area ($DArea$), in cubic feet per second per square mile.
$Gage\boldsymbol{Q}50$	The measured index flow defined as the minimum of median flows for the months of July, August, and September measured at a long-term streamgage.
$Gage\boldsymbol{rY}50$	The measured index water yield defined as the square root of water yield corresponding to the square root of the quantity $Gage\boldsymbol{Q}50$ divided by the upstream drainage area, in cubic feet per second per square mile.
$Indx\boldsymbol{Q}50$	The index regression estimate of index flow computed as the $Indx\boldsymbol{rY}50$ estimate squared times the corresponding drainage area.
$Indx\boldsymbol{rY}50$	The index regression estimate of water yield.
$DArea$	Drainage Area, in square miles.
$RMSE$	Root mean square error, which is a common measure of the standard deviation of the model residuals adjusted for the number of estimated parameters.
r_s^2	The estimated coefficient of determination computed by squaring the Spearman (rank) correlation coefficient.
J^o	A target streamflow site or streamgage.
$\{J'\}$	A set of auxiliary streamgages used to estimate index flow at J^o.
$\hat{\sigma}^2_{Indx\boldsymbol{rY}50(J^o)}$	The estimated prediction variance for site J^o based on the $Indx\boldsymbol{rY}50$ estimator.
$\boldsymbol{Q}dv$	A daily value of flow, in cubic feet per second.
$\boldsymbol{rY}dv$	A daily value of the square root of water yield computed as $\sqrt{Qdv/DArea}$.
$\boldsymbol{rY}mm$	An estimate of the square root of water yield based on an instantaneous flow measurement.
SLR	Simple linear regression, which implies a linear-regression equation with one explanatory variable and one response variable.
$Site\boldsymbol{rY}50$	An estimate of index water yield based on the Site Estimator.
$Site\boldsymbol{Q}50$	An estimate of the index flow computed as $Site\boldsymbol{rY}50^2 \cdot DArea$.
$Disc\boldsymbol{rY}50$	An estimate of index water yield based on the Discrete Estimator.
$Disc\boldsymbol{Q}50$	An estimate of index flow computed as $Disc\boldsymbol{rY}50^2 \cdot DArea$.
$Intg\boldsymbol{rY}50$	An estimate of index water yield based on the Integrated Estimator.
$Intg\boldsymbol{Q}50$	An estimate of the index flow computed as $Intg\boldsymbol{rY}50^2 \cdot DArea$.
$Augm\boldsymbol{rY}50$	An estimate of index water yield based on the Augmented Estimator.
$Augm\boldsymbol{Q}50$	An estimate of index flow computed as $Augm\boldsymbol{rY}50^2 \cdot DArea$.
I_q	A set of 12 flow-measurement intensities $\{i_{q_1}, i_{q_2}, \cdots, i_{q_{12}}\}$ where the number of measurements, $\#\{I_q\}$, equals $\{1, 2, \cdots, 7, 10, 15, 20, 50, 100\}$.

Symbol	Name
MHS	Michigan Hydrologic Subregion, which is referred to as subregion for brevity where ambiguity can be avoided.
$\{b_o, b_1, \cdots, b_p\}$	A set of regression parameters estimated by ordinary least squares.
ε	An error term indicating an unexpected difference between measured and estimated values.
$\{Disc\boldsymbol{rY}50_{t,J^o}\}$	A set of discrete estimates of flow indexed by t for site J^o.
$\hat{\sigma}^2_{p(t,J^o)}$	Estimated prediction error variances indexed by t for site J^o.
$\hat{\sigma}^2_w$	The within measurement-day estimated variance.
$\hat{\sigma}^2_b$	The between measurement-day estimated variance.
$ciDisc\boldsymbol{rY}50$	A 95-percent confidence interval about the $Disc\boldsymbol{rY}50$ estimate.
$ciIntg\boldsymbol{rY}50$	A 95-percent confidence interval about the $Intg\boldsymbol{rY}50$ estimate.
$ciSite\boldsymbol{rY}50$	A 95-percent confidence interval about the $Site\boldsymbol{rY}50$ estimate.
	For all, as in for all elements in a set.
$\in$	Is an element of, as an element of a set or a matrix.
$\nabla RMSE_{\kappa,\kappa+1}$	Percent decrease in the root mean-square error of two estimators indexed by κ .
rY_t	The (square) root water yield estimate at time t.
rY_o	A multiplicative root water yield (base flow) recession parameter corresponding approximately to the root water yield during recession periods beginning after direct runoff and early time ground-water response have passed.
τ	Time constant for a linear system.
e	The mathematical constant e is an irrational number approximately equal to 2.71828.
$\#\{A\}$	The cardinality (number of elements) of set A.

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By David J. Holtschlag

Abstract

In Michigan, index flow ***Q50*** is a streamflow characteristic defined as the minimum of median flows for July, August, and September. The state of Michigan uses index flow estimates to help regulate large (greater than 100,000 gallons per day) water withdrawals to prevent adverse effects on characteristic fish populations. At sites where long-term streamgages are located, index flows are computed directly from continuous streamflow records as *Gage**Q**50*. In an earlier study, a multiple-regression equation was developed to estimate index flows *Indx**Q**50* at ungaged sites. The index equation explains about 94 percent of the variability of index flows at 147 (index) streamgages by use of six explanatory variables describing soil type, aquifer transmissivity, land cover, and precipitation characteristics. This report extends the results of the previous study, by use of Monte Carlo simulations, to evaluate alternative flow estimators, *Disc**Q**50*, *Intg**Q**50, Site**Q**50*, and *Augm**Q**50*. The Monte Carlo simulations treated each of the available index streamgages, in turn, as a miscellaneous site where streamflow conditions are described by one or more instantaneous measurements of flow. In the simulations, instantaneous flows were approximated by daily mean flows at the corresponding site. All estimators use information that can be obtained from instantaneous flow measurements and contemporaneous daily mean flow data from nearby long-term streamgages. The efficacy of these estimators was evaluated over a set of measurement intensities in which the number of simulated instantaneous flow measurements ranged from 1 to 100 at a site.

The discrete measurement estimator *Disc**Q**50* is based on a simple linear regression developed between information on daily mean flows at five or more streamgages near the miscellaneous site and their corresponding *Gage**Q**50* index flows. The regression relation then was used to compute a *Disc**Q**50* estimate at the miscellaneous site by use of the simulated instantaneous flow measurement. This process was repeated to develop a set of *Disc**Q**50* estimates for all simulated instantaneous measurements, a weighted *Disc**Q**50* estimate was formed from this set. Results indicated that the expected value of this weighted estimate was more precise than the *Indx**Q**50* estimate for all measurement intensities evaluated.

The integrated index-flow estimator, *Intg**Q**50*, was formed by computing a weighted average of the index estimate *Indx**Q**50* and the *Disc**Q**50* estimate. Results indicated that the *Intg**Q**50* estimator was more precise than the *Disc**Q**50* estimator at low measurement intensities of one to two measurements. At greater measurement intensities, the precision of the *Intg**Q**50* estimator converges to the *Disc**Q**50* estimator. Neither the *Disc**Q**50* nor the *Intg**Q**50* estimators provided site-specific estimates. In particular, although expected values of *Disc**Q**50* and *Intg**Q**50* estimates converge with increasing measurement intensity, they do not necessarily converge to the site-specific value of ***Q**50.*

The site estimator of flow, *Site**Q**50,* was developed to facilitate this convergence at higher measurement intensities. This is accomplished by use of the median of simulated instantaneous flow values for each measurement intensity level. A weighted estimate of the median and information associated with the *Intg**Q**50* estimate was used to form the *Site**Q**50* estimate. Initial simulations indicate that the *Site**Q**50* estimator generally has greater precision than the *Intg**Q**50* estimator at measurement intensities greater than 3, however, additional analysis is needed to identify streamflow conditions under which instantaneous measurements will produce estimates that generally converge to the index flows.

A preliminary augmented index regression equation was developed, which contains the index regression estimate and two additional variables associated with base-flow recession characteristics. When these recession variables were estimated as the medians of recession parameters computed from daily values defining recession events at index streamgages, this augmented estimator (*AugIndx**rY**50*), had 66.3-percent lower root mean square error (*RMSE)* than the *Indx**rY**50* estimator. For the *Augm**rY**50* estimator, in which the recession parameters for the augmented regression were estimated by Monte Carlo analysis, error characteristics decreased monotonically between an upper limit defined by error characteristics of the *Indx**rY**50* and *AugIndx**rY**50* estimators, depending on sampling intensity. Characterizing the uncertainty of new predicted values from the *AugIndx**rY**50* estimator is problematic, however, because two of the key explanatory variables are uncertain.

Introduction

The Michigan Legislature (2006) passed Public Act 33 in 2006 (PA33–2006); this and related laws are the first state laws to regulate water withdrawals. The legislation seeks to prevent any new or increased large-capacity withdrawal (generally referring to withdrawals that average more than 100,000 gallons of water per day (0.1547 ft^3/s) in any consecutive 30-day period) from causing an adverse resource effect. This effect is defined as decreasing the flow of a stream by part of the index flow such that the ability of the stream to support characteristic fish populations is functionally impaired. PA33–2006 further defines index flow as the 50 percent exceedance (median) flow for the lowest flow month of the flow regime (year), as determined over the period of record or extrapolated from analyses of U.S. Geological Survey (USGS) streamflow-gaging-station (hereafter termed streamgage) records in Michigan.

In Michigan, the index flow ***Q50*** is defined as the minimum of median flows during the months of July, August, or September. From this definition, index flow is computed from daily mean flows measured at selected (index) streamgages operated in Michigan for the periods of systematic record available for July, August, and September. Although this calculated index flow *Gage**Q**50* varies somewhat with the period of record, it is an unbiased estimator of ***Q50*** (which is never observed), and, if ***Q50*** is stationary, *Gage**Q**50* converges to ***Q50*** as the period of record increases. The summer months of July, August, and September are considered the time of greatest stress on characteristic fish populations because of lower flow rates and higher water temperatures than other months.

Purpose and Scope

The purpose of this report is to develop and evaluate four techniques for increasing the accuracy of index flow estimates in Michigan by integrating information from one or more instantaneous flow measurements with continuous flow information at streamgages. The utilities of the methods are assessed on the basis of the existing daily flow data and index flow characteristics computed at index streamgages that were used to develop the multiple-regression equation for estimating index flows.

Description of the Study Area

Michigan is in the eastern north-central part of the United States and is surrounded by four of the five Great Lakes. Ontario, Canada, lies to the north and east of Michigan. To the west and south, bordering states include Wisconsin, Indiana, and Ohio. Michigan is the 10th largest state in the Union with a total land area of 57,000 mi^2, 38,600 mi^2 of Great Lakes waters, and 1,190 mi^2 of inland waters (Michigan's State Facts, 2010). According to the U.S. Census Bureau (2008), the population of Michigan in 2008 was estimated to be slightly more than 10 million.

Michigan has a humid continental climate in which the average precipitation (rainfall plus water-equivalent snowfall depths) varies from about 28 to 38 in/yr. December through March tends to have slightly less precipitation, whereas July through September tends to have slightly more precipitation, than is typical for the rest of the year. Greater evapotranspiration during the summer, however, generally causes summer streamflows to be lower than those at other times of the year.

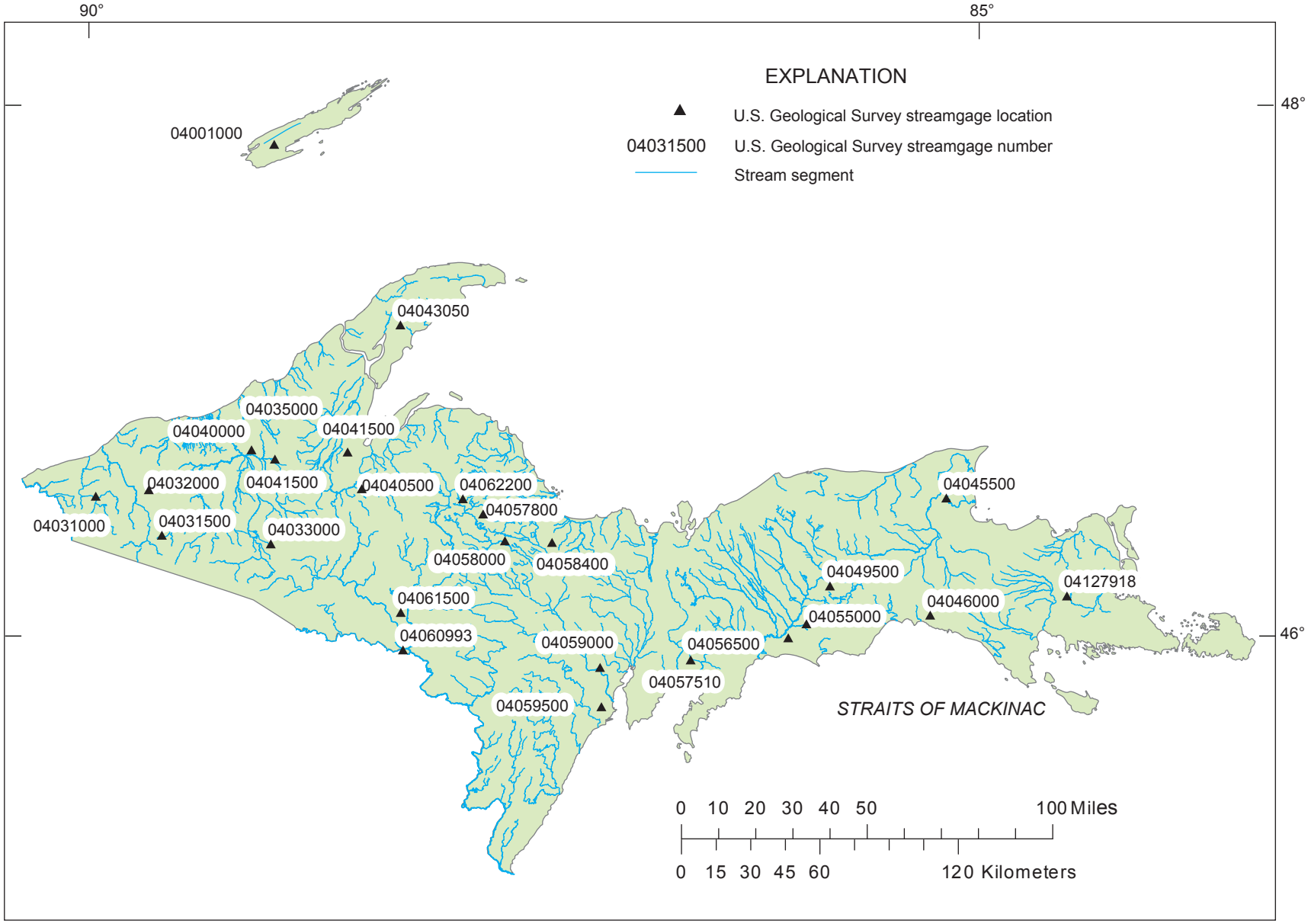
Michigan consists of two peninsulas (figs. 1 and 2) separated by the Straits of Mackinac, a body of water that connects Lake Michigan with Lake Huron. The straits are spanned by the Mackinac Bridge, where the northern tip of the Lower Peninsula is within about 5 mi of the southern coast of the eastern Upper Peninsula. The Upper Peninsula is heavily forested and somewhat mountainous in the west. Bedrock is at or near the surface in much of the Upper Peninsula.

The Lower Peninsula is covered by a thick layer of glacial drift. The northern part is characterized by sandy material and is heavily forested. Trout streams, sustained by plentiful base flow, are common in that area. Much of the southeastern part of the Lower Peninsula is flat lakebed plains that are extensively agricultural or urban; base flow is meager. The southwestern part of the Lower Peninsula has a wide mixture of landforms, soil types, land uses, and stream types.

An Existing Regression Model for Estimating Index Flows

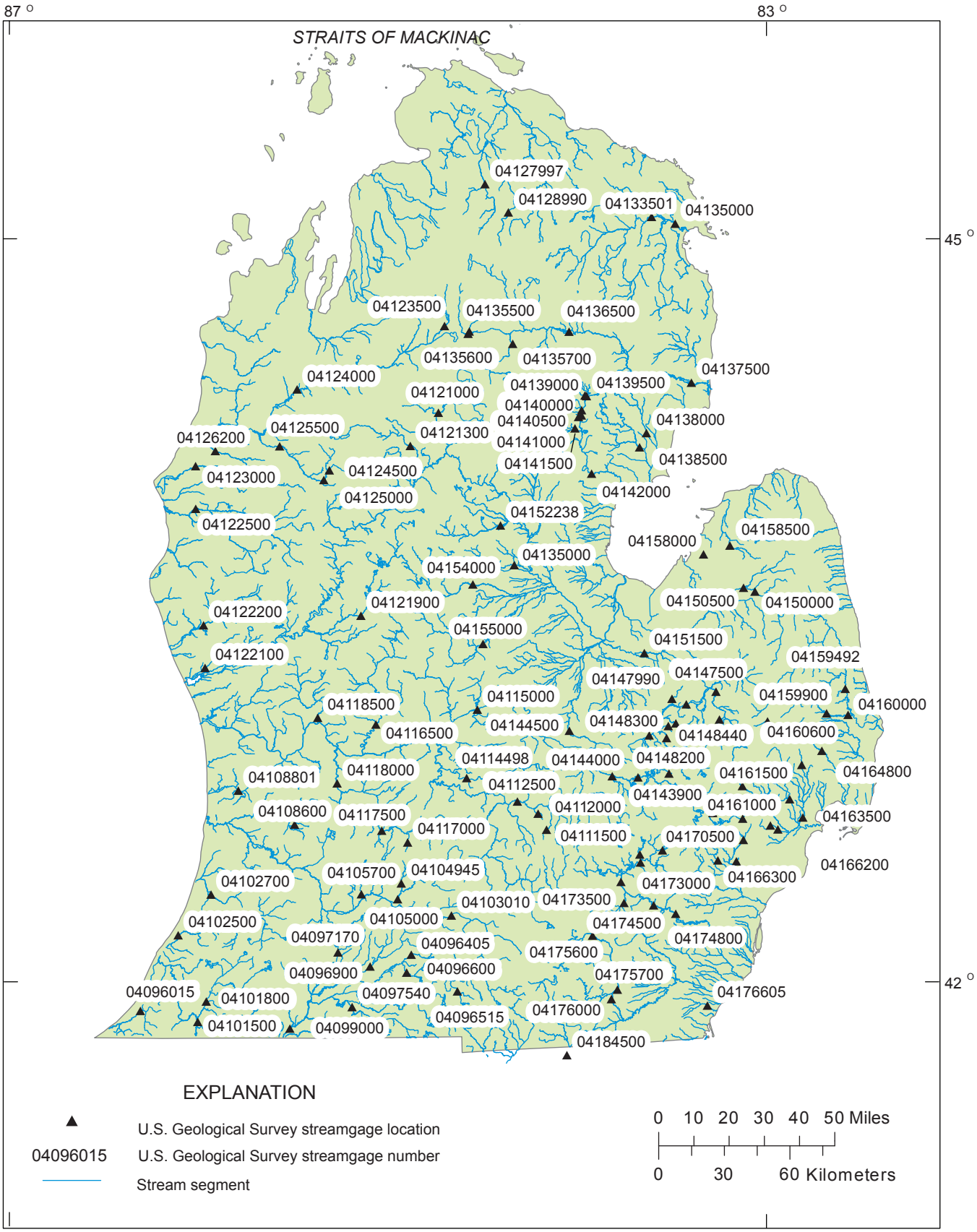
Hamilton and others (2008) developed a multiple-linear-regression model for use in estimating index flow ***Q50*** at ungaged sites. Regression models are based on the assumption that the errors (residuals or differences between measured and estimated values) are normally distributed. This assumption is unlikely to be satisfied if the response variable is not normally distributed. To more nearly satisfy this normality assumption, the *Gage**Q**50* estimate computed from streamgage records was divided by the upstream drainage areas *DArea* contributing to flow to compute index water yields *Gage**Y**50*. For simplicity, this notation will be abbreviated to ***Y50***, where ambiguity can be avoided. Hamilton and others (2008) found that the square ***r***oots of index water yields *Gage**rY**50* were approximately normally distributed, so this metric was used as the response variable in the index regression model. The regression estimate *Indx**rY**50* is an estimator that generally is applicable within Michigan. Squaring *Indx**rY**50* and multiplying by the drainage area of the upstream basin produces the index regression estimate of flow *Indx**Q**50*.

The index regression was based on a set of 147 USGS streamgages (referred to as index streamgages in this report) in or near Michigan that had (1) a minimum of 10 years of continuous-record flow data; (2) no appreciable effect on daily mean flows from water withdrawal, diversion, or augmentation; and (3) no effects of regulation, either natural storage in lakes or retention in regulated surface-water bodies, that would substantially mask the daily flow response from



Base map from U.S. Geological Survey, 1:100,000, 1980

Figure 1. Distribution of streamgages used to develop the regression model for index flows in the Upper Peninsula of Michigan.



Base map from U.S. Geological Survey, 1:100,000, 1980

Figure 2. Distribution of streamgages used in this analysis in the Lower Peninsula of Michigan.

precipitation. For index streamgages, the average length of record was 40.2 years, with a range of 11 to 91 years. Drainage areas among selected streamgages ranged from 1.1 mi² at South Branch Shepards Creek near Selkirk, Mich., (04141000) to 3,666 mi² at St. Joseph River at Niles, Mich. (04101500). The first year of record used in the regression analysis was 1901; 88 streamgages included data from water year 2005, which was the last water year used in the analysis.

The index regression model contains six explanatory variables including two soil properties associated with runoff potential, two measures of aquifer transmissivity, percent forest cover, and normal annual precipitation for the period 19712000. The regression estimate *Indx**rY**50* accounts for 70.8 percent of the variability in Gage***rY**50*. Based on Spearman's rank correlation coefficient squared r_s^2 (Conover, 1980), *Indx**Q**50* accounts for 94.0 percent of the variability in ***Q**50* (Hamilton and others, 2008). Owing to the nonlinear transformation (squaring) of *Indx**rY**50*, associated with the computation of flow, *Indx**Q**50* values are not strictly unbiased estimators of ***Q**50,* although the bias is thought to be small because the unexplained variability is small.

The *Indx**rY**50* estimate at site J^o, $Indx\boldsymbol{rY}50_{J^o}$, is associated with an estimated prediction variance $\hat{\sigma}^2_{Indx\boldsymbol{rY}50(J^o)}$. This variance is a function of the mean square error (*MSE*) of the index regression and the particular set of explanatory variables. The *MSE* represents the variance of the regression for a site where all the explanatory variables were equal to the mean of corresponding variables used to develop the regression equation. This variance increases, however, with the magnitude of differences between site values of the explanatory variables and the average values of those explanatory variables used to develop the regression equation (Draper and Smith, 1966).

In addition to predicting index water yields, the index regression model can be used to estimate the uncertainty of the response for any site. This uncertainty is sometimes expressed as a prediction interval that under repeated experiment is likely to contain the true index yield at a particular stream site with a specified probability. The 95-percent prediction interval is commonly used.

Hamilton and others (2008) defined seven hydrologic subregions in Michigan to check for potential bias in the spatial distribution of *Indx**rY**50* error (fig. 3). The seven subregions are composed of the same cataloging units shown on USGS hydrologic unit maps (Seaber and others, 1987), although the units in this report are grouped to form subregions somewhat differently than Seaber and others (1987) to conform to local understanding of hydrologic conditions. No spatial bias was detected in the distribution of *Indx**rY**50* errors (Hamilton and others, 2008). In this report, the seven subregions are referred to as Michigan hydrologic subregions (*MHS*).

Flow Data

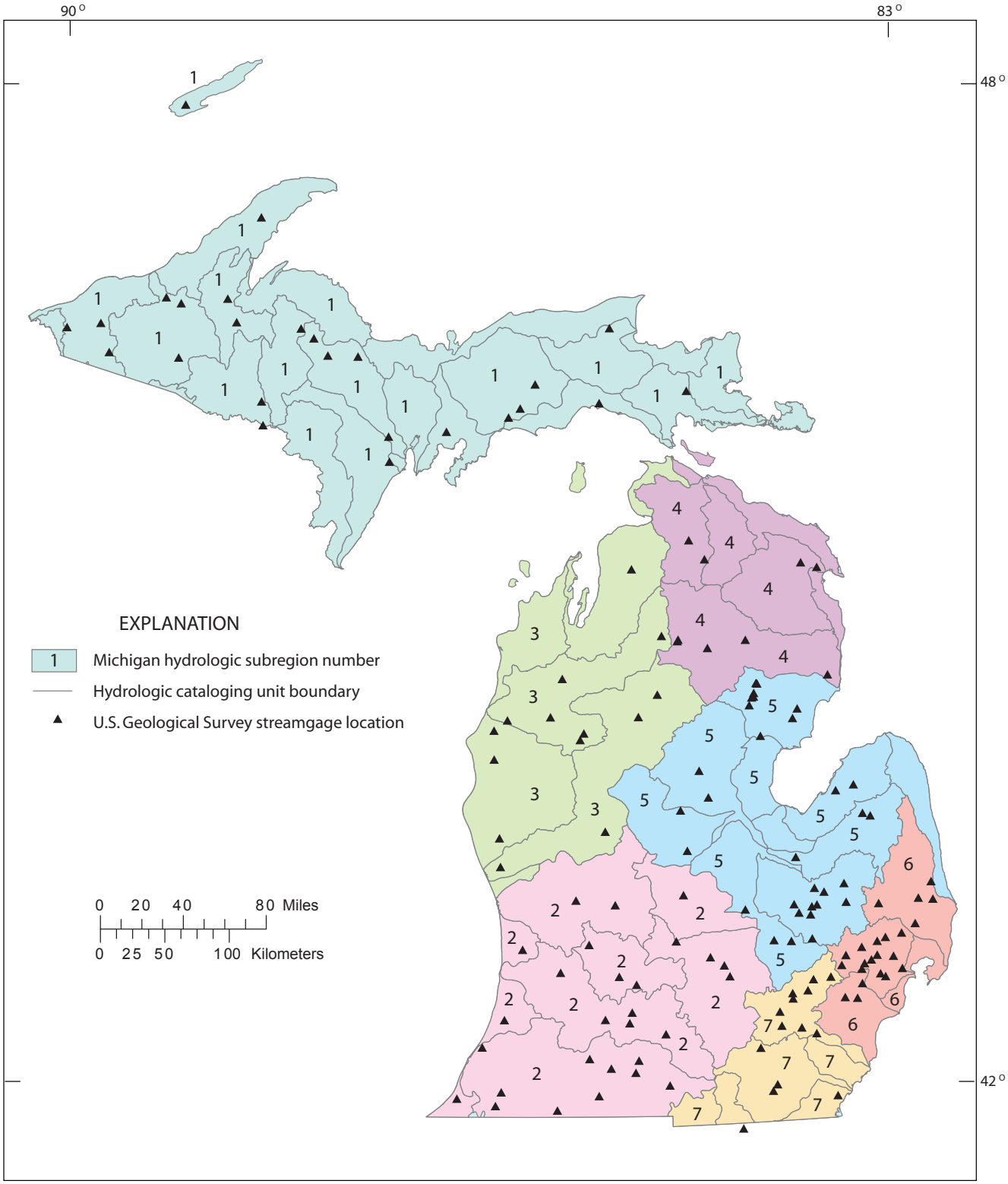
The USGS operates a network of streamgages that provides continuousflow and water-level data at hourly or smaller time intervals. Continuousflow data are computed from direct measurements of water levels, which are applied to a water-level-flow (stage-discharge) relation that is updated periodically by use of paired direct measurements of water level and flow (Rantz and others, 1982). Daily mean flows for streamgages are computed from hourly or smaller time-interval data. Direct measurements of flow also are obtained nonsystematically at miscellaneous stream sites. In this report, these measurements are referred to as instantaneous measurements, which describe streamflow conditions at the time of measurement, but are not commonly used to statistically characterize streamflow.

Continuous-Flow Data at Streamgages

Continuous-flow data used in this analysis were obtained from the same 147 streamgages (figs. 1 and 2) that were used to develop the index regression report (Hamilton and others, 2008). Daily mean flows from 1901 to 2005 were analyzed to compute the index flows *Gage**Q**50*, which were used again in this report (appendix 1, table 1–1). A subset of this daily flow data, plus information from water years 2006 to 2008, was used to form a sampling frame for the analysis described in this report. The sampling frame provided a basis for relating daily mean flows to index flows.

The sampling frame is a matrix of daily mean flows formed by 17,532 rows corresponding to consecutive days from October 1, 1960 to September 30, 2008 (the 48 water years from 19612008), and 147 columns corresponding to the 147 index streamgages. Of these index streamgages, a maximum of 126 streamgages were operated in 1970 and a minimum of 80 streamgages were operated in 1999 (fig. 4). An average of 98 index streamgages per year was operated during this period. Daily flows during periods when index streamgages were not operated are represented by a missing value, indicated within the matrix by the arithmetic representative for Not-a-Number (*NaN*). About 33.8 percent of the elements in the sampling frame were missing values.

The 48 years spanned by the sampling frame is considerably less than the 105 years of data used to compute index flows (fig. 5). Thus, the maximum period of record in the sampling frame is 48 years for the 47 index streamgages that were operated continuously from 1961 to 2008, compared with a maximum period of record of 91 years of record used to compute index flows. Also, the first year of daily mean flows represented by the sampling frame was 1961, which post-dated the last year of record used to compute index flows at five index streamgages. These index streamgages include (04056000) West Branch Manistique River near Manistique, Mich.; (04110000) Orchard Creek at Munith, Mich., (04145500) Bad River near Brant, Mich., (04157500)



Base map from U.S. Geological Survey, 1:100,000, 1980

Figure 3. Michigan hydrologic subregions used in the analysis of index flows.

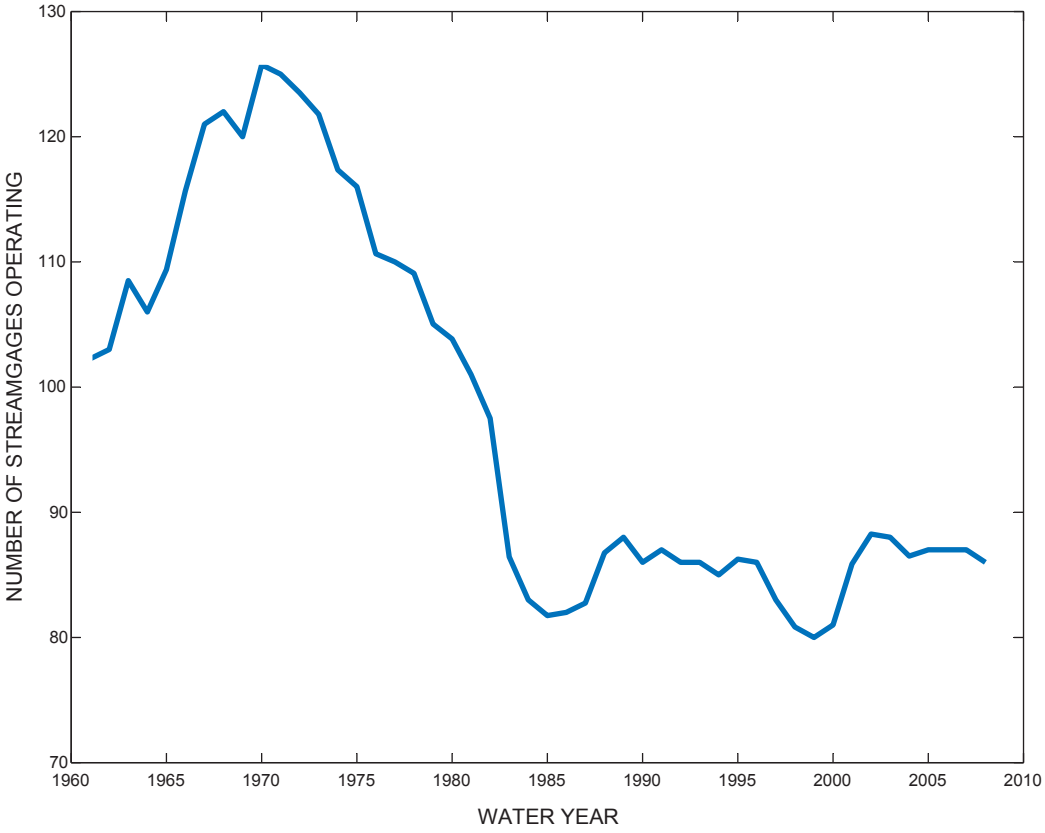


Figure 4. Number of streamgages operated during water years 19612008 from the 147 index streamgages in Michigan.

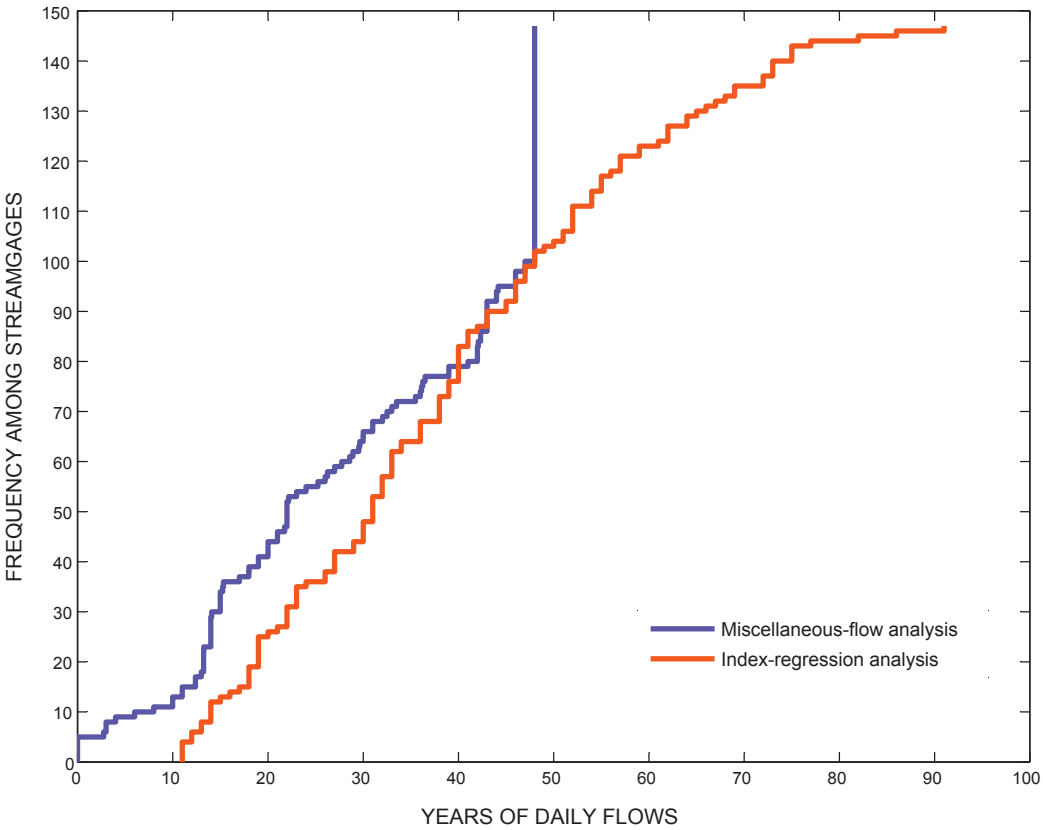


Figure 5. The empirical cumulative distribution function of the number of years of flow data used from the 147 index streamgages in Michigan.

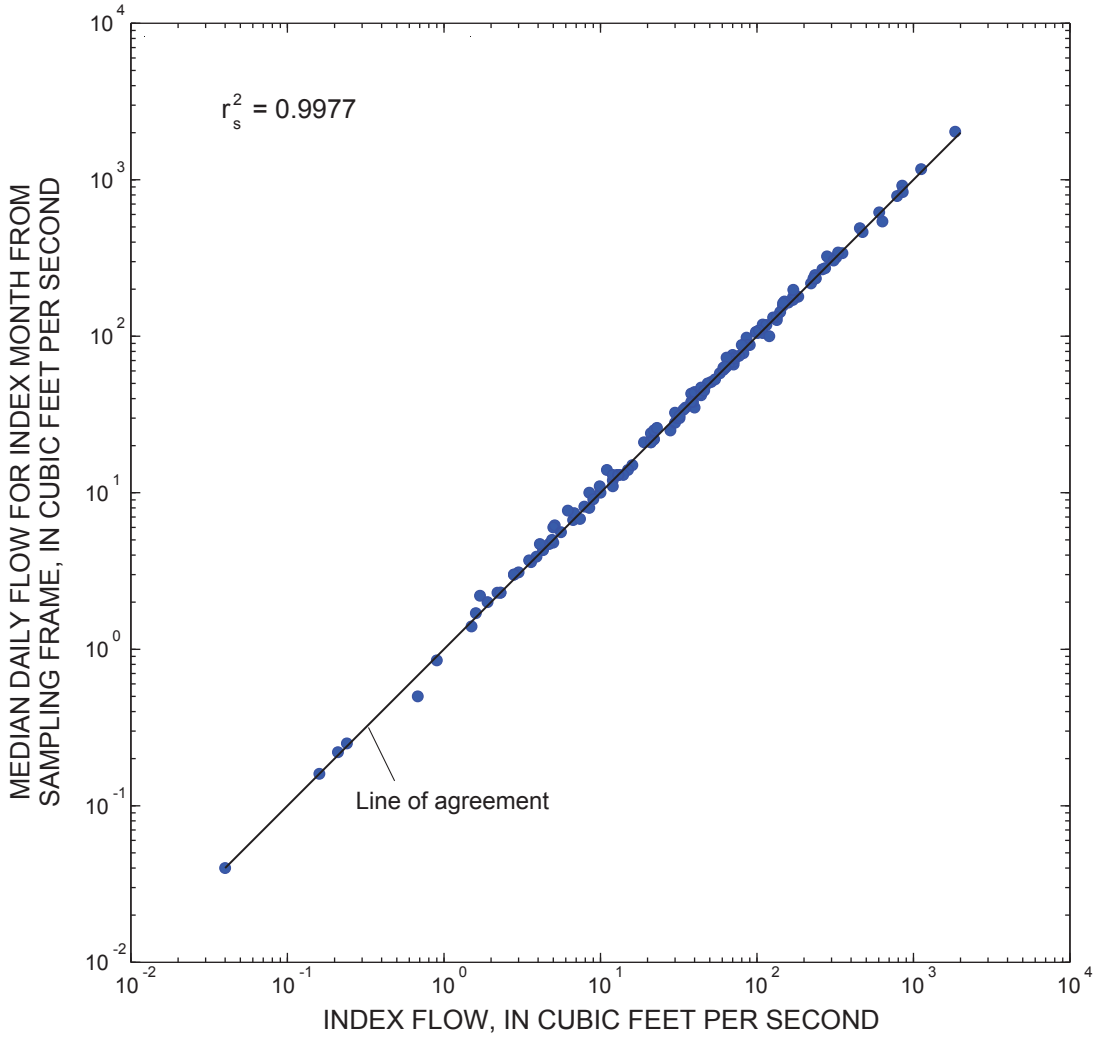


Figure 6. Relation between the index flow and the median daily flow in the sampling frame for the month of minimum flow. The sampling frame is a matrix of daily mean flows formed by 17,532 rows, corresponding to consecutive days from October 1, 1960 to September 30, 2008 (the 48 water years from 1961 to 2008), and 147 columns corresponding to the 147 index streamgages.

State Drain near Sebewaing, Mich., and (04160050) Black River near Port Huron, Mich. Daily values from these five streamgages were not included in the sampling frame, thus streamgages were effectively eliminated from this analysis.

For this analysis, it is not considered essential that the period of flow record represented by the sampling frame correspond exactly to the period of record used to compute index flows. This is because the index-flow statistics, which were computed on the full period-of-streamflow record used to develop the index regression report, are thought to be stationary, such that at least the means and variances of the series do not change with time. Thus, the expected values of index flows are assumed to be independent of the period of record used in their computation. In confirmation, the relation between index flows and the median flows computed by use of the daily values in the sampling frame for the month of minimum flow (appendix 1, table 1–1) were found to be in close agreement (fig. 6).

Data in the sampling frame were used to relate daily flow values to index flows. This relation is not expected to have changed during the period of streamflow record used by Hamilton and others (2008) nor that used in the compilation of this report. The sampling frame is thought to span a sufficient period of time to characterize the variability in the relation between daily mean flows (or instantaneous flows) and index flows.

Instantaneous Flow Measurements at Miscellaneous Stream Sites

In contrast to continuous-flow data at streamgages, instantaneous measurements provide a set of direct-flow measurements, generally obtained at irregular intervals. Instantaneous measurements are not sufficient to directly compute the index flow of a site. These data do, however, provide site-specific information

on current streamflow conditions. The utility of instantaneous flow measurements can be enhanced by comparison with daily mean flows at nearby active streamgages on the same day and their corresponding streamflow characteristics. In this report, miscellaneous sites are discussed as sites where existing or newly acquired flow data could be applied with the methods developed in this report to improve the local estimates of index flow from the regression-model estimate. For the purpose of method development, however, instantaneous flow measurements are approximated by use of daily average flow at index streamgages.

Direct measurements of flow commonly are obtained by subdividing a stream cross section into about 25 sections of approximately equal flow (Rantz and others, 1982). One or more point measurements of flow velocity along a vertical line in the center of each section are used along with section width and depth measurements to compute the flow within each section. Section flows are added to compute the total flow. Direct measurements of flow usually can be completed in about 1 hour.

Although instantaneous measurements quantify flow only during the measurement interval, they often are used to approximate daily mean flow. Compilations of instantaneous flow measurements in Michigan (Knutilla, 1974; Holtschlag and Eagle, 1985), for example, specify the date but not the time interval of the measurement. This approximation to daily mean flow is more reliable during base-flow conditions associated with index flows, than during periods of rapidly varying flow conditions commonly associated with storm events.

Holtschlag and Eagle (1985) document 17,607 flow measurements at 2,897 miscellaneous sites in Michigan. Since then, this dataset has expanded to 22,368 instantaneous flow measurements at 3,692 stream sites (fig. 7) (Marlio Lesmez, Michigan Department of Natural Resources and Environment, written commun., 2009). Thus, each miscellaneous site has an average of about six flow measurements, although the median is two and the mode is one measurement. A single measurement is available at 1,350 of the 3,692 miscellaneous sites; the maximum number of flow measurements available at a miscellaneous site is 213.

The instantaneous flow set includes measurements from 1931 to 2006 (fig. 8). During this period, the average number of measurements per year increased rapidly after 1945 and remained steady at about 321 measurements per year until about 1961. From 1962 to 1982, the average number of measurements per year increased to about 558 per year. From 1983 to 2006, the average number of measurements per year decreased to about 214. August has the highest frequency of instantaneous flow measurements (fig. 9), and it is the month where the median flow was lowest at 92 of the 147 streamgages used to define index flow (Hamilton and others, 2008). The lowest average instantaneous flow magnitudes are in August (fig. 10) and the highest average discrete flows are in April.

Improving Regression Estimates of Index Flow by use of Instantaneous Flow Measurements

Monte Carlo simulations were used to investigate the utility of information contained in instantaneous flow measurements for improving the accuracy of index flows estimated by the index regression equation. Repeated simulations were used to randomly select streamgages from the sampling frame, where the columns in the frame correspond to the set of streamgages used in the development of the index regression model. Rows in the sampling frame correspond to days within the period from October 1, 1960 to September 30, 2008. Where measured, elements in the frame are daily values (**Q***dv*) of mean flow.

Once a streamgage was selected for estimation as a target site, 1 or more days, depending on the measurement intensity being investigated, were randomly selected from the sampling frame during July, August, or September within the period of included record for that streamgage. In this report, measurement intensities represent the number of simulated instantaneous measurements used in the estimation process. The set of 12 measurement intensities was {1, 2,…, 7, 10, 15, 20, 50, 100}. Selection also was restricted to those days when the daily mean flow represented low- and medium-flow conditions. The selected daily mean flow at the target site was treated as a simulated instantaneous flow measurement.

For each initially selected day, other active index streamgages were identified in the same Michigan hydrologic subregion as the target site. Streamgages where daily mean flows represented low- and medium-flow conditions were chosen as possible auxiliary streamgages. If the set included five or more streamgages, the auxiliary streamgages were used to develop a simple linear regression (*SLR*) relation between the square ***r***oot of water ***y***ields from *d*aily mean flow *v*alues (***rY****dv)* and index water yields ***rY****50* for the selected day. Otherwise, a different day was randomly selected and re-evaluated so that a minimum of five auxiliary streamgages were available for relating daily mean flow to index flow. Index water yields at the target site were then estimated on the basis of this relation using the simulated instantaneous measurement as the explanatory variable in the simple linear-regression equation.

In this report, index water yields estimated at target sites on the basis of these simple linear-regression equations and simulated (discrete) instantaneous measurements are the basis of the Discrete Estimator, which is symbolized as *Disc****rY****50* for root index water yields and *Disc****Q****50* for index flows. The uncertainty of a discrete estimate of water yield is the prediction variance of the linear regression relating daily yields to index yields at the simulated instantaneous measurement. For measurement intensities greater than one, a weighted average of *Disc****rY****50* estimates is computed. Weights are inversely

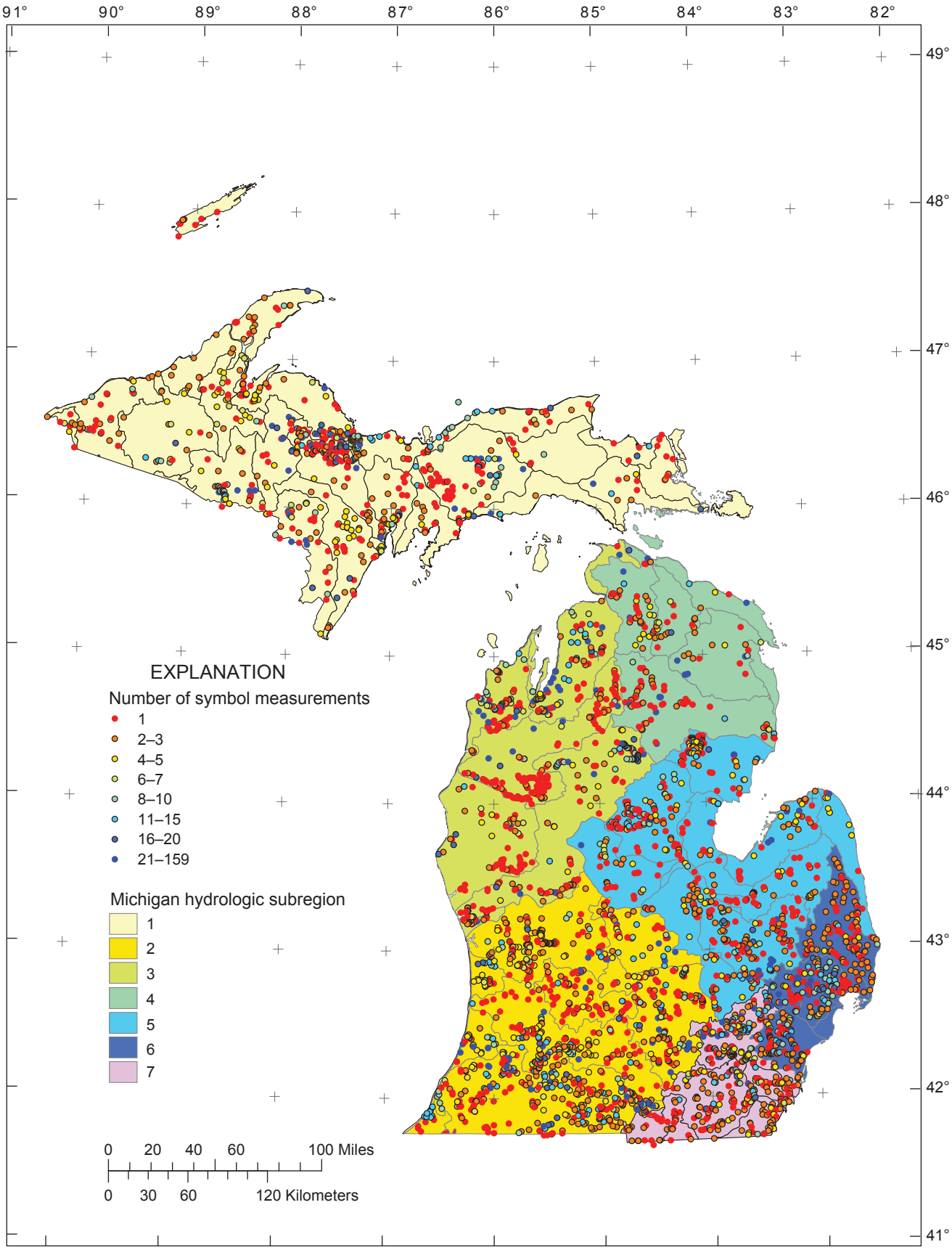


Figure 7. Distribution of sites with instantaneous streamflow measurements in Michigan.

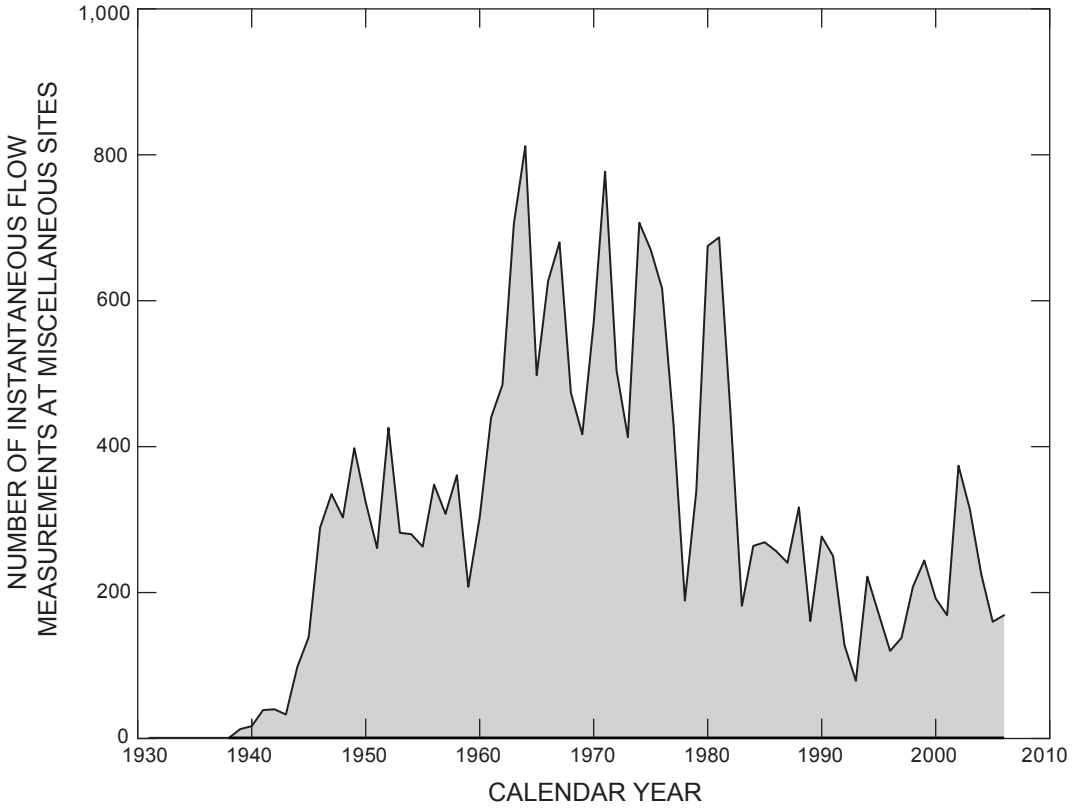


Figure 8. Number of streamflow measurements per year at miscellaneous sites, 1931 to 2006, in Michigan.

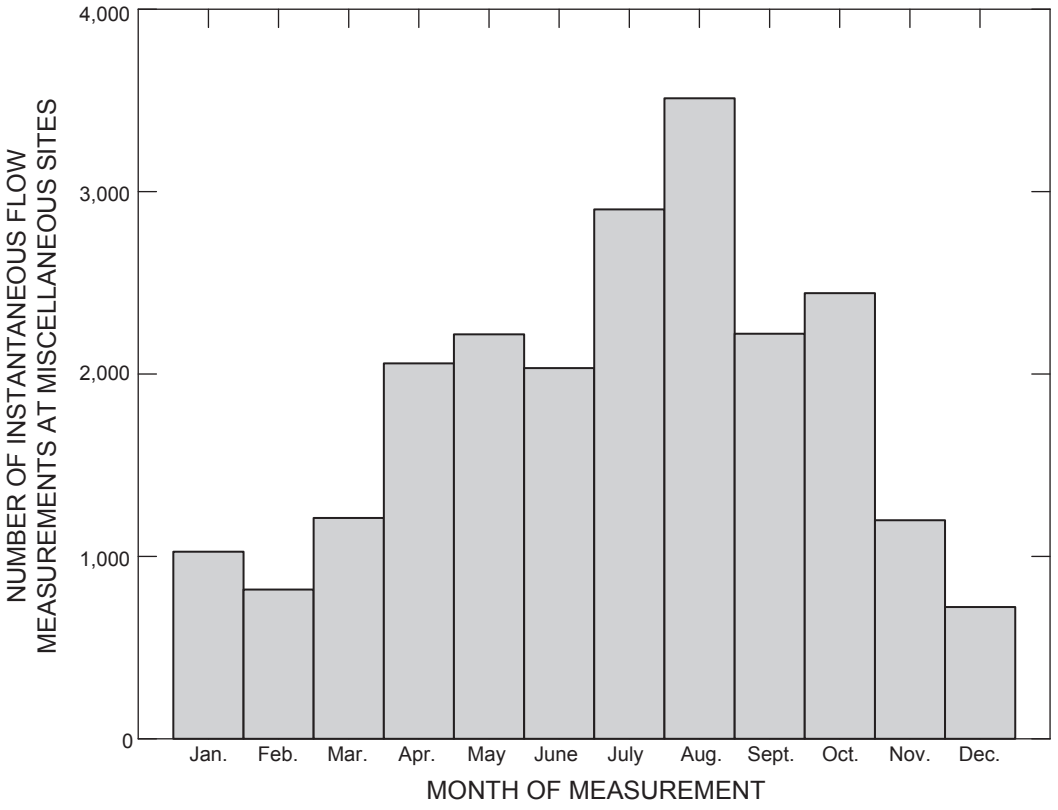


Figure 9. Distribution of instantaneous streamflow measurements, per month, in Michigan.

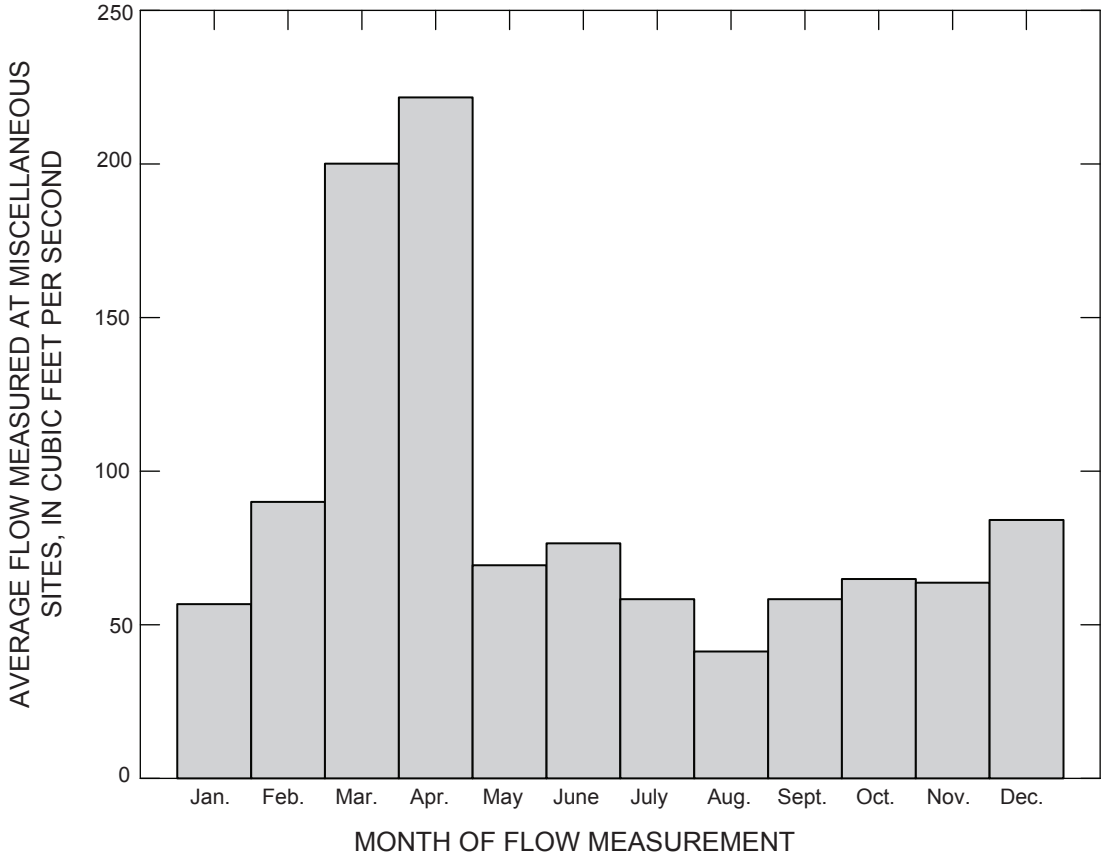


Figure 10. Average measured streamflow, by month, at miscellaneous sites in Michigan.

proportional to each estimate's corresponding prediction variances from the linear regression between daily mean ***rY****dv* and index water yields *Gage**rY**50*. Estimates are assumed to be independent; an assumption that is increasingly plausible as the time between instantaneous measurements increases. A weighted average of the *Disc**rY**50* estimate and the index regression estimate *Indx**rY**50* forms the Integrated Estimator of index water yields symbolized as *Intg**rY**50* and index flows as *Intg**Q**50*.

Like *Indx**rY**50*, both *Disc**rY**50* and *Intg**rY**50*, are generalized estimators, which refers to an estimator that is applicable to all sites that can be approximated by a regional norm. Generalized estimates, however, are not expected to converge[1] to site-specific characteristics with increasing measurement intensity. Convergence, however, is desirable where site-specific estimates of index water yield and flow are needed, and where measurement intensity can be varied to help achieve this objective. The third estimator, referred to as the Site Estimator, combines the Integrated Estimate *Intg**rY**50* with estimates of median water yield from simulated instantaneous measurements to estimate index water yield. Site Estimates of index water yield are symbolized as *Site**rY**50* and index flow as *Site**Q**50*. Following is a more formal discussion of the Monte Carlo methods used to test estimators *Disc**rY**50*, *Intg**rY**50*, and *Site**rY**50*.

[1] Converge (and convergence) is used in this report to indicate the tendency for a sample statistic to approach the corresponding population statistic with increasing sample size.

Development of the Discrete, Integrated, and Site Estimators using Monte Carlo Methods

Monte Carlo methods were used to assess the likely utility of instantaneous flow measurements for improving *Indx**Q**50* estimates of index flow ***Q**50*. Monte Carlo methods are a broadly defined set of numerical techniques that use repeated random sampling to characterize complex phenomena (Sobol´, 1994), four elements are common in Monte Carlo methods. First, a sampling frame is defined as a set of possible inputs. Second, random samples are repeatedly drawn from the sampling frame. Third, the results of the sample are used to compute an individual result. Fourth, individual results are aggregated to characterize the phenomena. These four elements are discussed in detail, with respect to the analysis described, in the following section.

1. As previously described, the sampling frame used in this analysis was based on a table of daily mean flow values $\boldsymbol{Q}dv_{t,j}$, with *t* corresponding to a day within the 48-year period Oct. 1, 1960–Sept. 30, 2008 (water years 1961–2008), and *j* corresponding to an index streamgage from 1 to 14

   a. Only 142 of the 147 index streamgages used in the development of the index flow regression model could be used in the Monte Carlo analysis, because the first day of flow values in the sampling frame postdated the last day of daily values at five index streamgages. Of these five streamgages, one streamgage was omitted in each of *MHS* 1, 2, and 6, and two streamgages were omitted from *MHS* 5 (fig. 3).

   b. Daily mean flows, $\boldsymbol{Q}dv_{t,j}$, in cubic feet per second, were converted to daily mean water yields $\boldsymbol{Y}dv_{t,j}$, in cubic feet per second per square mile, by dividing the flows by the drainage areas, $DArea_j$, in square miles, at the corresponding index streamgage. For consistency with assumptions underlying linear regression models, a square root transformation was applied to the daily water yields to form a matrix of square roots of daily water yields indicated as $\boldsymbol{rY}dv_{t,j}$.

2. A series of 84 random-sampling experiments were conducted as part of the Monte Carlo analysis. Each experiment was conducted over 1 of the 7 Michigan hydrologic subregions, and 1 of 12 levels of measurement intensity. The number of random trials for each measurement intensity was specified as the number of index streamgages within the subregion times 10,00 Thus, on the average, there were 10,000 trials for each streamgage, although this number varied slightly because of random sampling. Within each hydrologic subregion, measurement-intensity levels correspond to the number of instantaneous measurements simulated by using ***rY**dv* as instantaneous measurements. Each simulated instantaneous measurement generated a set of paired ***rY**dv* and ***rY**50* values from data at the auxiliary sites. These data were used to estimate the slope and intercept parameters for the *SLR* equation. The equation was used to estimate index water yield at the target site given the simulated instantaneous measurement of water yield at the target site. Measurement-intensity levels $I_q = \{i_{q_1}, i_{q_2}, \cdots, i_{q_{12}}\}$ were {1, 2, …, 7, 10, 15, 20, 50, 100}, corresponding to 1 to 100 simulated instantaneous measurements. For example, the number of simulated instantaneous measurements corresponding to $i_{q_{12}}$ is 100.

   a. Each sampling experiment consisted of an average of 10,000 trials per streamgage. In each trial, one of the index streamgages was randomly selected with replacement[2] from among the index streamgages in the target Michigan hydrologic subregion (*MHS*). This selected streamgage would be temporarily treated as the target (miscellaneous) site where the index water yield ***rY**50* (and flow ***Q**50*) could not be estimated directly from continuous-records. The remaining streamgages in the subregion were potential auxiliary streamgages for use in estimation. Selection of auxiliary streamgages was limited geographically to the same subregion as the target site so that auxiliary streamgages would likely have similar weather and flow conditions.

   b. Once the target site J^o was selected, a set of one or more days, depending on the measurement intensity i_q, was randomly selected with replacement from all days at J^o meeting the water yield selection criterion. This criterion specified that $\boldsymbol{rY}dv_{t,J^o} \le Gage\boldsymbol{rY}50_{J^o} + 1.96 \cdot \hat{\sigma}_{Indx\boldsymbol{rY}50_{J^o}}$. Otherwise, a different day was randomly selected. Such a one-side confidence interval would be expected to contain ***rY**50* for the site with 95-percent probability. Note that the confidence interval is based on *Gage**rY**50*, which is a more precise estimator of ***rY**50* than *Indx**rY**5* As an unbiased estimator of ***rY**50*, *Indx**rY**50* could be substituted for sites where flow records are unavailable, although the index estimate is sensitive to this criterion.

[2] Sampling with replacement is a sampling technique in which a randomly selected unit is returned to the sampled population for possible reselection in subsequent random samples.

Discrete Estimator

3. Once a day was selected initially; all other operating index streamgages in the subregion were potential auxiliary streamgages. A set of auxiliary streamgages $\{J'\}$ was identified, meeting the same daily water-yield criterion at their respective locations as the target site. The minimum number of auxiliary streamgages for use in estimation was five $(\#\{J'\} \geq 5)$. Otherwise, an alternative day was randomly selected and evaluated. Additional days were selected for the target site to form a set where the number of elements in the set corresponded to the measurement intensity being evaluated.

   a. Using daily values of water yield measured at auxiliary streamgages, a set of simple linear regression $\{SLR\}$ equations was developed as

   $$Gage\boldsymbol{rY}50_{J'} = b_o + b_1 \cdot \boldsymbol{rY}dv_{t,J'} + \varepsilon_{t,J'} \quad (1)$$

   where

   $Gage\boldsymbol{rY}50_{J'}$ is a vector of water yield estimates of $\boldsymbol{rY}50$ based on flow records at streamgages used as auxiliary sites J' on a day indexed by t,

   $\boldsymbol{rY}dv_{t,J'}$ is the corresponding vector of the square roots of daily mean water yields at the auxiliary sites J' on day t,

   b_o, b_1 are the intercept and slope parameters, respectively,of a simple linear regression estimated by ordinary least squares to relate daily yields to index yields, and

   $\varepsilon_{t,J'}$ is a vector of regression errors (residuals) of length $\#\{J'\}$ for time t.

   b. This regression equation then was used to estimate the index water yield at target site J^o for time t as

   $$Disc\boldsymbol{rY}50_{t,J^o} = b_0 + b_1 \cdot \boldsymbol{rY}dv_{t,J^o} \quad (2)$$

   c. In future applications, an actual instantaneous measurement indicating water yield $\boldsymbol{rY}mm_{t,J^o}$ might be used to approximate $\boldsymbol{rY}dv_{t,J^o}$. These measurements, subject to the same selection criterion as the simulated measurements, would provide the explanatory variable needed to estimate $Disc\boldsymbol{rY}50_{t,J^o}$, using an equation of the form [2].

   d. $Disc\boldsymbol{rY}50_{t,J^o}$ estimates from all sample days t for the measurement intensity i_q being investigated were weighted inversely proportionally to their corresponding prediction variances and averaged to estimate $Disc\boldsymbol{rY}50_{i_q,J^o}$ at the target site (Taylor, 1982). The estimated prediction variance of the regression estimate of water yield at the target site for the selected day was computed as:

   $$\begin{aligned}\hat{\sigma}^2_{p(t,J^o)} &\equiv \widehat{Var}\left(\boldsymbol{rY}50_{t,J^o} - Disc\boldsymbol{rY}50_{t,J^o}\right) \quad (3)\\ &= \hat{\sigma}^2_{SLR(t,J')}\left[1 + \frac{1}{\#\{J'\}} + \frac{\left(\boldsymbol{rY}dv_{t,J^o} - \overline{\boldsymbol{rY}dv}_{t,J'}\right)^2}{\sum_{\forall j \in J'}\left(\boldsymbol{rY}dv_{t,j} - \overline{\boldsymbol{rY}dv}_{t,j}\right)^2}\right] + \cdots\\ &\quad + \widehat{Var}\left(\boldsymbol{rY}50_{t,J^o} - \boldsymbol{rY}mm_{t,J^o}\right)\end{aligned}$$

   where

   $\hat{\sigma}^2_{SLR(t,J')}$ is the mean square error of the linear regression for the simulated measurement, which is estimated as $\frac{1}{\#\{J'\}-2}\sum_{\forall j \in J'}\left(\boldsymbol{rY}50_{t,j} - Disc\boldsymbol{rY}50_{t,j}\right)^2$, and

   $\#\{J'\}$ is the number of axilary streamgages in J', and

   $\widehat{Var}\left(\boldsymbol{rY}50_{t,J^o} - \boldsymbol{rY}mm_{t,J^o}\right)$ is the estimated variance of the ***m***iscellaneous (instantaneous) ***m***easurement $\boldsymbol{rY}mm$ as an estimator of the daily mean $\boldsymbol{rY}dv$. For these simulations, the uncertainty was estimated by use of a one percent coefficient of variation of the gaged daily mean yield, which results in a variance of $\left(0.01 \cdot Gage\boldsymbol{rY}dv_{t,J^o}\right)^2$. In future applications, the variance might be based on $\boldsymbol{rY}mm_{t,J^o}$.

   e. Once all selected days for the specified measurement intensity level were evaluated, a weighted average index water yield estimate was computed as (Taylor, 1982)

   $$Disc\boldsymbol{rY}50_{i_q,J^o} = \sum_{\forall t \in \{i_q\}} \frac{Disc\boldsymbol{rY}50_{t,J^o}}{\hat{\sigma}^2_{p(t,J^o)}} \Bigg/ \sum_{\forall t \in \{i_q\}} \frac{1}{\hat{\sigma}^2_{p(t,J^o)}} \quad (4)$$

   f. The estimate of the index flow based only on a set i_q of simulated discrete instantaneous measurements at target site J^o and the relation between daily mean and index water yields at the auxiliary streamgages is:

   $$Disc\boldsymbol{Q}50_{i_q,J^o} = \left(Disc\boldsymbol{rY}50_{i_q,J^o}\right)^2 \cdot DArea_{J^o}. \quad (5)$$

   g. For a measurement intensity of one, the variance of the weighted average water yield estimate within each simulated measurement-day set of length $\#\{t \in i_q\}$ was computed as (Taylor, 1982)

   $$\hat{\sigma}^2_{w_1(i_q,J^o)} = 1 \Bigg/ \sum_{\forall t \in \{i_q\}} \frac{1}{\hat{\sigma}^2_{p(t,J^o)}} \quad (6)$$

h. For measurement intensities greater than one, this variance was adjusted to

$$\hat{\sigma}^2_{w_1(i_q,J^o)} = 1 \Big/ \sum_{\forall t \in \{i_q\}} \frac{1}{\hat{\sigma}^2_{p(t,J^o)}} \cdot \max\left(1, \chi^2_v\right) \qquad (7)$$

where,

χ^2_v is a correction for possible underdispersion of the variance of the weighted mean, and is computed as

$$\frac{1}{\left(\#\{t \in i_q\} - 1\right)} \sum_{\forall t \in \{i_q\}} \frac{\left(Discr\boldsymbol{Y}50_{t,J^o} - Discr\boldsymbol{Y}50_{i_q,J^o}\right)^2}{\hat{\sigma}^2_{p(t,J^o)}}.$$

i. When measurement intensities greater than one were evaluated, a between-measurement variance was included as

$$\hat{\sigma}^2_{b_1(i_q,J^o)} = \frac{1}{1 - v_{i_q,J^o}} \sum_{\forall t \in \{i_q\}} u_{i_q,J^o} \left(Discr\boldsymbol{Y}50_{t,J^o} - Discr\boldsymbol{Y}50_{i_q,J^o}\right)^2 \qquad (8)$$

where,

u_{i_q,J^o} is a vector of standardized weights computed as

$$\frac{1}{\hat{\sigma}^2_{p(t,J^o)}} \Big/ \sum_{\forall t \in \{i_q\}} \frac{1}{\hat{\sigma}^2_{p(t,J^o)}}$$

v_{i_q,J^o} is the sum of squared standardized weights $\sum_{\forall t \in \{i_q\}} u_{i_q,J^o}$

j. The estimated variance of the *Discr**Y**50* for measurement intensity i_q at site J^o is

$$\hat{\sigma}^2_{Discr\boldsymbol{Y}50(i_q,J^o)} = \hat{\sigma}^2_{w_1(i_q,J^o)} + \hat{\sigma}^2_{b_1(i_q,J^o)} \qquad (9)$$

k. A 95-percent confidence interval was developed for each estimate of index water yield based on the set of simulated instantaneous measurements as

$$ciDiscr\boldsymbol{Y}50_{i_q,J^o} = Discr\boldsymbol{Y}50_{i_q,J^o} \pm t_{0.025,nReg+nMeasQ-1} \sqrt{\hat{\sigma}^2_{Discr\boldsymbol{Y}50(i_q,J^o)}} \qquad (10)$$

where,

$t_{0.975,nReg+nMea}$ is the critical value of the t distribution for a two-sided test at the 95-percent level of probability with $nReg+nMea-1$ degrees of freedom, where *nReg* is the minimum number of streamgages available in the regression relating index to daily water yields, and *nMeasQ* is the measurement intensity being evaluated. This random interval contains *Gager**Y**50* about 95 percent of the time.

Integrated Estimator

l. An integrated estimate of the index water yield was computed by combining information from both the index regression equation and the simulated instantaneous measurements as

$$Intgr\boldsymbol{Y}50_{i_q,J^o} = \frac{Indxr\boldsymbol{Y}50_{J^o} \Big/ \hat{\sigma}^2_{Indxr\boldsymbol{Y}50(J^o)} + Discr\boldsymbol{Y}50_{i_q,J^o} \Big/ \hat{\sigma}^2_{Discr\boldsymbol{Y}50(i_q,J^o)}}{1 \Big/ \hat{\sigma}^2_{Indxr\boldsymbol{Y}50(J^o)} + 1 \Big/ \hat{\sigma}^2_{Discr\boldsymbol{Y}50(i_q,J^o)}}, \qquad (11)$$

where

$Intgr\boldsymbol{Y}50_{i_q,J^o}$ is the integrated estimate of water yield based on $\#\{t \in i_q\}$

$Indxr\boldsymbol{Y}50_{J^o}$ is the index regression estimate (Hamilton and others, 2008) of water yield at streamgage J^o, and

$Discr\boldsymbol{Y}50_{i_q,J^o}$ is the discrete estimate of water yield based on all available miscellaneous measurements at site J^o,

$\hat{\sigma}^2_{Indxr\boldsymbol{Y}50(J^o)}$ is the estimated prediction variance of the index regression estimate of water yield at streamgage J^o, and

$\hat{\sigma}^2_{Discr\boldsymbol{Y}50(i_q,J^o)}$ is the estimated prediction variance of the discrete estimator with i_q instantaneous measurements at site J^o.

m. The Integrated estimate of the index flow based on combining information from the Index Regression Estimator *Indxr**Y**50* with the Discrete Estimator *Discr**Y**50* and drainage area at the target site is

$$Intg\boldsymbol{Q}50_{i_q,J^o} = \left(Intgr\boldsymbol{Y}50_{i_q,J^o}\right)^2 \cdot DArea_{J^o} \qquad (12)$$

n. The within estimator variability of the integrated estimate was estimated as

$$\hat{\sigma}^2_{w_2(i_q,J^o)} = 1 \Bigg/ \left(\frac{1}{\hat{\sigma}^2_{Indxr\boldsymbol{Y}50(J^o)}} + \frac{1}{\sigma^2_{w_2(i_q,J^o)}} \right) \left\{ \left(Indxr\boldsymbol{Y}50_{J^o} - \frac{Indxr\boldsymbol{Y}50_{J^o} + Intgr\boldsymbol{Y}50_{i_q,J^o}}{2} \right)^2 + \left(Intgr\boldsymbol{Y}50_{i_q,J^o} - \frac{Indxr\boldsymbol{Y}50_{J^o} + Intgr\boldsymbol{Y}50_{i_q,J^o}}{2} \right)^2 \right\}. \qquad (13)$$

o. Similarly, the between estimator variability of integrated estimate was computed as

$$\hat{\sigma}^2_{b_2(i_q,J^o)} = \frac{1}{1-v}\left[u_1\left(Indx\boldsymbol{rY}50_{J^o} - Intg\boldsymbol{rY}50_{i_q,J^o}\right)^2 + \ldots \right. \quad (14)$$
$$\left. u_2\left(Disc\boldsymbol{rY}50_{i_q,J^o} - Intg\boldsymbol{rY}50_{i_q,J^o}\right)^2\right],$$

where,

u is a 2-vector of standardized weights computed as:

$$\left[\frac{1}{\hat{\sigma}^2_{Indx\boldsymbol{rY}50(J^o)}}, \frac{1}{\sigma^2_{w_1(i_q,J^o)}}\right] \Big/ \left[\frac{1}{\hat{\sigma}^2_{Indx\boldsymbol{rY}50(J^o)}} + \frac{1}{\sigma^2_{w_1(i_q,J^o)}}\right],$$

$\hat{\sigma}^2_{Indx\boldsymbol{rY}50(J^o)}$ is the prediction variance of the index regression at site J^o,

$\sigma^2_{w_1(i_q,J^o)}$ is the variance defined in equation 6, and

v is the sum of squared weights $u_1^2 + u_2^2$.

p. The estimated variance of *Intg**rY**50* at measurement intensity i_q at site J^o is

$$\hat{\sigma}^2_{Intg\boldsymbol{rY}50(i_q,J^o)} = \hat{\sigma}^2_{w_2(i_q,J^o)} + \hat{\sigma}^2_{b_2(i_q,J^o)} \quad (15)$$

q. A 95-percent confidence interval was developed for each integrated estimate of index water yield based on the index regression estimate and the set of simulated instantaneous measurements as

$$ciIntg\boldsymbol{rY}50_{i_q,J^o} = Intg\boldsymbol{rY}50_{i_q,J^o} \pm t_{0.025,nReg+nMeasQ-1}\sqrt{\hat{\sigma}^2_{Intg\boldsymbol{rY}50(i_q,J^o)}}, \quad (16)$$

where,

$Intg\boldsymbol{rY}50_{i_q,J^o}$ is the Integrated estimate of ***rY***50 for the measurement intensity i_q at site J^o,

$\hat{\sigma}^2_{Intg\boldsymbol{rY}50(i_q,J^o)}$ is the prediction variance associated with the Integrated estimate of ***rY***50 for measurement intensity i_q at site J^o,

$t_{0.025,nReg+nMeasQ-1}$ is the critical value of the t distribution for a two-sided 95-percent level of probability with $nReg+nMeasQ-1$ degrees of freedoms where *nReg* is the minimum number of streamgages available in the regression relating index to daily water yields, and *nMeasQ* is the measurement intensity being evaluated. This random interval is expected to contain the *Gager**Y**50* about 95 percent of the time.

Site Estimator

r. Both the *Indx**rY**50* and *Disc**rY**50* estimators that form the *Intg**rY**50* estimator likely would improve as generalized estimators of index water yield with increasing numbers of streamgages in the index regression estimator and increasing numbers of instantaneous flow measurements in the discrete estimator. Thus, over the population of stream sites that are applicable, both index streamgages and instantaneous flow measurements are helpful. Increases in streamgages or instantaneous measurements, however, would not be likely to result in site-specific estimates of water yield that converge to water yield values computed from long-term streamflow records at every site. Convergence is not expected because site-specific conditions result in unique index water yield and flow conditions.

s. To better ensure convergence to site-specific conditions, a third estimator of index water yields *Site**rY**50* and flows *Site**Q**50* was developed and evaluated. This estimator weighs estimates of *Intg**rY**50* estimates with estimates of the median water yield computed from three or more simulated instantaneous flow measurements obtained during July, August, or September. Again, simulated instantaneous flow measurements ***rY**dv* that were associated with water yields that were less than $\boldsymbol{rY}dv_{t,J^o} \le Gage\boldsymbol{rY}50_{J^o} + 1.96 \cdot \hat{\sigma}^2_{Indx\boldsymbol{rY}50(J^o)}$ were included; otherwise, a different day t was selected. This estimator requires an estimate of the variability of the median water yield as a function of the number of instantaneous measurements. The assumption that index water yields and flows are equally likely to occur in July, August, and September, and that median flows are similar to the minimum of median flows during these months is implicit in the simulation procedure described in this report. This procedure readily could be modified, however, to target the month more likely to contain the minimum flow.

t. An estimate of the variance of the median flow $\sigma^2_{\tilde{x}}(n)$ from a small set of n instantaneous flow measurements is needed to implement the Site Estimator. Rider (1960) reported that variances of the median for samples of size 1, 3, 5, and 7 from a standard normal distribution (with mean zero and variance 1) are 1, 0.4487, 0.2868, and 0.2104, respectively. Rider (1960) did not compute variances for other odd-numbered sample sizes or for any even-numbered sample sizes. Medians of even numbered samples are not strictly defined, although they are conventionally computed as the average of the two middle values in an ordered list. In this report, Rider's (1960) estimates of variances of medians for selected sample sizes were confirmed and extended to include all investigated measurement intensities by use of resampling techniques involving

1 million trials for each measurement intensity (table 1). By extension, the variances of sample medians from normal populations is:

$$\hat{\sigma}^2_{\tilde{x}}(n) = \hat{\sigma}^2_{\tilde{x}}(n) \cdot \hat{\sigma}^2_{\tilde{x} \in N(0,1)}, \tag{17}$$

where,

$\hat{\sigma}^2_{\tilde{x}}(n)$ is the sample estimate of the variance of the median based on n measurements,

$\hat{\sigma}^2_{\tilde{x}}(n)$ is the sample estimate of the variance of the mean, and

$\hat{\sigma}^2_{\tilde{x} \in N(0,1)}$ is the sample estimate of the variance of the median of a standard normal variate (table 1).

$$Site\boldsymbol{rY}50_{i_q,J^o} = \frac{\dfrac{Intg\boldsymbol{rY}50_{i_q,J^o}}{\hat{\sigma}^2_{Intg\boldsymbol{rY}50(i_q,J^o)}} + \dfrac{Med\left(\boldsymbol{rY}dv_{i_q,J^o}\right)}{\hat{\sigma}^2_{Med\boldsymbol{rY}dv\left(i_q,Mhs(J^o)\right)}}}{1\Big/\hat{\sigma}^2_{Intg\boldsymbol{rY}50(i_q,J^o)} + 1\Big/\hat{\sigma}^2_{Med\boldsymbol{rY}dv\left(i_q,Mhs(J^o)\right)}}, \tag{18}$$

where

$Site\boldsymbol{rY}50_{i_q,J^o}$ is the estimator of square root of water yield based on $Intg\boldsymbol{rY}50_{i_q,J^o}$ and the median Med of water yields computed from a corresponding set of miscellaneous measurements and drainage area at the target site,

$\hat{\sigma}^2_{Intg\boldsymbol{rY}50(i_q,J^o)}$ is the estimated variance of the Integrated estimate,

$\hat{\sigma}^2_{Med\boldsymbol{rY}dv\left(i_q,Mhs(J^o)\right)}$ is the estmated variance of the median water yield in Michigan hydrologic subregion Mhs containing J^o. This variance was computed as the average variance for all index streamgages (table 1–2) in the corresponding Mhs and is listed in table 1–3 for each measurement intensity.

u. Estimated variances of the median $\boldsymbol{rY}dv$ values $\hat{\sigma}^2_{Med\boldsymbol{rY}dv\left(i_q,Mhs(J^o)\right)}$ for each of the 142 index streamgages j and each of the 12 measurement intensities investigated in this report were estimated through a set of resampling experiments. Each experiment included 500,000 trials in which median values were computed from random samples of $\boldsymbol{rY}dv$ of size i_q drawn with replacement from the sampling frame during the months of July, August, and September and low- and medium-flow conditions.

Table 1. Variances of the median from sampled measurement intensities for a standard normal probability distribution.

Sample size	Variance of the median
1	1.000
2	.5006
3	.4491
4	.2985
5	.2866
6	.2145
7	.2108
10	.1683
15	.1665
20	.1382
50	.1374
100	.1175

v. Estimated standard deviations of median water yields are listed as a function of measurement intensity $\hat{\sigma}^2_{\hat{\sigma}^2_{Med\boldsymbol{rY}dv\left(i_q,Mhs(J^o)\right)}}$ by streamgage in appendix 1, table 1–2, and by Michigan hydrologic subregion in appendix 1, table 1–3.

w. The estimated variance of the $Site\boldsymbol{rY}50$ estimator for measurement intensity i_q at site J^o is:

$$\hat{\sigma}^2_{Site\boldsymbol{rY}50(i_q,J^o)} = \frac{1}{1\Big/\hat{\sigma}^2_{Med\boldsymbol{rY}dv\left(i_q,Mhs(J^o)\right)} + 1\Big/\hat{\sigma}^2_{Intg\boldsymbol{rY}50(i_q,J^o)}} \tag{19}$$

x. An approximate 95-percent confidence interval is estimated as

$$ciSite\boldsymbol{rY}50_{i_q,J^o} = Site\boldsymbol{rY}50_{i_q,J^o} \pm t_{0.025,nReg-1} \cdot \hat{\sigma}_{Site\boldsymbol{rY}50(i_q,J^o)} \tag{20}$$

where,

$t_{0.025,nReg-1}$ is the quantile from the t distribution with $nReg-1$ degrees of freedom, where $nReg$ is the minimum number of streamgages available in the regressions relating index to daily water yields. The random interval that is expected to contain $\boldsymbol{rY}50$ at site J^o about 95 percent of the time.

4. Estimator accuracies were determined from Monte Carlo simulations and compared on the basis of the root mean square error (*RMSE*) criterion. The *RMSE* is computed for each of the four estimators in the ordered set $\widehat{\boldsymbol{rY}50} = \{Indx\boldsymbol{rY}50, Disc\boldsymbol{rY}50, Intg\boldsymbol{rY}50, Site\boldsymbol{rY}50\}$. The RMSE is the square root of the sum of the mean error squared plus the variance of the errors, which can be written as

$$RMSE_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right)} = \sqrt{\left(\frac{\sum_{\tau} e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\tau}}{\#\left\{e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\cdot}\right\}}\right)^2 + \frac{e'_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\cdot} \cdot e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\cdot}}{\#\left\{e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\cdot}\right\}}} \quad (21)$$

where

$e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\tau}$ is a column vector of errors associated with κ estimator $\widehat{\boldsymbol{rY}50}_{\kappa\left(i_q,j\right),\tau}$ indexed by measurement intensity i_q and streamgage j during trial τ,

$\left(\frac{\sum_{i} e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),i}}{\#\left\{e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\cdot}\right\}}\right)$ is the average error, and

$\frac{e'_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\cdot} \cdot e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\cdot}}{\#\left\{e_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,j\right),\cdot}\right\}}$ is the variance of the errors, and

κ indexes the four estimators in the ordered set $\widehat{\boldsymbol{rY}50}$.

An *RMSE* equal to zero would imply a perfect estimator; an *RMSE* equal to the standard deviation of ***rY50*** would imply that only the mean is being estimated. The *RMSE* of the index regression, which equals 0.12377 $ft^{1.5}/s^{0.5}$-mi, will be used as a basis to compare alternative estimators.

The *RMSE* was computed for each of the 84 simulations over the 10,000 trials used per simulation within all the Michigan Hydrologic Subregions, *MHS*, and over all levels of measurement intensities *I*. The change in accuracy of the index flow estimate by supplementing the index regression estimate with information from one or more simulated instantaneous measurements is described by the percent decrease in *RMSE*, which was computed as

$$\nabla RMSE_{\widehat{\boldsymbol{rY}50}_{\kappa+1,\kappa}}\left(i_q, \forall j \in MHS\right) = \frac{-\left(RMSE_{\widehat{\boldsymbol{rY}50}_{\kappa+1,\kappa}\left(i_q,\ MHS\right)} - RMSE_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,\ MHS\right)}\right)}{RMSE_{\widehat{\boldsymbol{rY}50}_{\kappa}\left(i_q,\ MHS\right)}} \cdot 100 \quad (22)$$

where,

$\nabla RMSE_{\widehat{\boldsymbol{rY}50}_{\kappa+1,\kappa}}\left(i_q, \forall j \in MHS\right)$ is the negative percent change in the RMSE between estimator $\widehat{\boldsymbol{rY}50}_{\kappa+1}$ and $\widehat{\boldsymbol{rY}50}_{\kappa}$ for measurement intensity i_q for all j streamgages in Michigan hydrolic subregion MHS. κ indexes the four estimators in $\{\widehat{\boldsymbol{rY}50}\}$

Positive *RMSE* values imply greater accuracy of the $\widehat{\boldsymbol{rY}50}_{\kappa+1}$ estimator relative to the $\widehat{\boldsymbol{rY}50}_{\kappa}$ estimator for ordered estimators indexed by $\kappa \in \{1,2,3\}$.

Results of the Monte Carlo Simulations for the Discrete, Integrated, and Site Estimators

A three-step estimation process was developed and evaluated by Monte Carlo simulations to evaluate the potential utility of instantaneous flow measurements for improving estimates of index water yields ***rY50*** and index flows ***Q50*** in Michigan from those based on the index regression equation, *Indx**rY50*** and *Indx**Q50***, respectively. Each step in the estimation process was evaluated within each of the seven hydrologic subregions spanning Michigan and at simulated instantaneous flow measurement intensities that ranged from 1 to 100 per site.

Discrete Estimator

Although the index regression estimator applies to all hydrologic subregions, the estimator *Disc**rY50*** is based on the relation between daily mean water yields during specified flow conditions in July, August, and September and index water yields at streamgages within the same hydrologic subregion (fig. 11). This linear relation commonly explained a substantial amount of the variability in index water yields (fig. 12). The coefficient of variation[3] of single measurement *Disc**rY50*** estimates varied from 9.15 percent in Michigan hydrologic subregion 4 to 21.7 percent in subregion 5, with a weighted average of 15.1 percent overall. The distribution of single measurement *Disc**rY50*** estimates for subregion 2, which had an average coefficient of variation of 15.5 is shown in fig. 13.

Results indicate that expected *Disc**rY50*** estimates averaged 32.6 percent lower *RMSE* than the *Indx**rY50*** for one simulated instantaneous measurement and 66.5 percent lower *RMSE* for 100 simulated measurements (fig. 14), for the 7 hydrologic subregions in Michigan For one simulated instantaneous measurement, subregion 5 had the least improvement

[3] The coefficient of variation is computed as the standard deviation divided by the mean, and expressed as a percent.

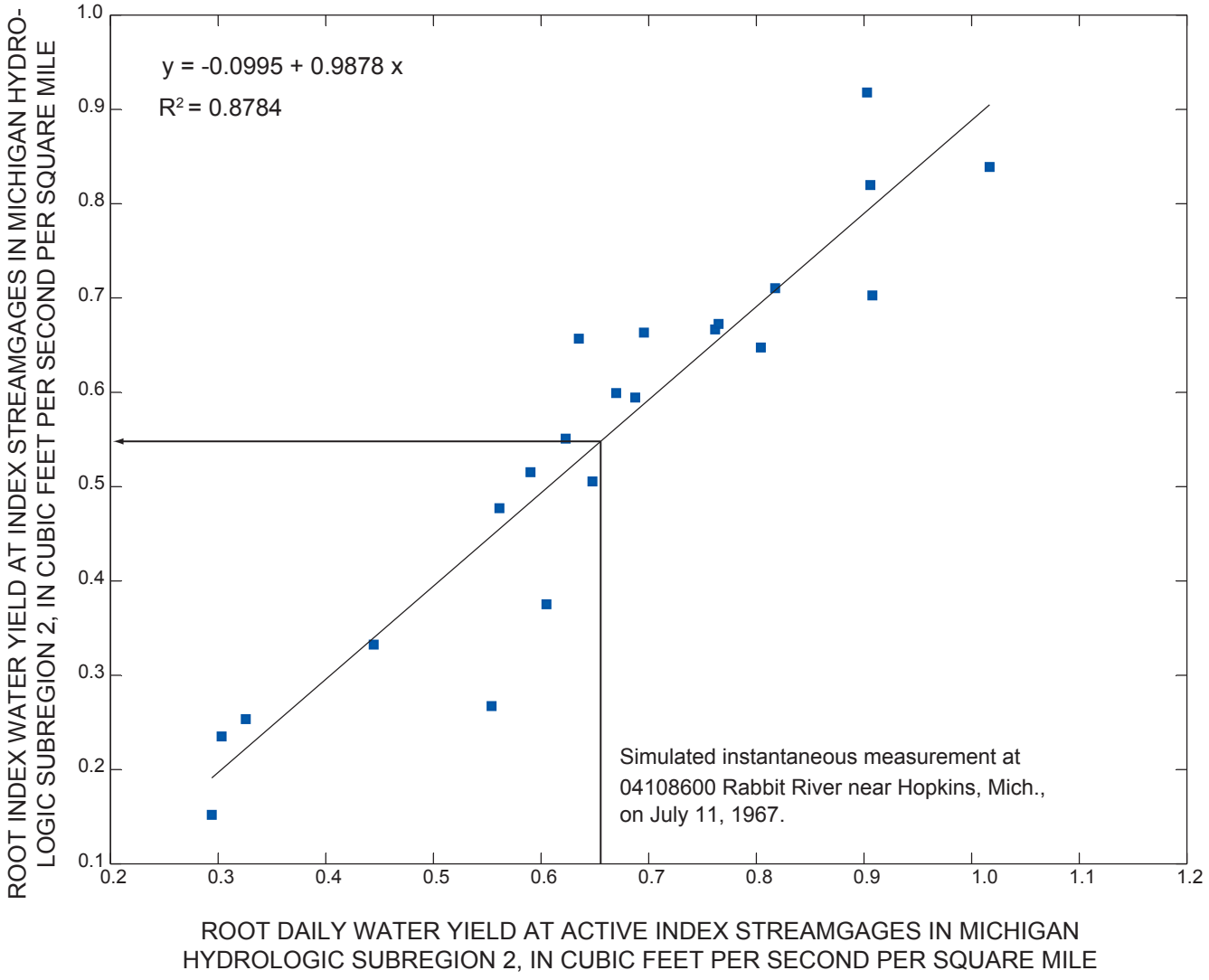


Figure 11. Typical relation between the square roots of daily mean water yields and index water yields, illustrated with data from active streamgages in Michigan hydrologic subregion 2 and a simulated instantaneous flow measurement at U.S. Geological Survey streamgage 04108600, Rabbit River near Hopkins, Mich., on July 11, 1967.

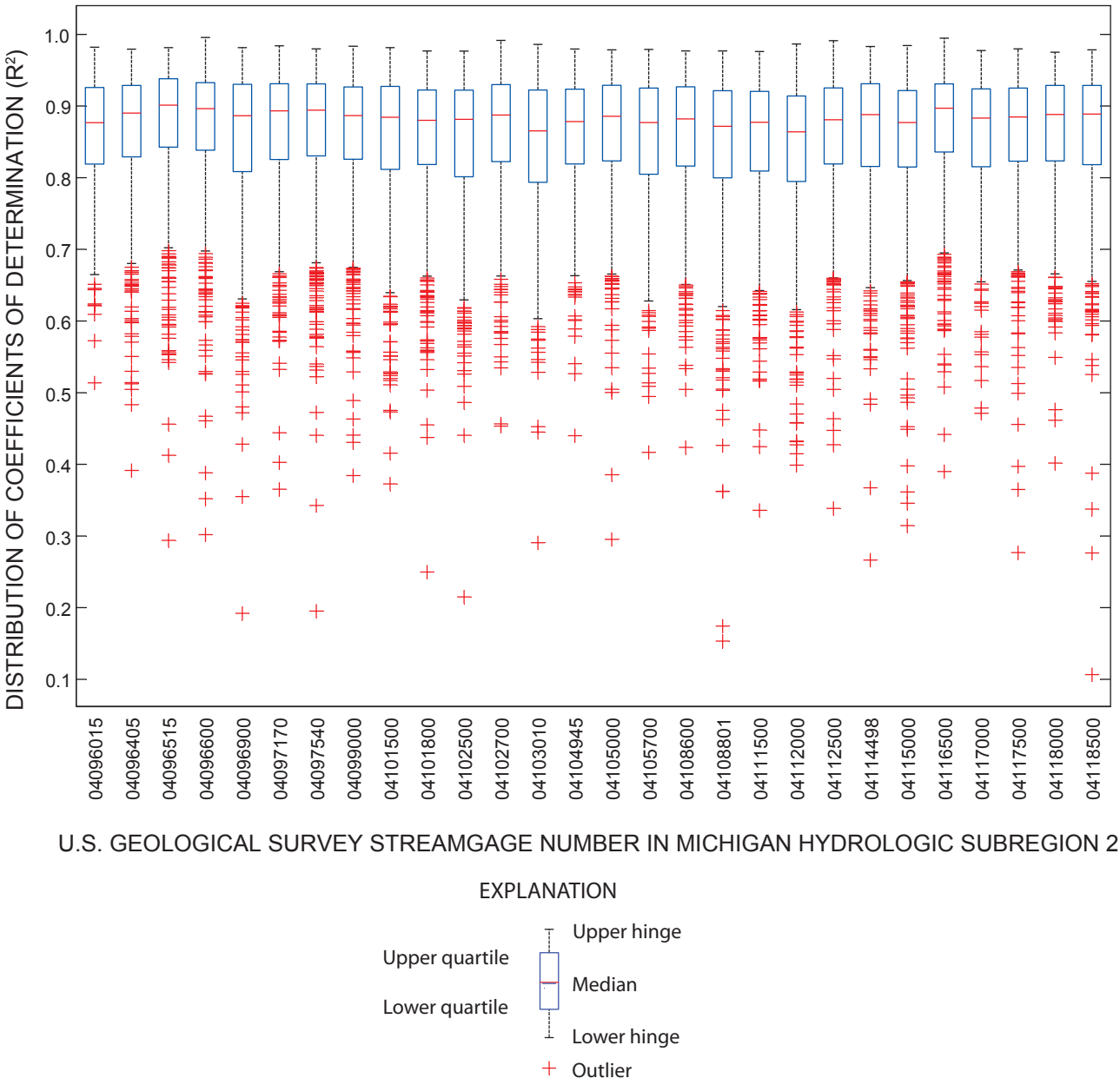


Figure 12. Distribution of coefficients of determination for regression relations between the square roots daily water yields and the square roots of index water yield, illustrated with results for index streamgages in Michigan hydrologic subregion 2.

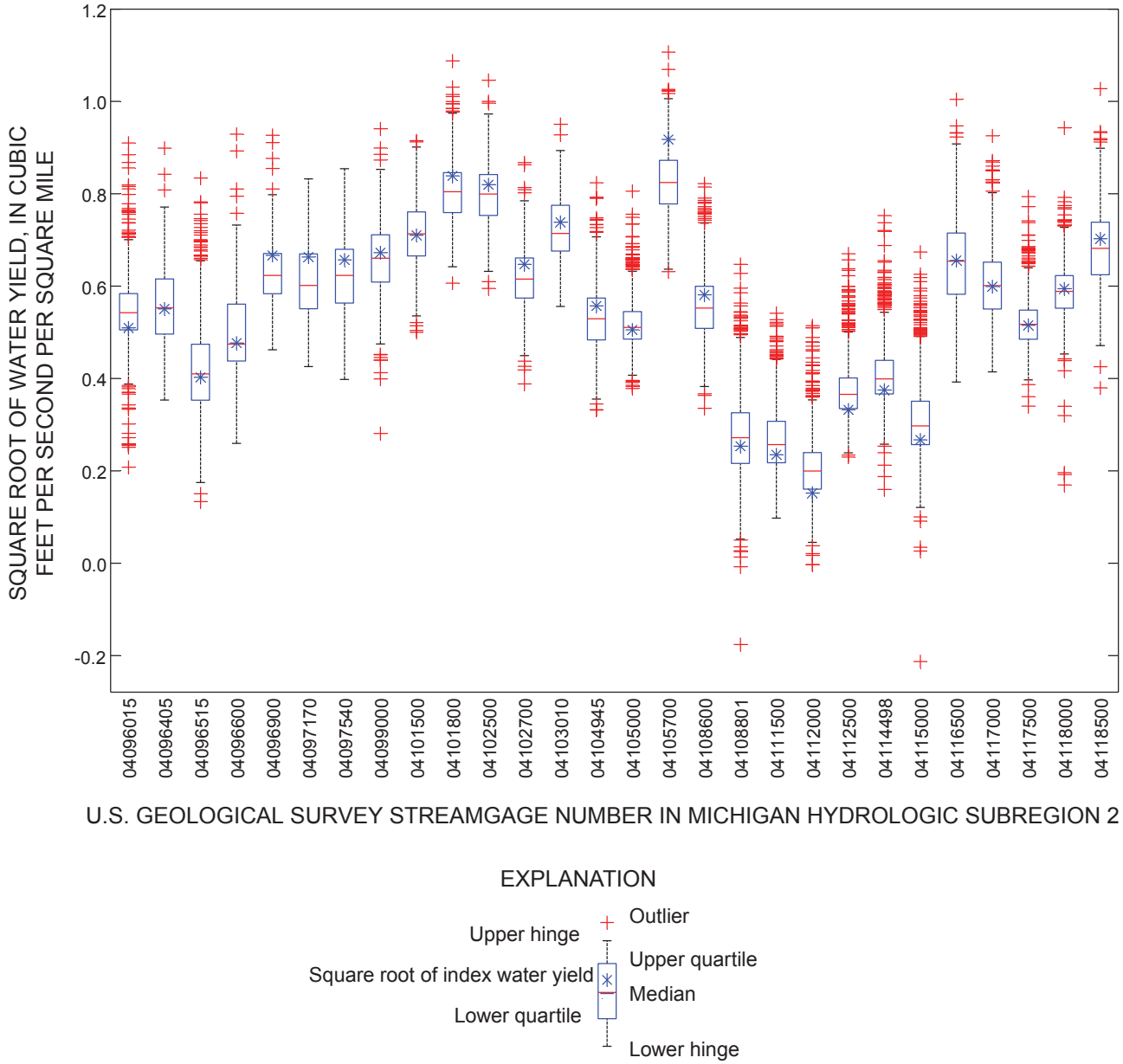


Figure 13. Distribution of single measurement *DiscrY50* estimates for index streamgages in Michigan hydrologic subregion 2.

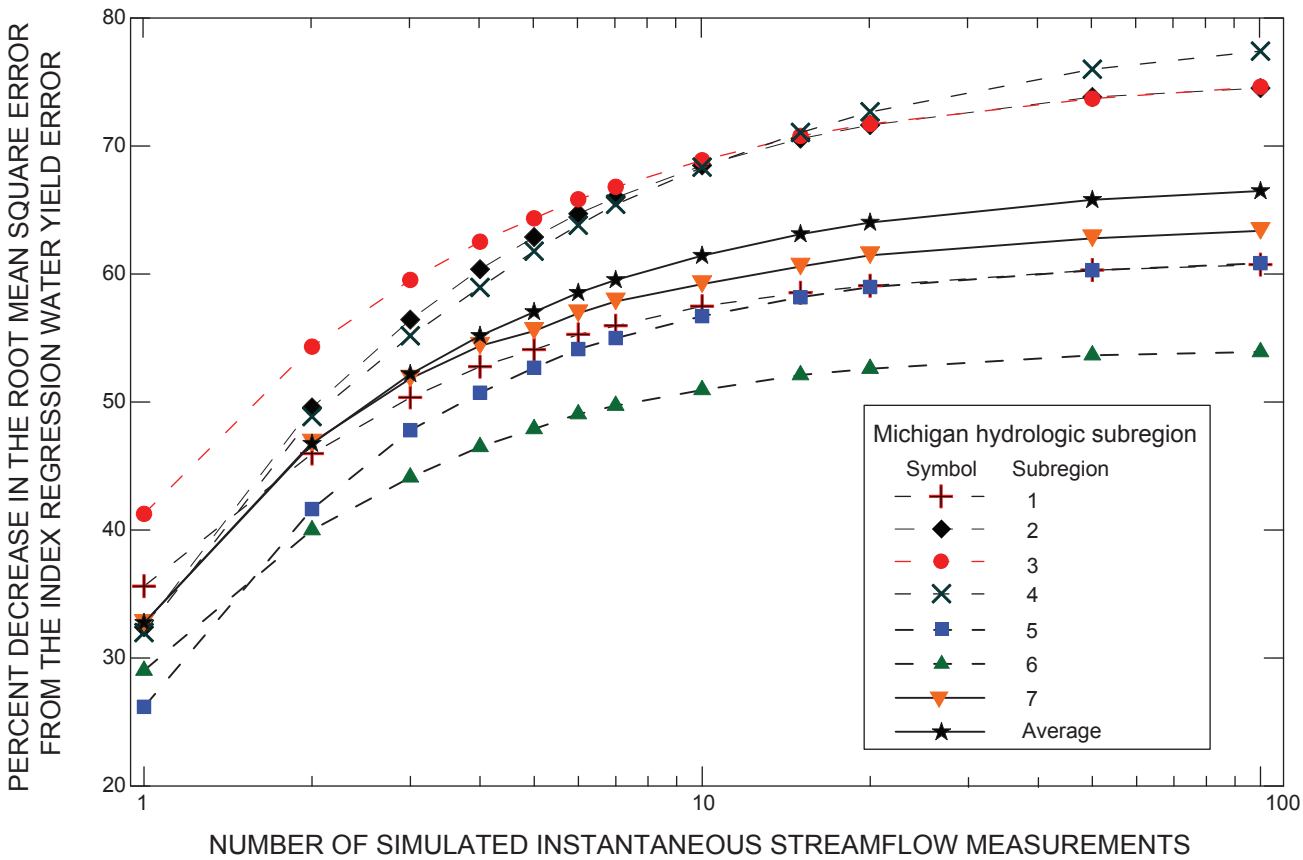


Figure 14. Percent decrease in the root mean square error of the *Discr**Y**50* estimator with respect to the *Indxr**Y**50* estimator with the number of simulated streamflow measurements by Michigan hydrologic subregion.

of 26.3 percent, and subregion 3 had the greatest improvement of 40.7 percent. Thus, even a single instantaneous flow measurement, obtained during July, August, or September under the appropriate flow conditions, is expected to provide a more precise estimate of index water yield and flow than the index regression equation. For 100 simulated measurements, subregion 6 had the least improvement of 54.3 percent, and subregion 4 had the greatest improvement of 77.6 percent. *Disc**Q**50* estimates formed by squaring *Discr**Y**50* estimates and multiplying by the corresponding drainage areas are expected to be consistent with the improvements in the *Discr**Y**50* estimates.

Variances were computed in addition to the magnitudes of the corresponding estimates. The variances were used to combine estimates from different days of simulated measurement and to generate nominal 95-percent confidence intervals about resulting weighted estimates. Over all measurement intensities and subregions, results indicate that the *Discr**Y**50* variances are slightly overestimated in that 96.9 percent of the *Gager**Y**50* values are contained within a nominal 95-percent confidence interval about *Discr**Y**50* estimates (table 2). At a measurement intensity of 1, however, average variances are slightly underestimated in that the nominal 95-percent confidence interval only contains 92.1 percent of *Gager**Y**50* values. There is a slight increase in the average *effective* confidence widths with increasing measurement intensities from 2 to 100. In subregions 1, 2, and 4, at measurement intensity 100, the maximum of 100 percent of ***rY**50* values were contained within the nominal 95-percent confidence interval formed about *Discr**Y**50*; in subregion 4 at measurement intensity 1, the minimum of 88.7 percent of ***rY**50* values were contained within the nominal 95-percent confidence interval.

Integrated Estimator

Integrated estimates are a weighted average of Index regression estimates and Discrete estimates. Integrated estimates of water yield *Intgr**Y**50* averaged 8.3 percent lower *RMSE* than expected *Discrete* values of estimates with one simulated instantaneous streamflow measurement (fig. 15) for the 7 hydrologic subregions. Among hydrologic subregions, improvements in *RMSE* ranged from 1.6 percent for subregion 5 to 19.1 percent for subregion 7. This difference, however, decreases with to zero after about five simulated measurements. Subregions 3 and 5 showed some increase

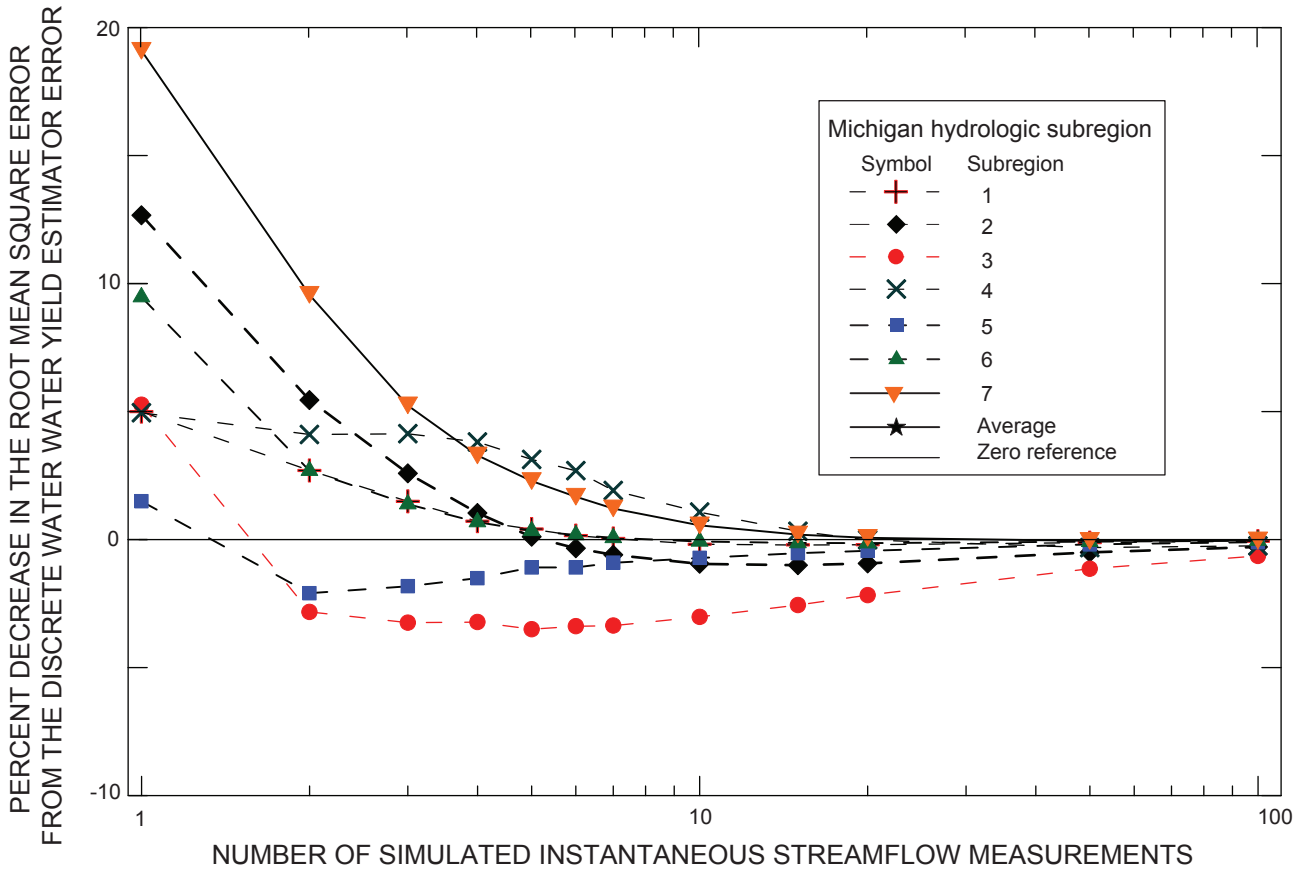


Figure 15. Percent decrease in the root mean square error of the *IntgrY50* estimator with respect to the *IndxrY50* estimator with the number of simulated streamflow measurements by Michigan hydrologic subregion.

Table 2. Percent of *GagerY50* values contained within a nominal 95-percent confidence interval about *DiscrY50*, *IntgrY50*, and *SiterY50* estimators.

Number of simulated instantaneous streamflow measurements	Percent of *GagerY50* values contained within a nominal 95-percent confidence interval about the corresponding estimator		
	DiscrY50	*IntgrY50*	*SiterY50*
1	92.10	90.62	88.17
2	96.03	94.62	91.16
3	96.65	95.35	92.49
4	96.98	95.81	92.45
5	97.15	96.05	93.01
6	97.31	96.28	92.87
7	97.38	96.37	93.09
10	97.57	96.61	92.71
15	97.72	96.78	92.34
20	97.80	96.87	91.31
50	97.99	97.02	85.52
100	98.09	97.07	77.66
Average	96.90	95.79	90.23

in the *RMSE* of the Integrated estimates with two or more simulated measurements; this increase also tended to zero with increasing numbers of simulated measurements. Variances developed for *Intg**rY**50* estimates were used to form nominal 95-percent confidence intervals for all subregions and measurement intensity levels. These confidence intervals effectively included about 95.8 percent of *Gage**rY**50* values for all measurement-intensity levels (table 2). The effective confidence intervals contained a smaller percentage of observations than the nominal confidence intervals would indicate at lower measurement frequencies, but a larger percentage than nominal confidence intervals would indicate at higher measurement frequencies.

Figures (fig. 16-22) show the distribution of errors for estimates of index water yield computed by use of the *Indx**rY**50* and *Intg**rY**50* estimators for each index streamgage within the seven Michigan hydrologic subregions. In these figures, *Intg**rY**50* estimates are based on one simulated instantaneous measurement. The *Indx**rY**50* residual is displayed as an asterisk, which is at the midpoint of a vertical line spanning the middle 50 percent of the probability distribution from the 75- to 25-percent points of the cumulative-distribution function. This interval is based on the standard error of prediction for the index regression equation. In comparison, the distribution of *Intg**rY**50* residuals is displayed as a boxplot in which the upper and lower limits of the box represent the 75th and 25th percentiles, respectively, of the error distribution. Thus, this interquartile range represents the middle 50-percent of the error distribution from the average of 10,000 Monte Carlo trials per streamgage used in the analysis. The mean and median errors for each boxplot show the central tendencies of the distribution.

For each paired boxplot of *Indx**rY**50* and *Intg**rY**50* values at a particular streamgage, the method in which the absolute value of the mean (expected) error is closer to zero provides the more accurate estimate. For all subregions with one simulated instantaneous measurement, the *Intg**rY**50* estimator outperformed the *Indx**rY**50* estimator 105 times for the 142 streamgages evaluated, or about 74 percent of the time. For 100 simulated measurements, the *Intg**rY**50* estimator outperformed the *Indx**rY**50* estimator 108 times out of 142 streamgages evaluated.

With one simulated instantaneous streamflow measurement, variations in the ranks of *Intg**rY**50* account for 94.9 percent of the variability in the ranks of *Gage**rY**50*, and variations in *Intg**Q**50* account for 98.0 percent of the variability in *Gage**Q**50* (fig. 23). With 100 simulated instantaneous measurements, variations in *Intg**rY**50* account for 96.7 percent of the variability in *Gage**rY**50*, and variations in *Intg**Q**50* account for 98.8 percent of the variations *Gage**Q**50*. Relative to the 70.8 percent of the variability in ***rY**50* associated with *Indx**rY**50* estimator, the *Intg**rY**50* estimator is more accurate, explaining 94.9-percent or more of the variability in ***rY**50*. The accuracy of the *Intg**rY**50* estimator, however, shows limited improvement with increased measurement intensity.

Site Estimator

The Site Estimator of index water yields *Site**rY**50* and flows *Site**Q**50* was developed to facilitate the convergence of index water yield and flow estimates to site-specific values of *Gage**rY**50* and *Gage**Q**50* with increasing numbers of instantaneous flow measurements. The technique is based on the premise that as the number of instantaneous flow measurements obtained during July, August, and September approaches the number of daily mean flow values used to compute *Gage**rY**50* and *Gage**Q**50*, the estimates should tend to converge to the computed values assuming the monthly medians are similar. Because median flows in July, August, and September differ somewhat, the median of instantaneous measurements taken with equal probability from these months likely will exceed the index flow, which is defined as the minimum of median flows during these 3 months.

Application of the Site Estimator is limited to sites where three or more instantaneous measurements are available. Determination of the variance of the median estimator requires that at least two instantaneous flow estimates be available for estimation. The median of two samples, by convention however, is indistinguishable from the mean of two samples, even though the mean generally has different probability distribution than the median. With increasing sample size, the variance anomaly diminishes between even- and odd-numbered sample sizes.

Simulation results indicate that the decrease in *RMSE* of the Site Estimator *Site**rY**50* relative to the Integrated Estimator *Intg**rY**50* generally increases with increasing numbers of simulated instantaneous measurements greater than or equal to 3 for all hydrologic subregions in Michigan (fig. 24). All Michigan hydrologic subregions except 2 have higher average precisions than the corresponding integrated estimator for all measurement intensities investigated. The reason for the anonymous results in subregion 2, which do not indicate increases in precision until more than 20 measurements are obtained, is not known.

Unlike the results for the Discrete and Integrated estimators, the average effective percentage of *Gage**rY**50* values contained with the nominal 95 percent confidence interval did not increase monotonically with increased sampling intensity. Instead, the maximum average of 93.1 percent occurred for a 7 simulated instantaneous measurements, while the minimum average percentage occurred for 100 simulated instantaneous measurements (table 2).

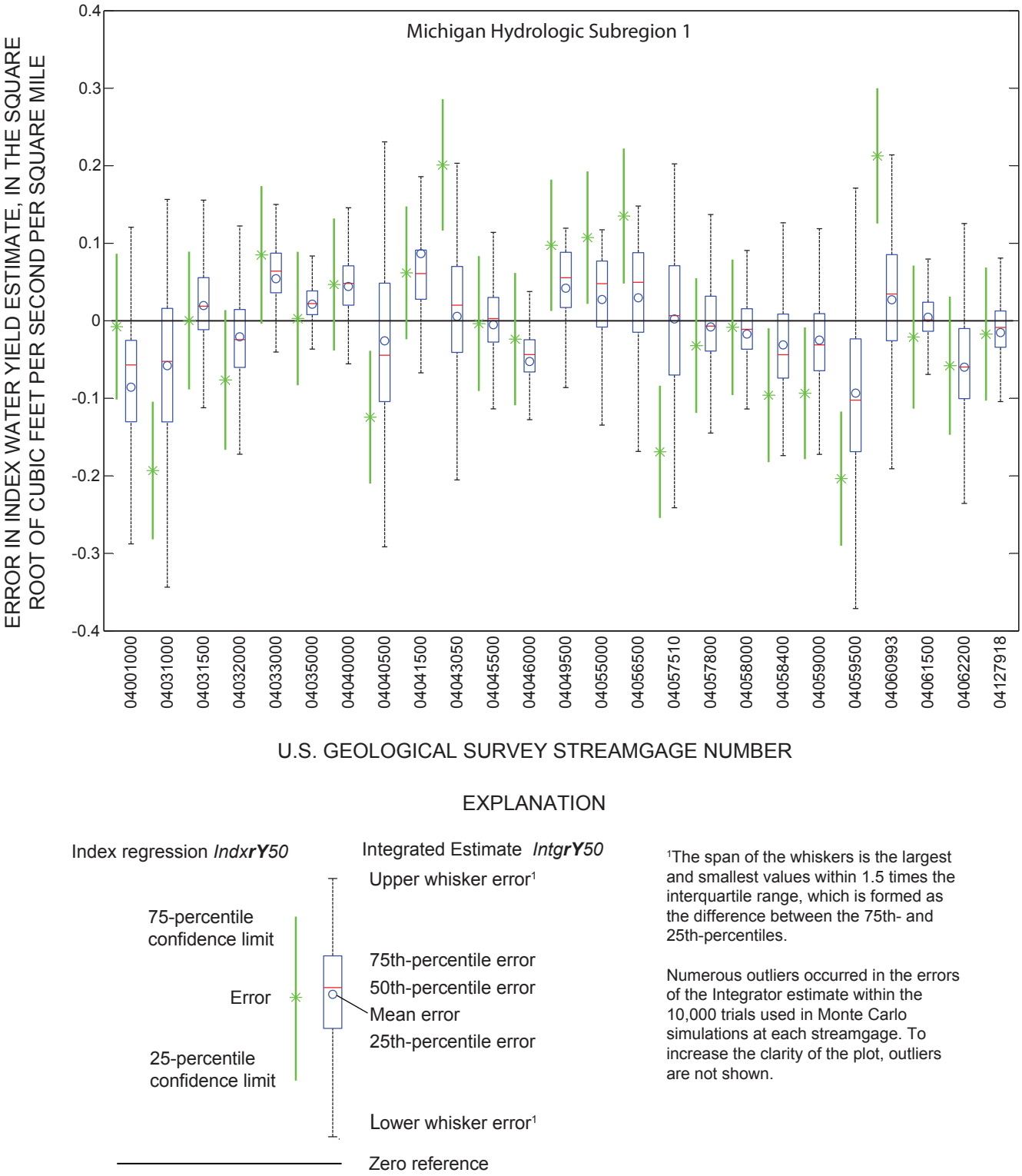
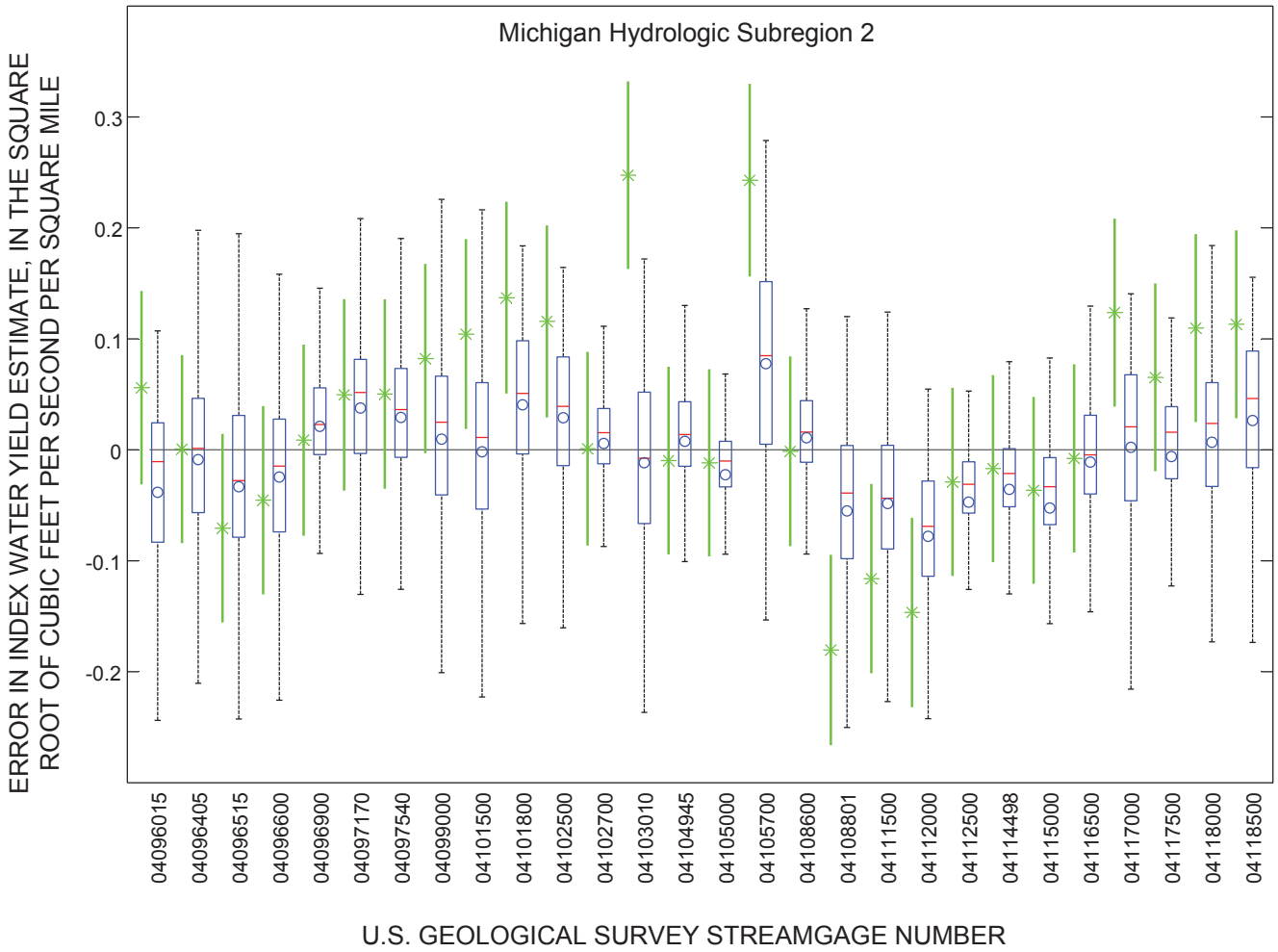


Figure 16. Error for the index regression estimator *IndxrY50* and the distribution of errors for the integrated water yield estimator *IntgrY50* with one simulated measurement for index streamgages within Michigan hydrologic subregion 1.



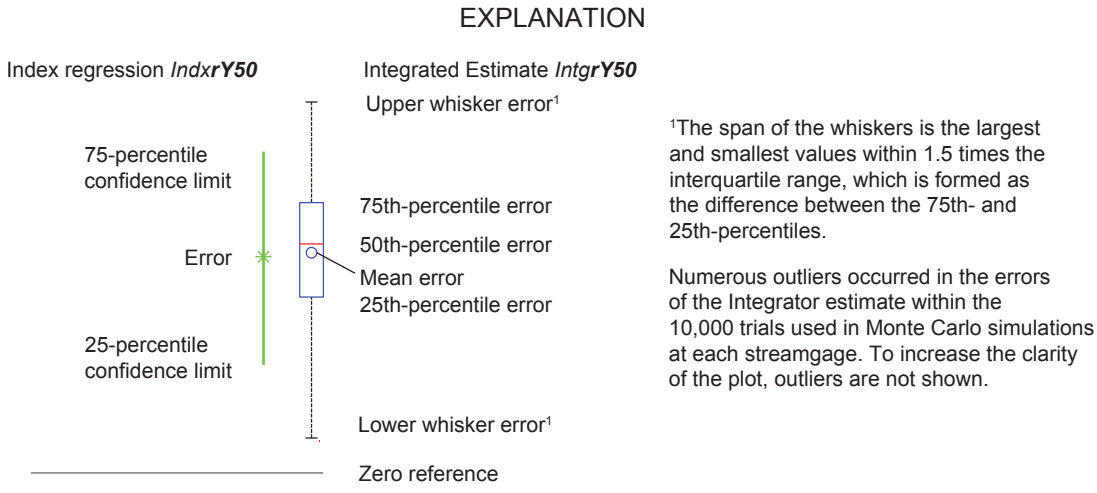
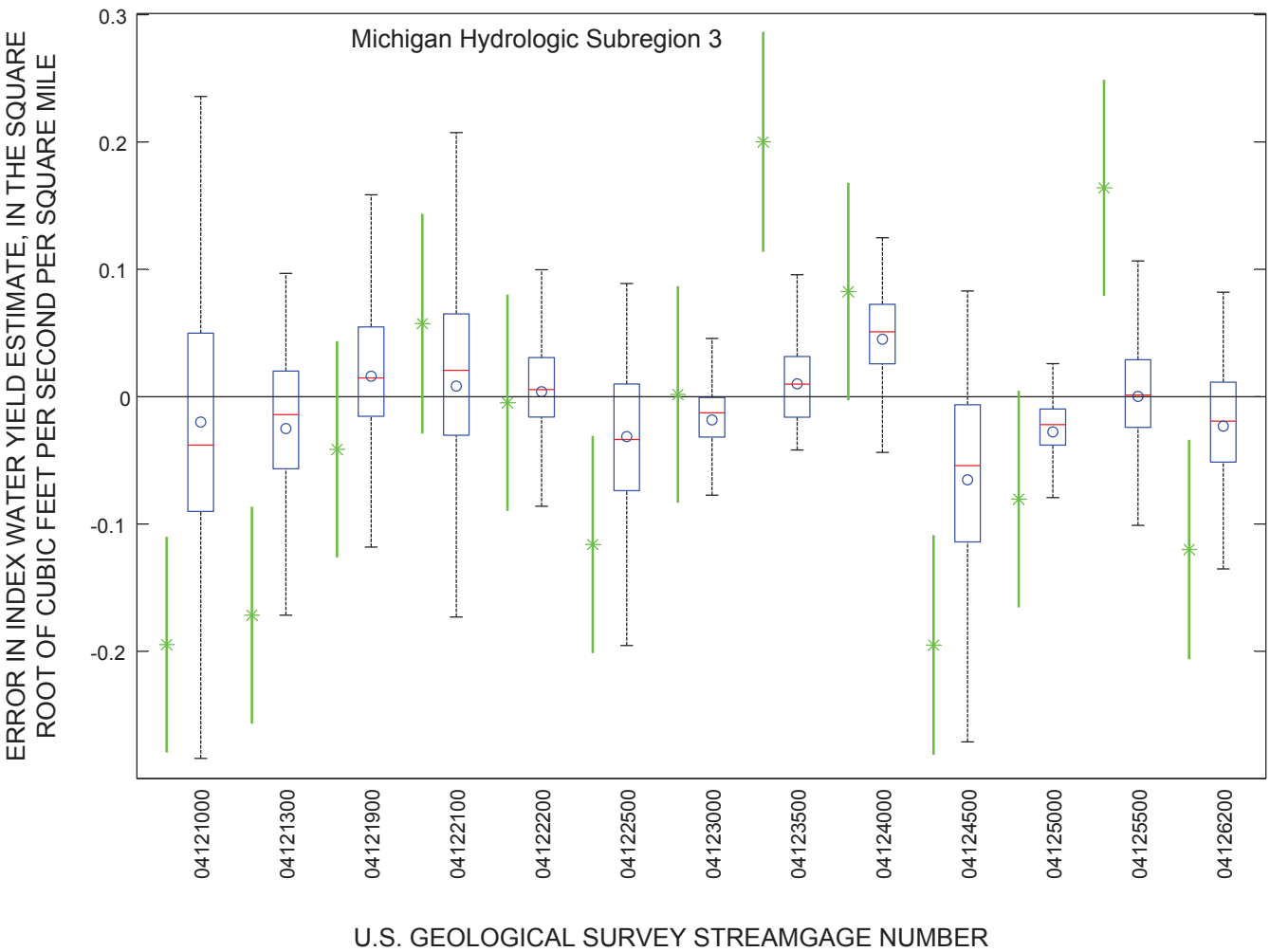
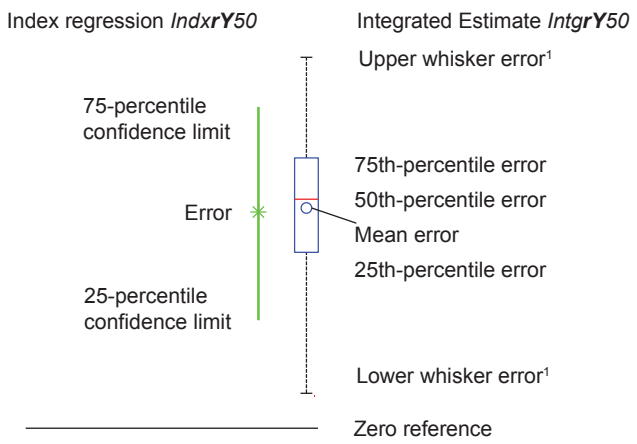


Figure 17. Error for the index regression estimator *IndxrY50* and the distribution of errors for the integrated water yield estimator *IntgrY50* with one simulated measurement for index streamgages within Michigan hydrologic subregion 2.



EXPLANATION



[1]The span of the whiskers is the largest and smallest values within 1.5 times the interquartile range, which is formed as the difference between the 75th- and 25th-percentiles.

Numerous outliers occurred in the errors of the Integrator estimate within the 10,000 trials used in Monte Carlo simulations at each streamgage. To increase the clarity of the plot, outliers are not shown.

Figure 18. Error for the index regression estimator *IndxrY50* and the distribution of errors for the integrated water yield estimator *IntgrY50* with one simulated measurement for index streamgages within Michigan hydrologic subregion 3.

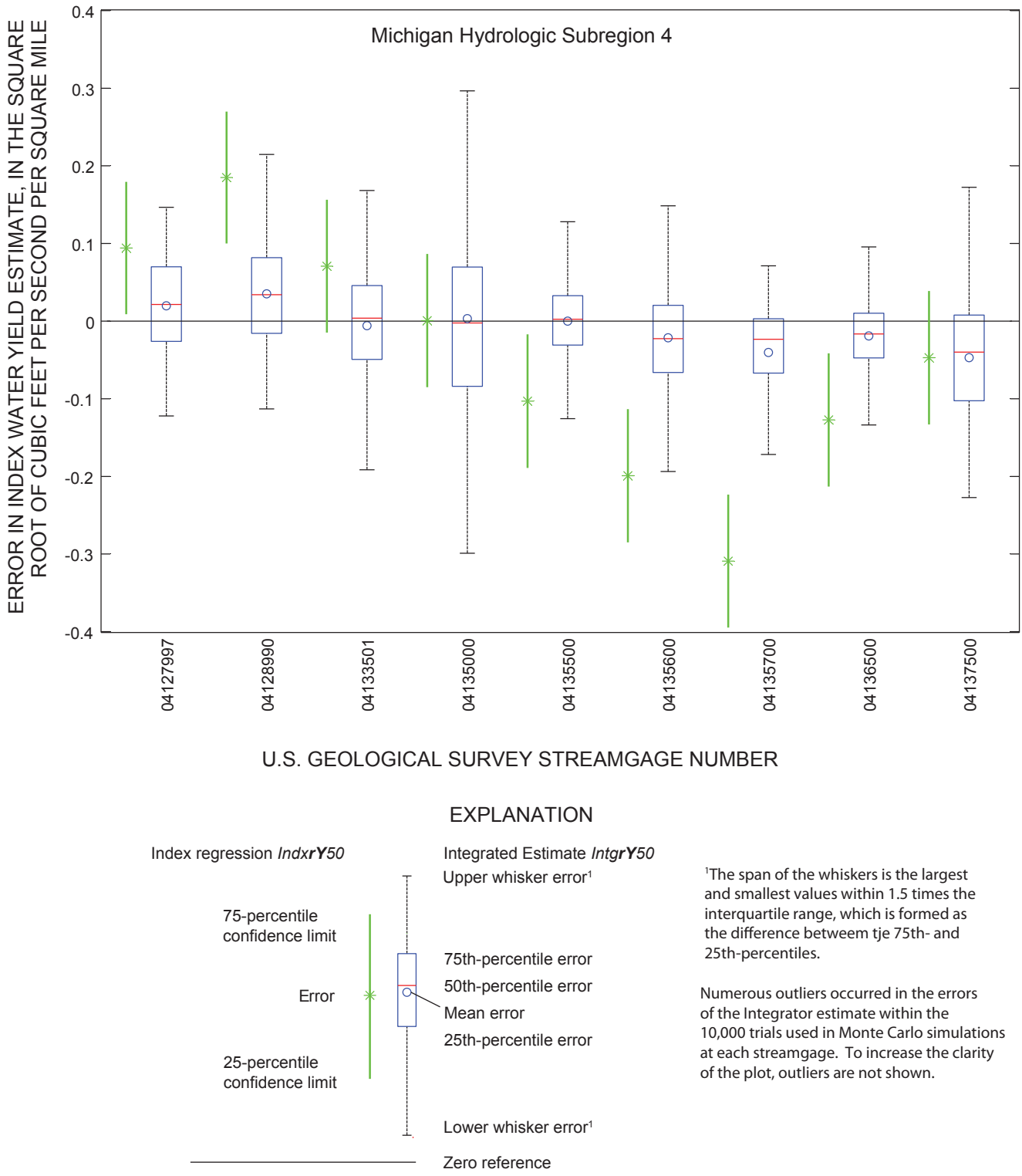


Figure 19. Error for the index regression estimator *IndxrY50* and the distribution of errors for the integrated water yield estimator *IntgrY50* with one simulated measurement for index streamgages within Michigan hydrologic subregion 4.

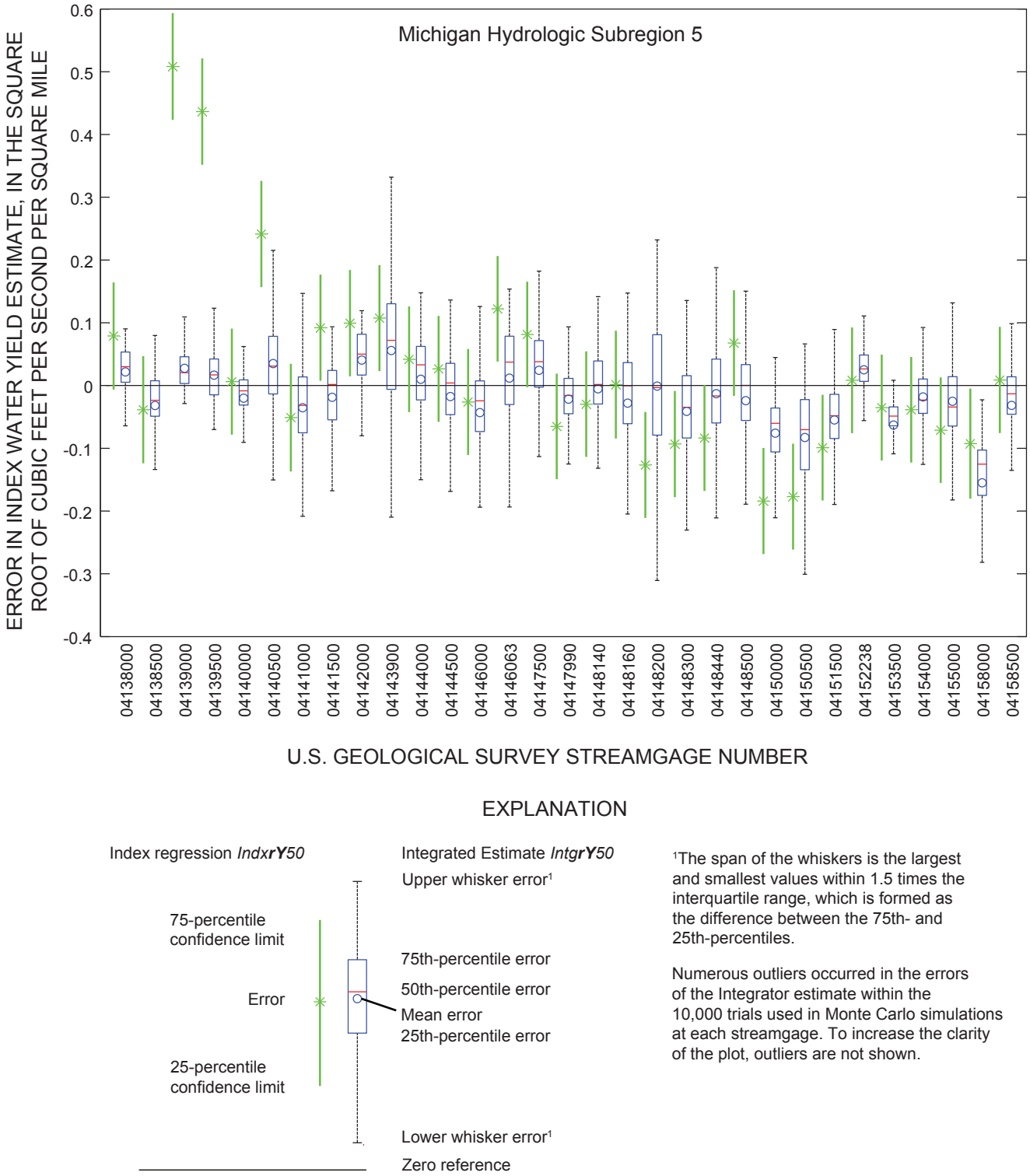
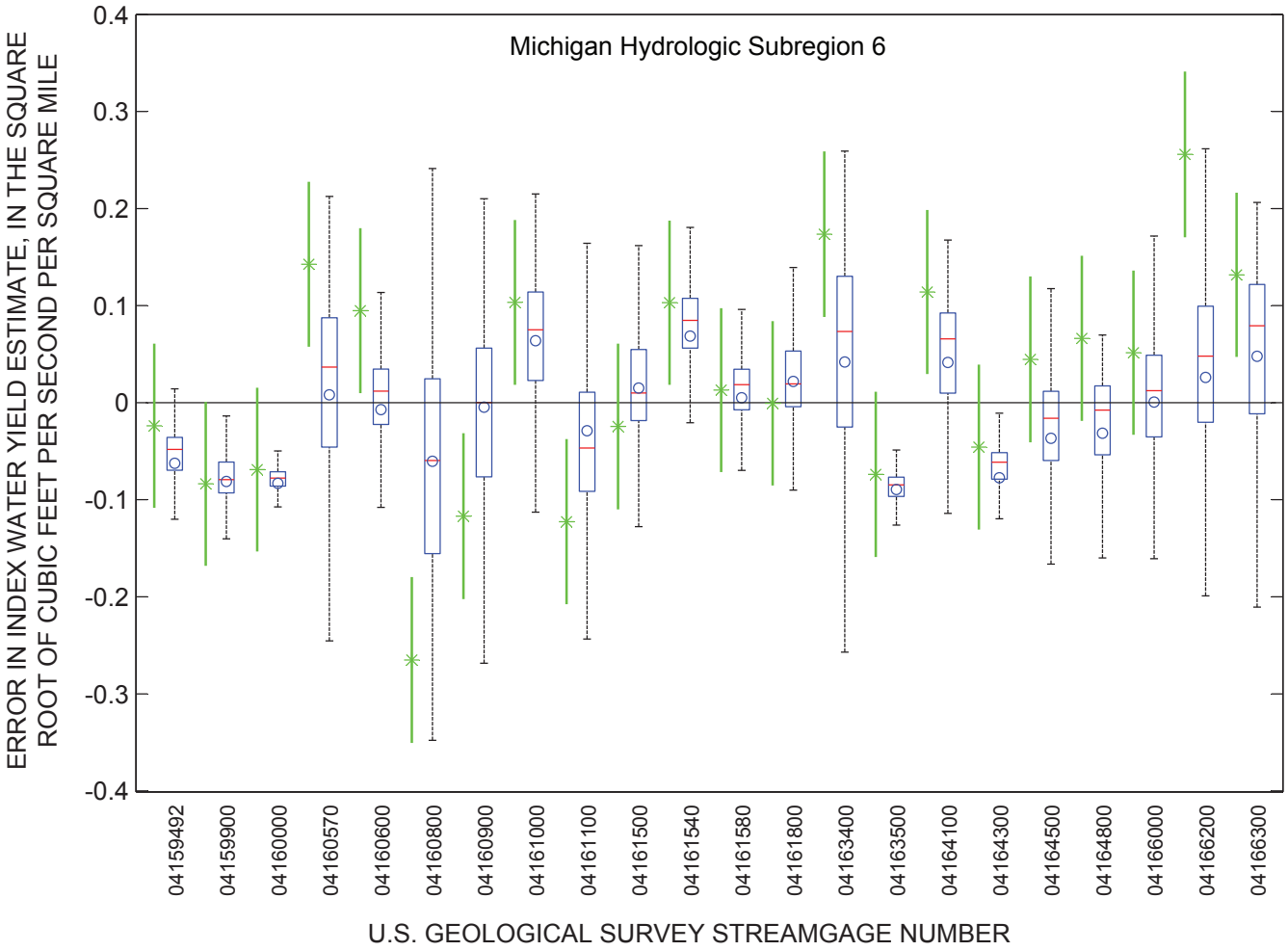


Figure 20. Error for the index regression estimator *IndxrY50* and the distribution of errors for the integrated water yield estimator *IntgrY50* with one simulated measurement for index streamgages within Michigan hydrologic subregion 5.



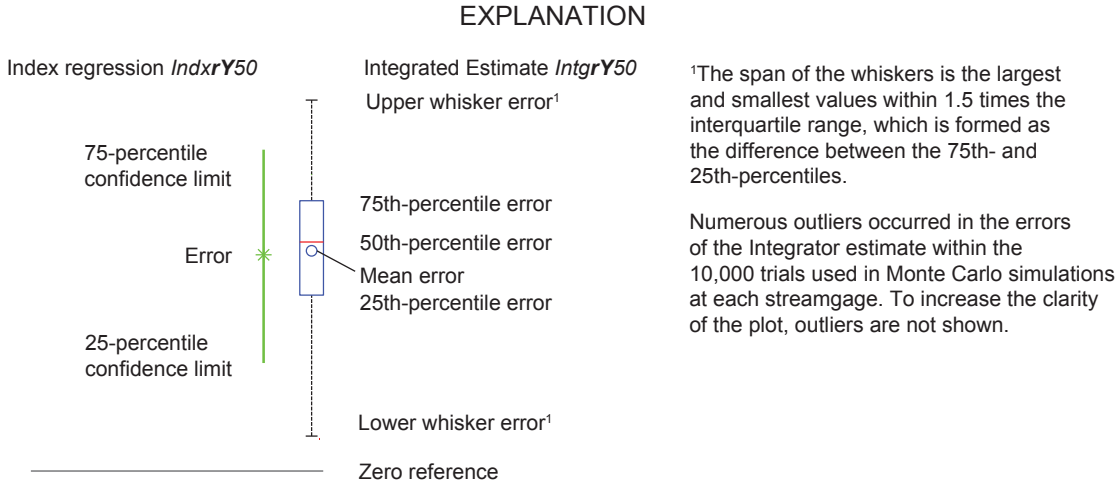


Figure 21. Error for the index regression estimator *IndxrY50* and the distribution of errors for the integrated water yield estimator *IntgrY50* with one simulated measurement for index streamgages within Michigan hydrologic subregion 6.

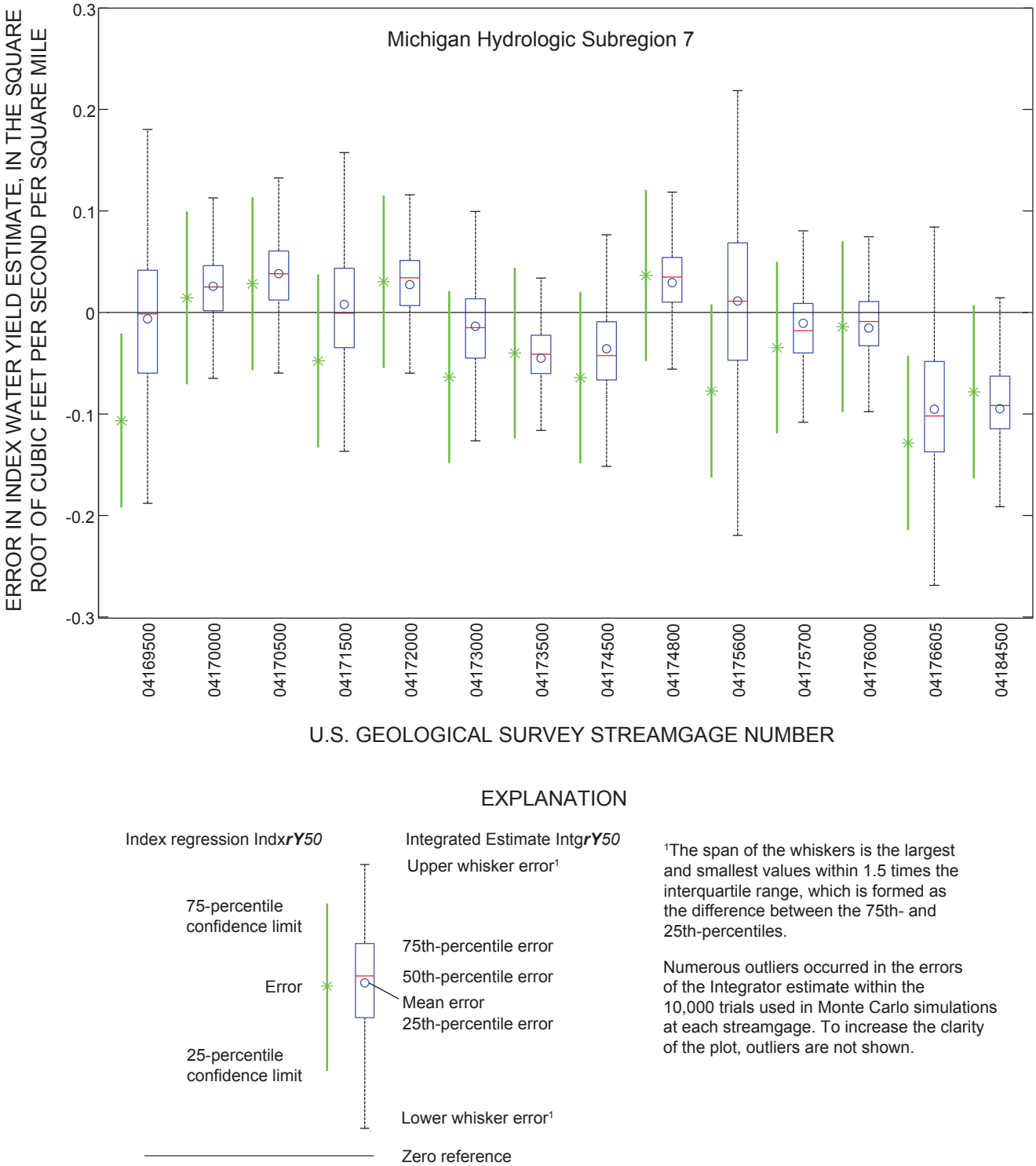


Figure 22. Error for the index regression estimator *Indx**rY**50* and the distribution of errors for the integrated water yield estimator *Intg**rY**50* with one simulated measurement for index streamgages within Michigan hydrologic subregion 7.

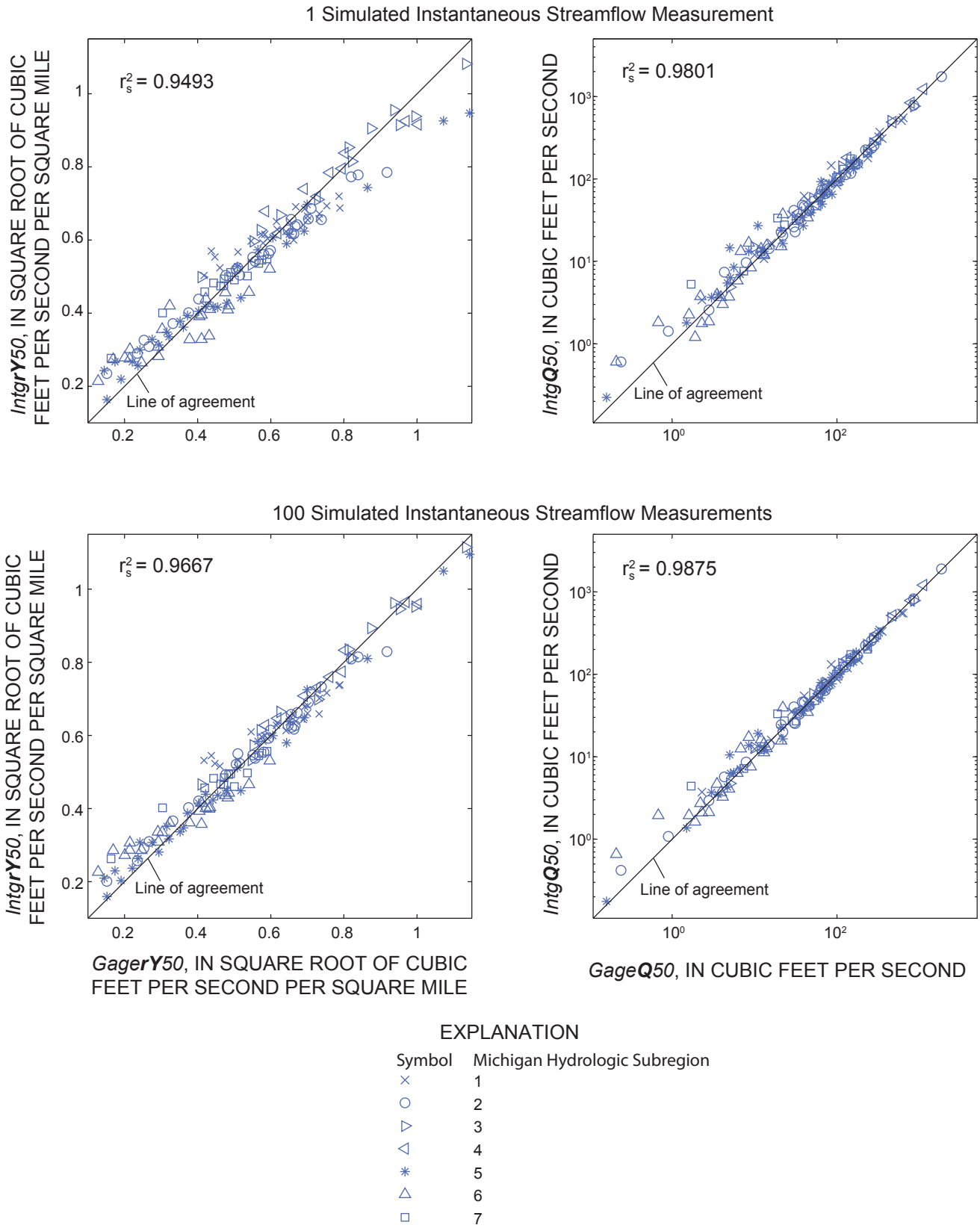


Figure 23. Relation of *Intg**rY**50* with *Gage**rY**50* and *Intg**Q**50* with *Gage**Q**50* for 1 and 100 simulated instantaneous streamflow measurements.

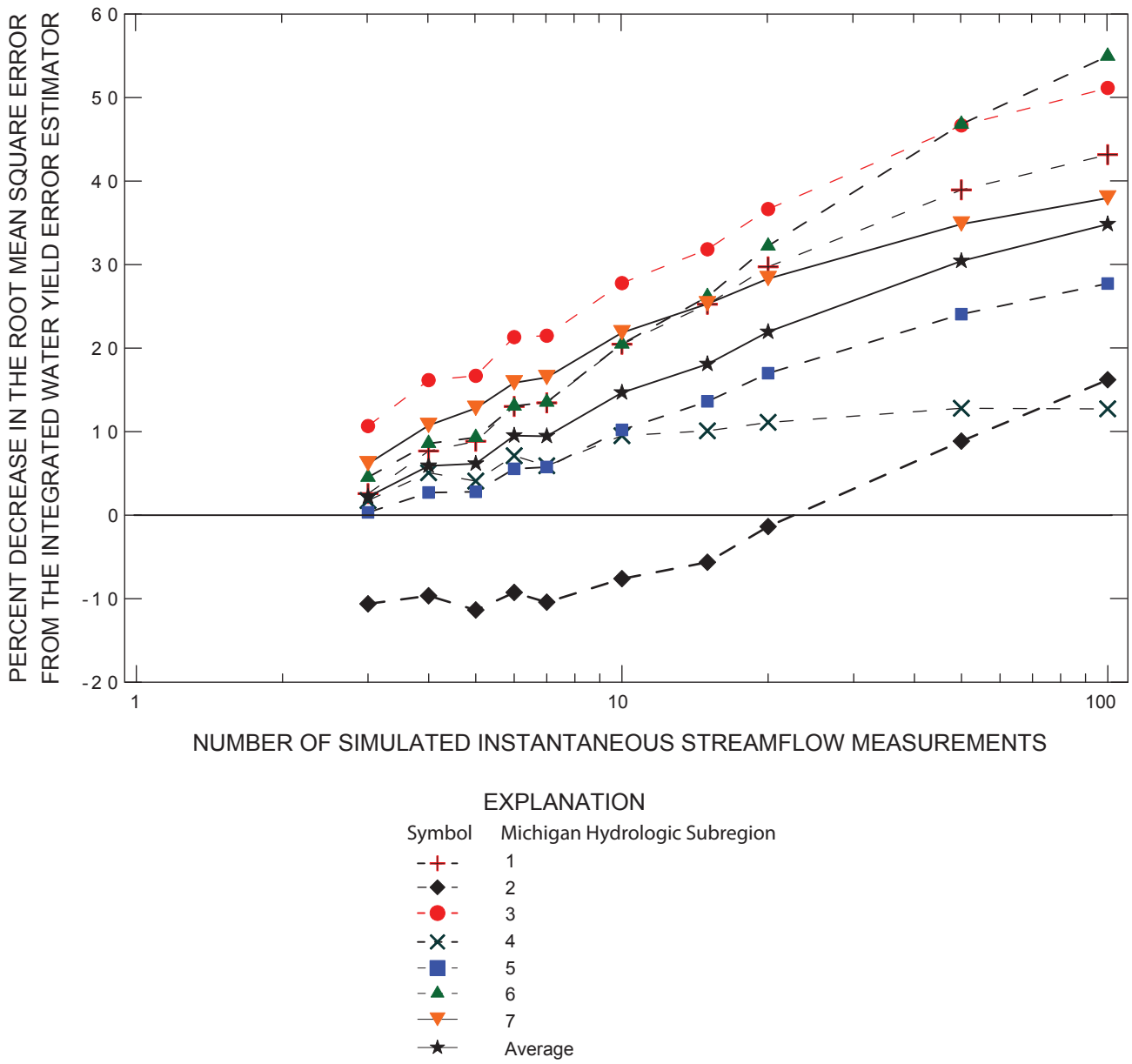


Figure 24. Decrease in root mean square error of the *SiterY50* estimator with increased number of simulated instantaneous streamflow measurements.

The three estimators developed in this report tended to converge to their final values with increasing numbers of simulated instantaneous measurements. Only *Site**rY**50* estimates, however, tended to converge to ***rY**50* values with increasing measurement intensity. Rates of convergence varied widely among streamgages where convergence occurred (fig. 25), from about 6 simulated instantaneous flow measurements to more than 100. At other streamgages, however, *Site**rY**50* estimates converged to values that were greater or less than *Gage**rY**50,* at least within the range of simulated measurement intensities. Convergence of *Site**rY**50* to *Gage**rY**50* values is thought to be sensitive to the month of flow measurement and the flow criterion under which instantaneous flow measurements are obtained.

The tendency for convergence to *Gage**rY**50* values was compared by computing the *RMSE* between ***rY**50* estimates and *Gage**rY**50* values for each estimator by measurement intensity (fig. 26). Results indicate that *RMSE* for *Disc**rY**50* decreased with measurement intensity and converged with *RMSE* values for *Intg**rY**50* at measurement intensities greater than 2. For measurement intensities greater than 10, the *RMSE* for either *Disc**rY**50* or *Intg**rY**50* showed occasional substantial decreases. At measurement intensities greater than 3, the *RMSE* for *Site**rY**50* estimates is consistently less than the *RMSE* for either *Disc**rY**50* or *Intg**rY**50*. Moreover, the *RMSE* for *Site**rY**50* decreases throughout the range of measurement intensities investigated. This continued decrease indicates that *Site**rY**50* estimates are converging to *Gage**rY**50* values. The *RMSE* for the *Indx**rY**50* estimator is plotted for reference, although it is not a function of measurement intensity.

To further improve convergence characteristics of the Site estimator, it may be appropriate to measure instantaneous flows during the month when minimum median flow is most likely to occur. Among the 147 index streamgages in Michigan, index flows occurred 62.9 percent of the time in August, 34.0 percent of the time in September, and 3.4 percent of the time in July. Among Michigan hydrologic subregions, August was the most frequent month of minimum flow, expect for subregion 2, where minimum flows more commonly occurred in September (fig. 27). All index flows occurred in August within subregion 1, which corresponds to Michigan's Upper Peninsula. July was the least frequent month of index flows among subregions. Convergence characteristics of the Site estimator also may be sensitive to the water-yield criterion used to restrict the selection of days for simulated instantaneous measurement.

Further experimentation with the criterion for selecting daily values as estimators of ***rY**50* values may facilitate convergence of the *Site**rY**50* estimator. For example, it may be helpful if a threshold percentage of index streamgages in the target hydrologic subregion are above and below their index flows, thus forming a mixed index-flow condition for the subregion. Real-time monitoring of daily flows at index streamgages might be used to select a day for instantaneous measurement that would be expected to meet this criterion. For instance, an instantaneous measurement might be planned on a day when 10 percent or more of index streamgages in the hydrologic subregion are flowing at a rate less than their index flows, and 10 percent or more of the index streamgages are flowing at a rate greater than their index flows. Further analysis would help identify the likelihood of these mixed index flow conditions for a range of threshold percentages. Additional Monte Carlo simulations may clarify how the threshold percentages affect the rate of convergence of the *Site**rY**50* estimates to ***rY**50* values.

Towards the Development of an Augmented Estimator from Base Flow Recession Characteristics

The augmented index regression estimator of water yield and flow (*Augm**rY**50* and *Augm**Q**50,* respectively) is based on an augmented form of the index regression equation, which contains two additional variables characterizing base-flow recession. The augmented equation is developed by analysis of 9,724 base-flow recession events identified from daily flow values at index streamgages. Base flow recession characteristics at index streamgages are described, and recession parameters for a simplified recession equation are estimated. Monte Carlo simulations are used to describe the relation between the number of recession events measured and the accuracy of the augmented regression estimator. A set of two or more instantaneous flow measurements during one or more base-flow recession events can be used to estimate the base-flow recession variables needed to apply the augmented index regression equation.

Eng and Milly (2007) proposed the use of base flow recession information to improve estimates of low-flow characteristics at partial-record stream sites. In their study, base flow recession segments were chosen only from the period April 1 to October 31, because low streamflow values generally occur during this period in their study region in the southeastern part of the United States. By use of daily flow data at streamgages, Eng and Milly (2007) identified sequences of eight or more daily values that (1) started with a peak daily streamflow that exceeded the median daily flow value, and (2) ended with the first day j for which both Q_{j+1} and Q_{j+2} exceed Q_j. Eng and Milly (2007) concluded that (1) introduction of a base-flow recession time constant, estimated from a single pair of strategically timed streamflow measurements, approximately halves the root-mean-square estimation error relative to that of a conventional drainage area regression, and (2) additional streamflow measurements can be used to further reduce the error. A similar analysis was conducted as part of this report to assess the utility of base-flow recession information on the estimation index water yield and flow.

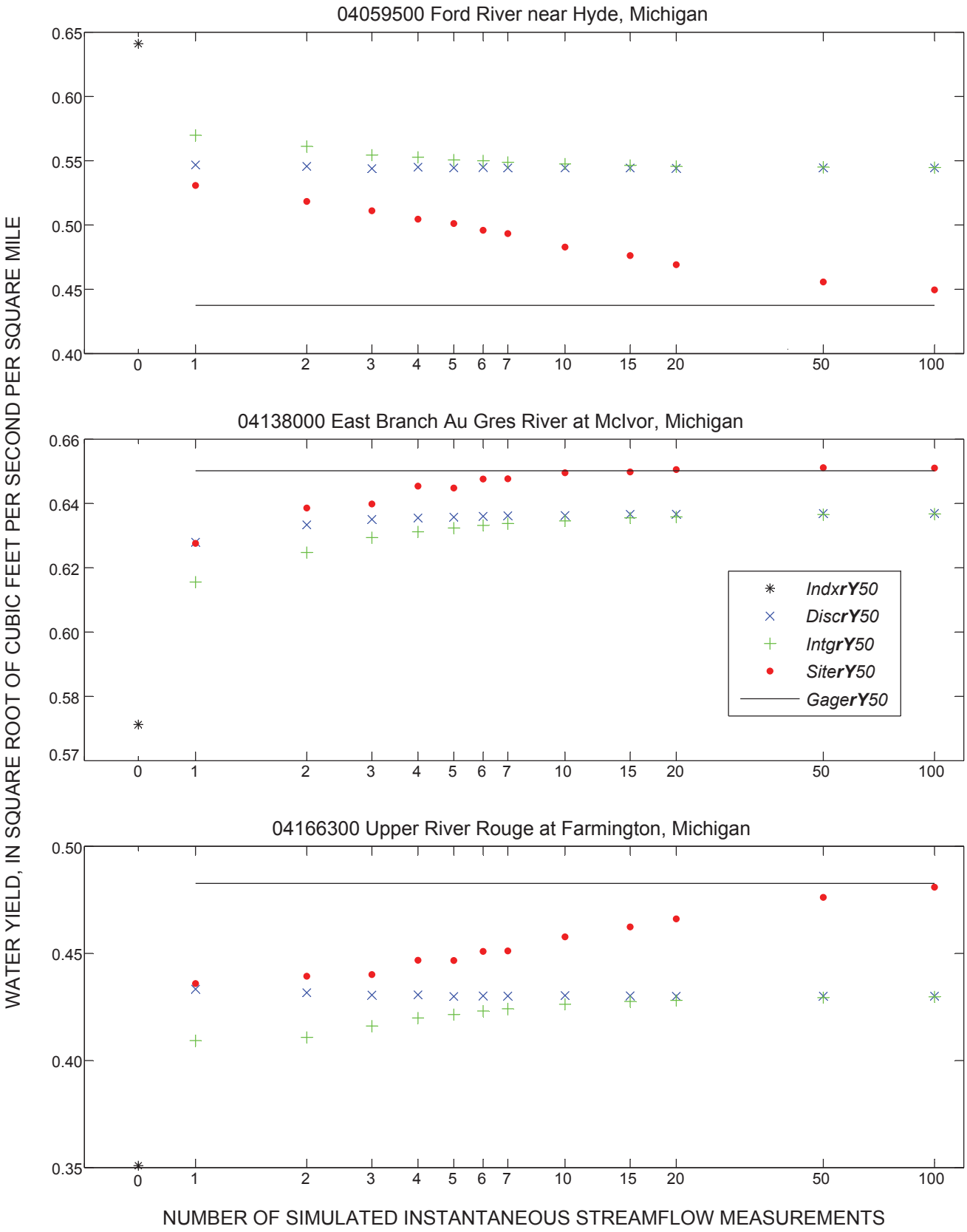


Figure 25. Convergence of *DiscrY50*, *IntgrY50*, and *SiterY50* estimates with increasing number of simulated instantaneous streamflow measurements.

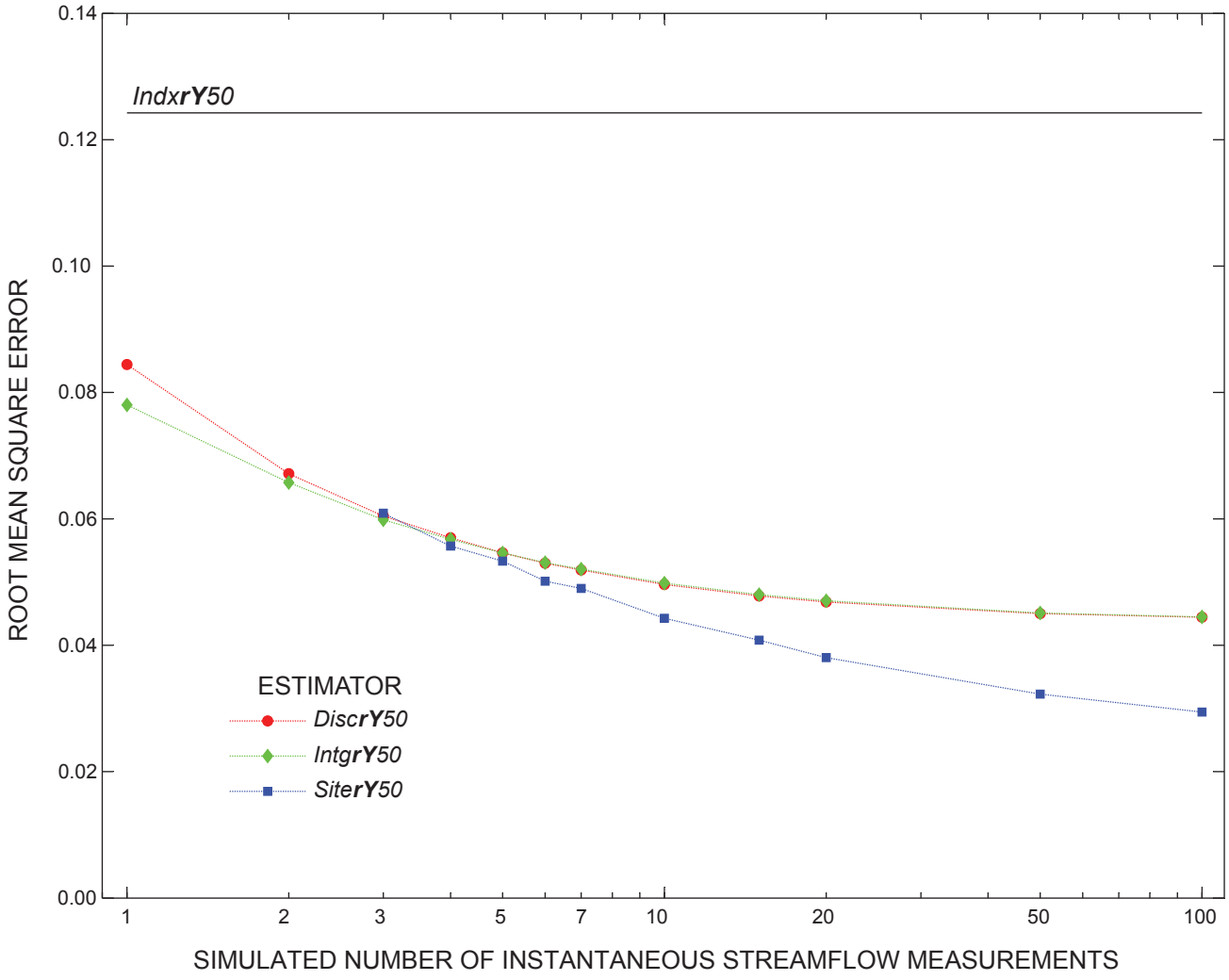


Figure 26. Relation between root mean square error and the number of simulated instantaneous streamflow measurements for estimators *Discr**Y**50*, *Intgr**Y**50*, and *Siter**Y**50*.

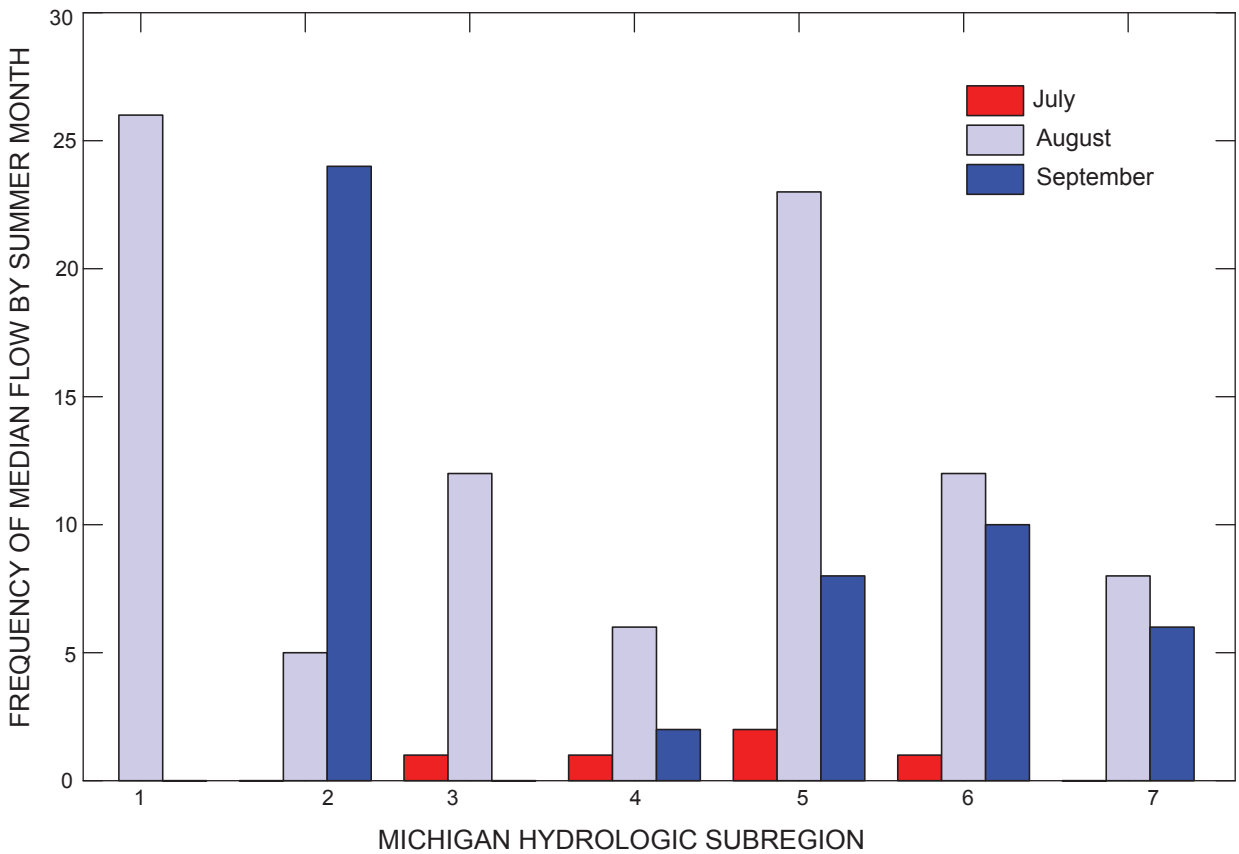


Figure 27. Frequency of summer month of minimum median flow by Michigan hydrologic subregion.

Base-Flow Recession

Consistent with the determination of index flows, recession segments were restricted to the period from July 1 to September 30. Like Eng and Milly (2007), flow on the first day of each base flow segment was required to exceed a minimum criterion, in this case the index flow ***Q50*** for the corresponding streamgage. For consistency with the metric used by Hamilton and others (2008), recession events were defined on the basis of daily mean water yields ***rYdv*** rather than daily flows ***Qdv***.

Daily values contained in the sampling frame were the primary source of information for identifying base-flow recession events for index streamgages. Periods of daily record were added, however, for the five streamgages that were discontinued before first day of values in the sampling frame and for selected other index streamgages with short periods of record within the sampling frame. A total of 9,724 base-flow recession events were identified at 147 index streamgages (appendix 1, table 1-4). No recession events were identified, however, at streamgage 04135000 Thunder Bay River near Alpena, Mich. The lack of recession events meeting the specified criteria may be associated with flow regulation by a hydroelectric plant 1,000 ft upstream from this gage (Blumer and others, 1994). One to two base-flow recession events commonly occurred each year (fig. 28) at each streamgage. About one-half of the identified recession events were 8 or 9 days long (fig. 29).

Following Eng and Milly (2007), parameters rY_o and *tau* for each recession event were estimated by least squares using all but the first five daily water yield values ***rYdv*** within the identified recession event as

$$rY_t = rY_o \cdot e^{-\frac{t}{tau}}, \tag{23}$$

where,

rY_t is the square root of the estimated water yield at time t,

rY_o is the initial square root of water yield of the target base flow recession segment, and

tau a parameter associated with the recession time constant.

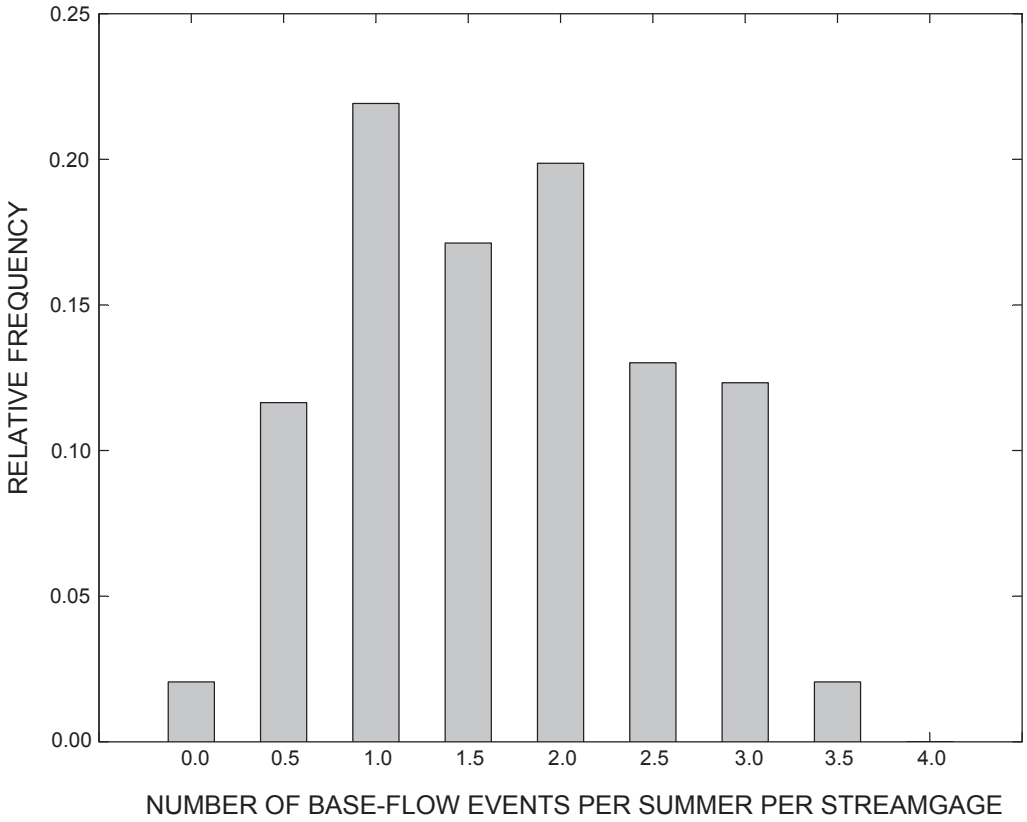


Figure 28. Relative frequency of the number of base-flow events per summer per index streamgage.

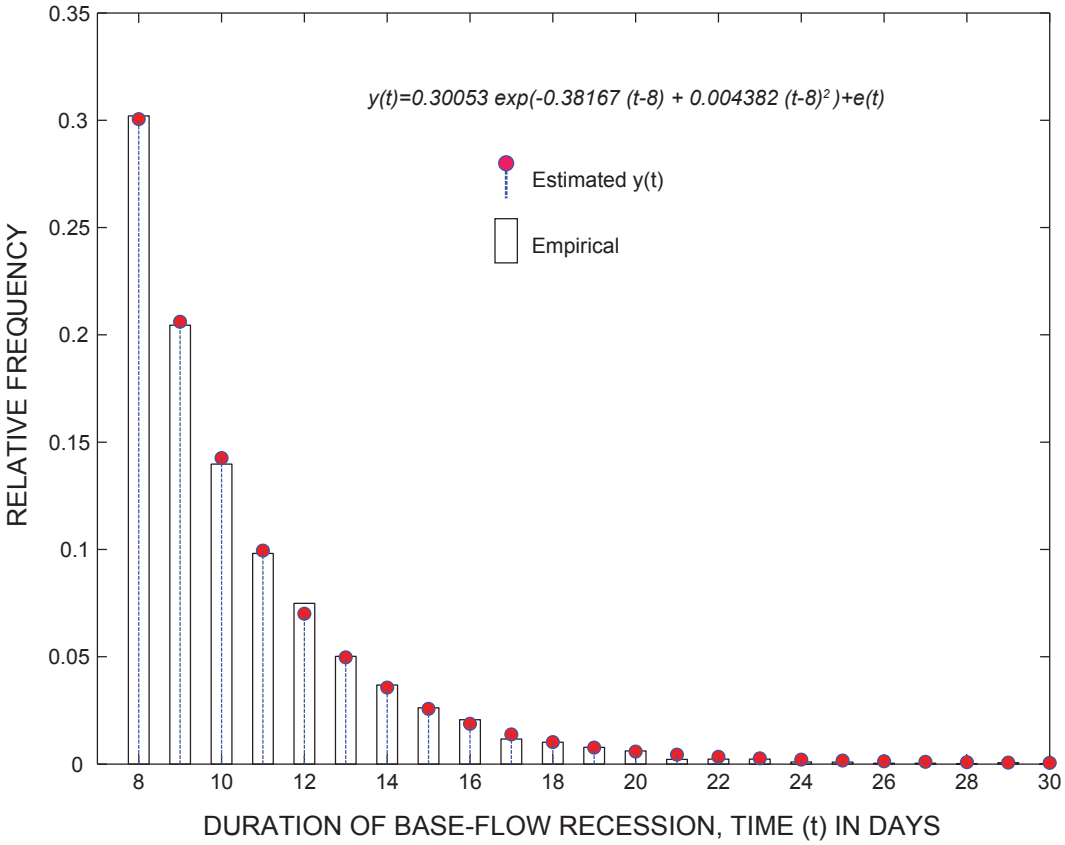


Figure 29. Relative frequency of the durations of base flow recession segments identified at index streamgages.

Two examples are provided to illustrate the ideal match between measured and recession equation estimates of root water-yield recession. Figure 30 illustrates the importance of omitting the first five daily root water-yield values from the estimation process. In particular, the magnitude of the multiplicative parameter $\boldsymbol{rY_o}$ would have been larger had the first five daily values been included in the estimation process. Also, the stepper gradient in the early period of the recession event would have been associated with a greater exponential term *tau*. Figure 31 shows that the fitted equation also can provide a close approximation to extended recession periods. The recession events selected as examples represent cases that are more consistent with the recession-equation estimates than perhaps are typical, but are not exceptional.

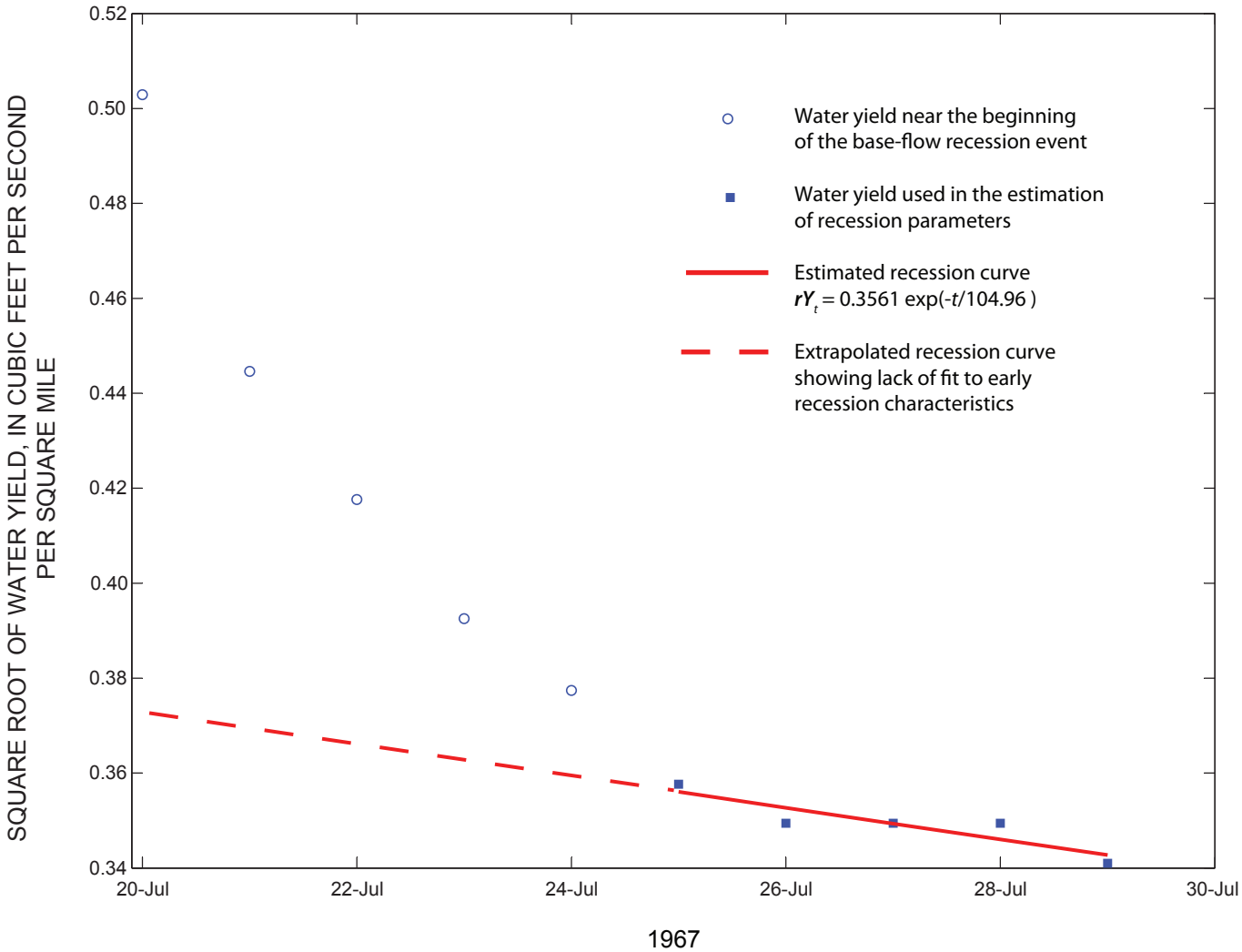


Figure 30. Base-flow recession starting July 20, 1967, at U.S. Geological Survey (USGS) streamgage 04112500 Red Cedar River at East Lansing, Mich. (*t*) is time in days since the beginning of the recession period used in parameter estimation.)

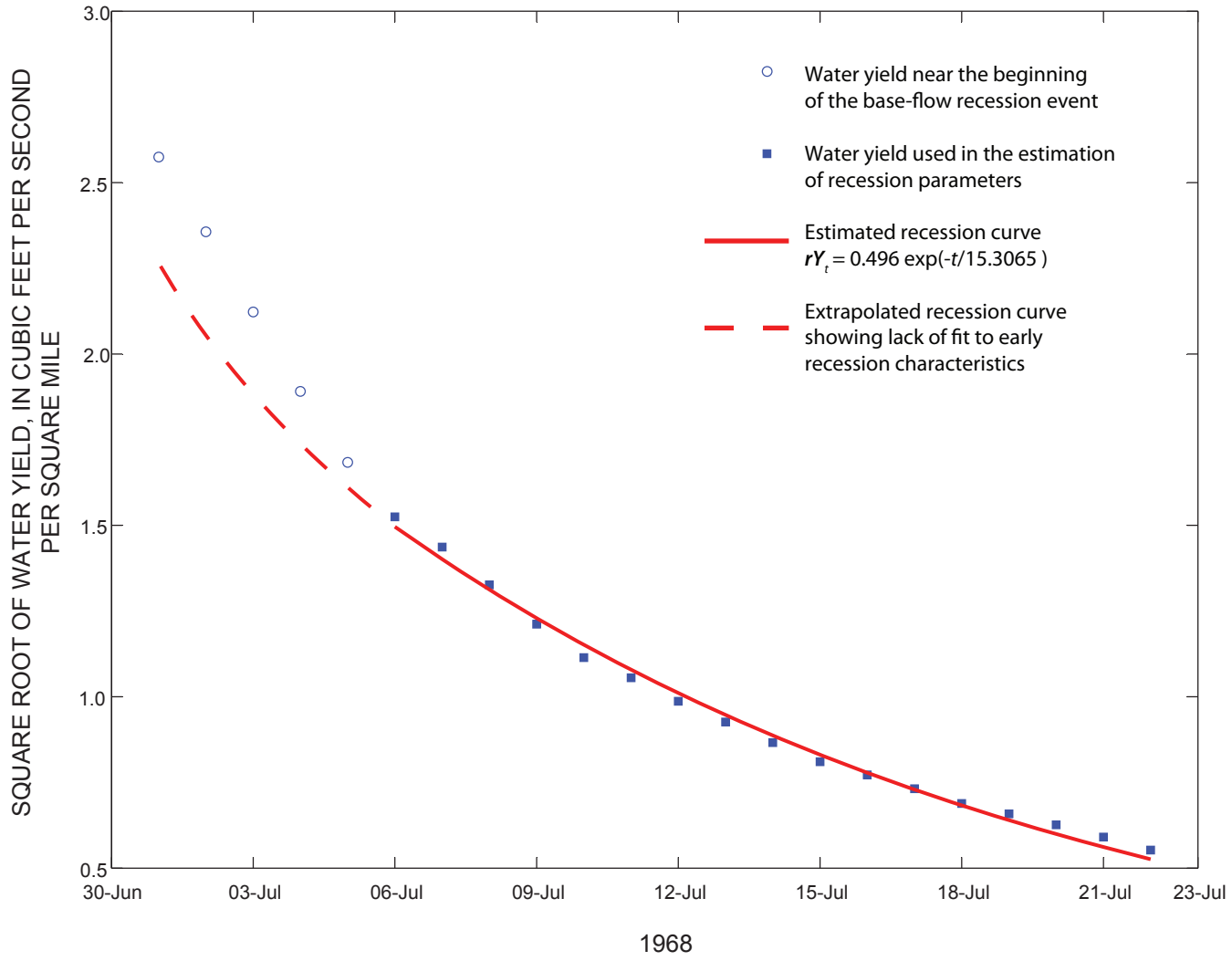


Figure 31. Base flow recession starting July 1, 1968, at the U.S. Geological Survey (USGS) streamgage 04112500 Red Cedar River at East Lansing, Mich. (*t* is time in days since the beginning of the recession used in estimating the parameters).

The medians of the two parameters for all recession segments were used to estimate the corresponding recession characteristics at each index streamgage (appendix 1, table 1–4). The distribution of median initial water yield recession mrY_o is approximately normal with mean 0.5968 and variance 0.05202 (fig. 32). The distribution of the common logarithms of the median *tau* value for each streamgage, designated as *lmtau*, is approximately normal with mean 1.612 and variance 0.09872 (fig. 33).

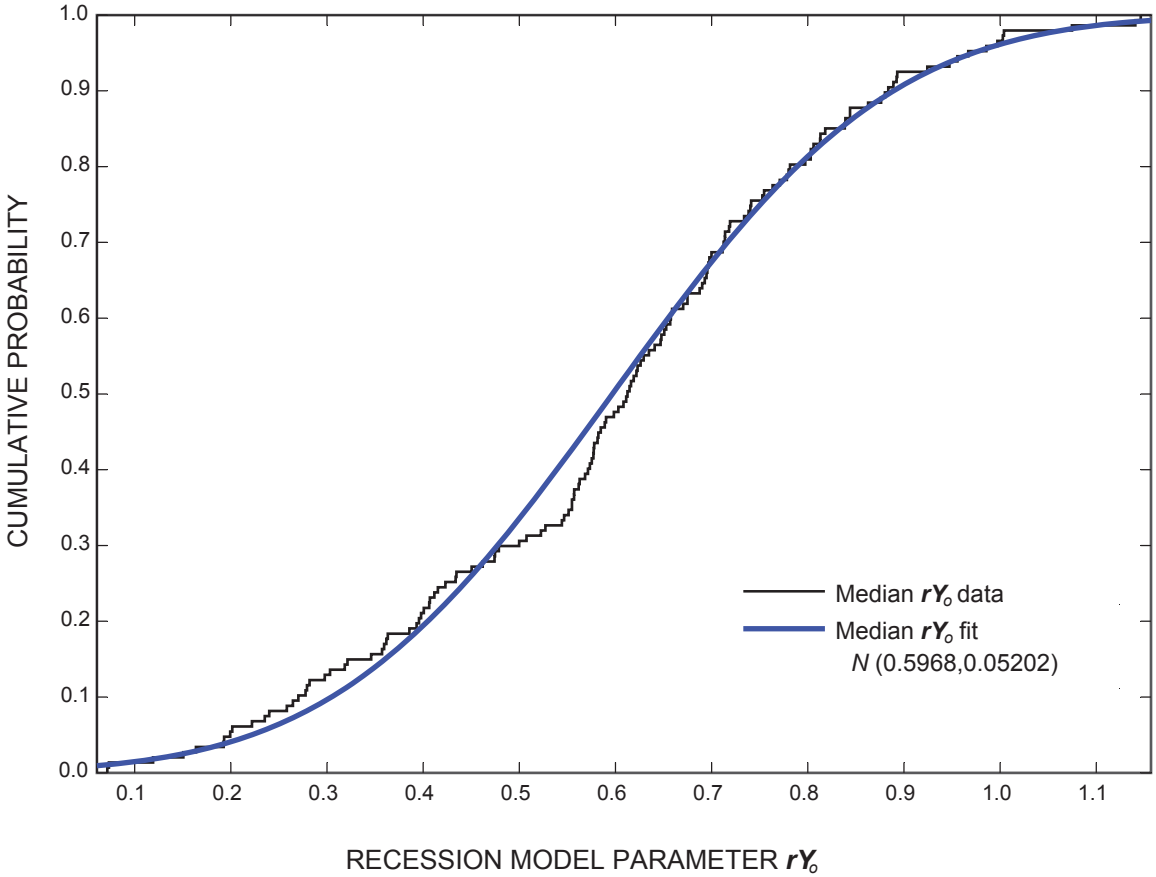


Figure 32. The cumulative distribution function of median recession parameter rY_o at index streamgages.

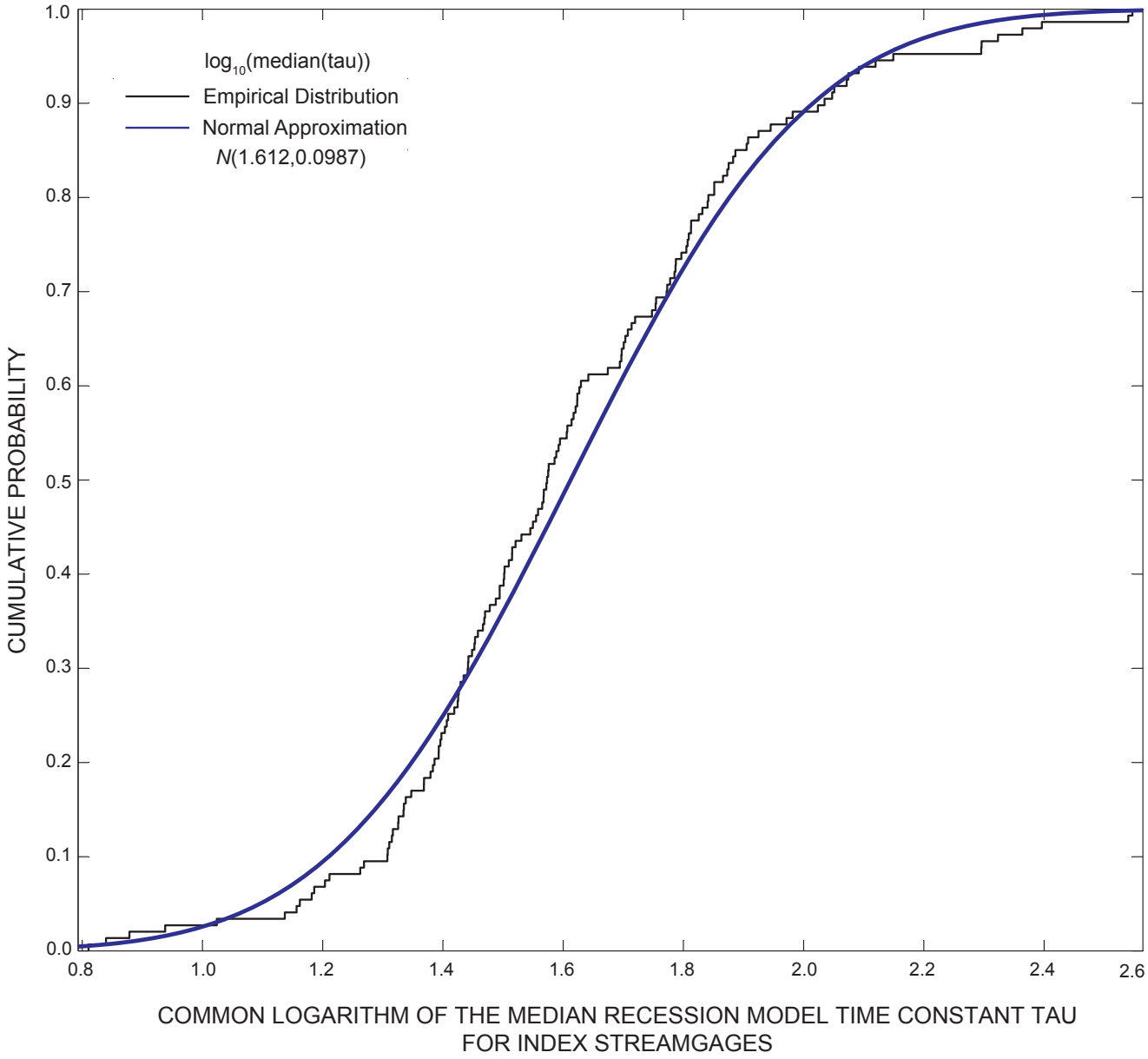


Figure 33. The cumulative distribution function of the common logarithm of the median recession parameter *tau* computed from base-flow recession events at index streamgages.

Recession Characteristics and Estimation of Index Water Yield

Hamilton and others (2008) developed an index regression model that contains six explanatory variables including measures of low and high aquifer transmissivities, *L_Trans* and *H_Trans*, respectively; percent forest covering the basin, *Forest*; normal annual precipitation for the period 1971–2000, *Precip*; and percent hydrologic soil groups A and B covering the basin, as *Soil_A* and *Soil_B*, respectively. The index regression estimate *Indx**rY50*** accounts for 70.8 percent of the variability in ***rY50*** and has an *RMSE* of 0.1238.

The utility of streamgage-specific recession parameters used as variables in an augmented form of the index regression equation was evaluated as an eight-variable regression model in this report. The variables were formed as the median of estimated $\boldsymbol{rY_o}$ values ($\boldsymbol{mrY_o}$) and common logarithms of the median recession time constants *tau* (*lmtau*) from all recession events identified at index streamgages. Results indicate that $\boldsymbol{mrY_o}$ and *lmtau* were highly significant (p-value <0.0001), and together with the original six variables, account for 97.4 percent of the variability in ***rY50*** with an *RMSE* of 0.03684 (table 3). However, parameter estimates changed and their statistical significances were lower for some of the original variables in the index regression.

To provide greater consistency with the index regression equation (Hamilton and others, 2008), a two-step least-squares form of the augmented equation is provided. The first step provides predicted values of root index water yield *Indx**rY50***, which are used together with base-flow recession parameters mrY_o and *lmtau* as explanatory variables computed from continuous streamflow data in the second step to estimate ***rY50***. Results for this estimator, identified as *AugIndx**rY50**,* indicate that second-stage estimates account for 97.3 percent of the variability in ***rY50*** with an *RMSE* of 0.03842. An analysis of variance and estimated parameters for the augmented index regression is shown in table 4. Parameters in the augmented index regression model may be biased because of the uncertainty in the explanatory variables, which are assumed to be known precisely under ordinary least squares regression. The large percentage of the total variability of ***rY50*** explained by the regression, however, mitigates some of the concern about bias in the corresponding estimate of flow. The *AugIndx**rY50*** has an *RMSE* that is 69 percent lower than the *Indx**rY50*** estimator.

An estimate of the variance of *AugIndx**rY50*** at site J^o can be computed as

$$\hat{\sigma}^2_{AugmrY50(J^o)} = s^2 \cdot \left(1 + x_{J^o} \cdot (X' \cdot X)^{-1} \cdot x_{J^o}\right), \tag{24}$$

where,

s^2 is the mean square error of the regression equation $\left(0.03842^2\right)$,

x_{J^o} is a column vector containing site-specific values of $\left[1 \;\; Indx\boldsymbol{rY}50 \;\; \boldsymbol{mrY_o} \;\; Imtau\right]$, and

$(X' \cdot X)^{-1}$ Inverse of the cross product matrix of the design matrix (table 5).

Monte Carlo analysis was used to assess the effect of uncertainty in base flow recession variables on the uncertainty of augmented index regression estimates of root index water yields *AugIndx**rY50***. For this estimator, identified as *Augm**rY50***, $\boldsymbol{mrY_o}$ and *lmtau* variables were estimated by random sampling with replacement from the set of recession parameters estimated for the identified base flow recession events at index streamgages. A set of 12 sampling intensities {1, 2,…, 7, 10, 15, 20, 50, 100} for recession events was evaluated. For each Monte Carlo trial, the medians of sampled recession parameters were used to estimate the $m\text{-}_o$ and *lmtau* variables.

Parameters estimated in the augmented index regression equation (table 4) were used to compute *Augm**rY50*** estimates for each index streamgage in each of 10,000 trials per sampling intensity. Residuals were computed as the difference between *Gage**rY50*** values and *AugIndx**rY50*** estimates. The interquartile range, the difference between the 75th and 25th percentiles of residuals, was used to provide a robust measure of the spread of the residuals.

The distribution of interquartile range for each sampling intensity was compared with the interquartile ranges of residuals for the index regression *Indx**rY50*** and augmented index regression *AugIndx**rY50*** estimates (fig. 34). Results show a monotonic decrease in the median interquartile range with increases in sampling intensity. The median interquartile range of residuals from *Augm**rY50*** estimates based on 1 sampled recession event was slightly less than the interquartile range of residuals from *Indx**rY50*** estimates, and the median interquartile range of residuals from *Augm**rY50*** estimates based on 100 sampled recession events was slightly more than the interquartile range of residuals from *AugIndx**rY50*** estimates.

Daily values from streamgages operated less than 10 years, but otherwise meeting the criteria for index streamgages, could be used instead of instantaneous flow measurements to estimate recession variables for the augmented index regression equation. Owing to the continuity of daily values throughout recession periods, data from short-term streamgages would provide a more complete description of recession characteristics than limited sets of instantaneous measurements.

Although the relation between the number of base-flow recession events used in the estimation of recession variables and estimation accuracy of *Augm**rY50*** was evaluated, the relation between the number of instantaneous measurements per base-flow recession event and the accuracy of recession parameters was not. Eng and Milly (2007) argue that a single pair of strategically timed streamflow measurements approximately halves the *RMSE* of low-flow estimates relative to that of a conventional drainage area regression relation. Presumably, this timing would be informed by real-time streamflow data from streamgages near the site of interest and weather forecasts, which could help avoid the early (5-day) period following the start of the recession period and increases in streamflow from subsequent precipitation events. Owing to the autocorrelation of streamflow values, instantaneous

Table 3. Analysis of variance table and parameter estimates for the index regression with base-flow recession parameters as variables.

{—, not applicable]

Source	Degrees of freedom	Sum of squares	Mean square	F value	Probability of great F statistic
Model	8	7.45288	0.93161	624.42	<0.0001
Error	138	.20589	.00149	—	—
Corrected Total	146	7.65877	—	—	—
Variable	**Degrees of freedom**	**Parameter estimate**	**Standard error**	**t value**	**Probability of a greater |t| statistic**
Intercept	1	-0.17634	0.05945	-2.97	0.0036
L_Trans	1	.00019954	.00017088	1.17	.2449
H_Trans	1	-.00014948	.00016943	-.88	.3792
Forest	1	-.00002788	.00019767	-.14	.8880
Precip	1	-.00149	.00185	-.80	.4223
A_Soils	1	.00041971	.0002672	1.57	.1185
D_Soils	1	.00005736	.00034072	.17	.8665
mrY_o	1	.82909	.02677	30.97	<.0001
lmtau	1	.15552	.01715	9.07	<.0001

Table 4. Analysis of variance table and parameter estimates for the augmented index regression *AugIndx**rY**50* equation.

[—, not applicable]

Source	Degrees of freedom	Sum of squares	Mean square	F value	Probability of great F statistic
Model	8	7.45510	0.93189	631.4	<0.0001
Error	138	.20367	.0014759	—	—
Corrected Total	146	7.65877	—	—	—
Variable	**Degrees of freedom**	**Parameter estimate**	**Standard error**	**t value**	**Probability of a greater |t| statistic**
Intercept	1	-0.22994	0.01698	-13.54	<0.0001
*Indx**rY**50*	6	.01082	.03145	.34	.7314
mrY_o	1	.81129	.02558	31.72	<.0001
lmtau	1	.16590	.01476	11.24	<.0001

Table 5. The inverse of the X'X matrix needed to compute prediction limits for the augmented index regression *AugIndx**rY**50* equation.

Explanatory variables in the *AugIndxrY50* regression equation			
Intercept	***IndxrY 50***	***mrY_o***	***lmtau***
0.202449	0.054157	0.023014	-0.147576
.054157	.694465	-.380471	-.119923
.023014	-.380471	.459334	-.059837
-.147576	-.119923	-.059837	.152903

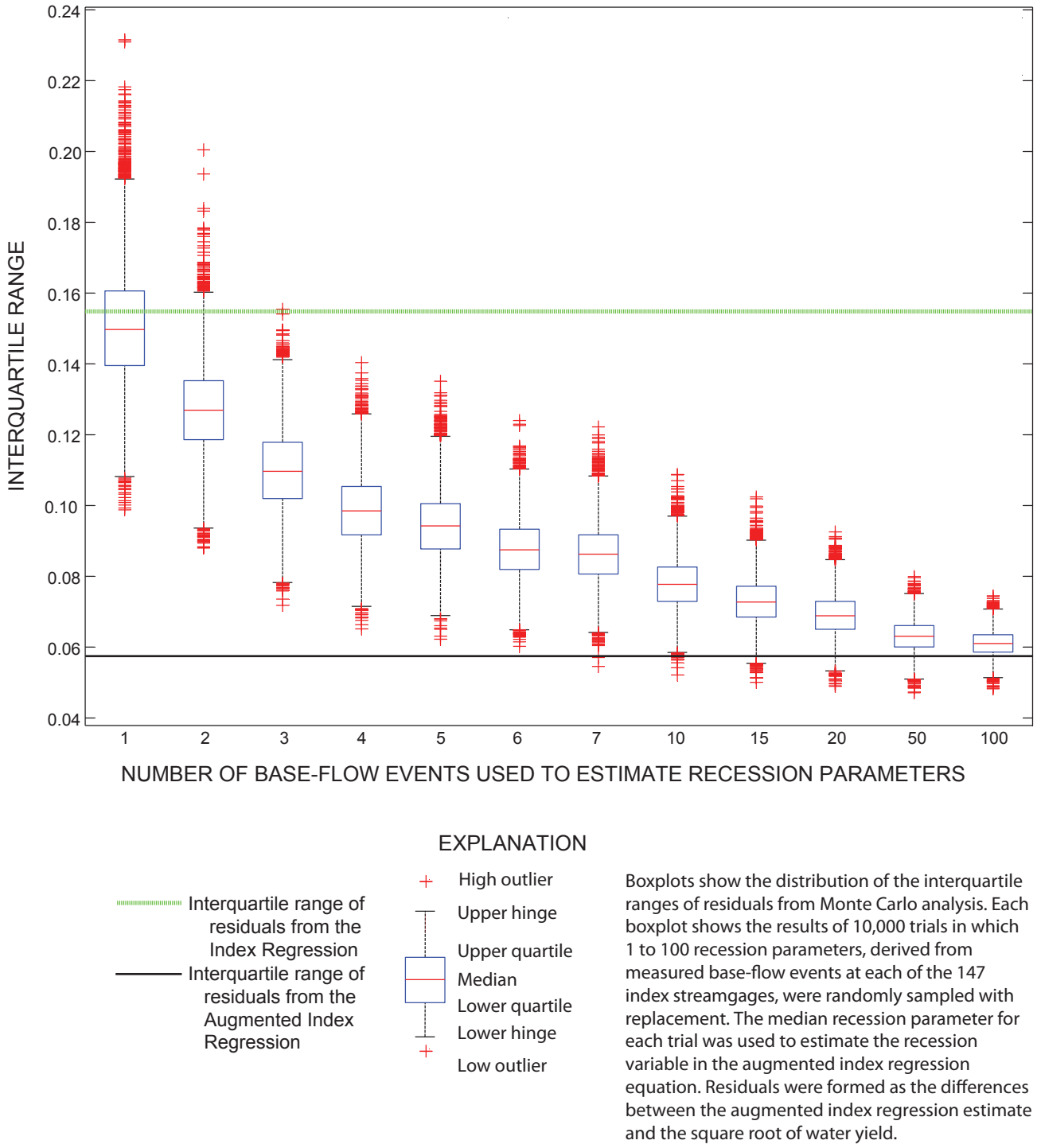


Figure 34. Boxplots showing the distribution of interquartile ranges of residuals of augmented regression estimates as a function of the number of sampled base-flow recession events.

measurements separated by more than 3 days, for example, would be more informative than measurements separated by fewer than 3 days, assuming all measurements were in the later period of the recession event. A minimum of three measurements per recession event would be needed to provide an assessment of the recession model fit or parameter uncertainty. If a sufficient number of grouped instantaneous measurements are available to reliably estimate the recession variables mrY_o and *lmtau* for the target site, the experimental *Augm**rY**50* estimator might be considered.

In regression analysis, explanatory variables like mrY_o and *lmtau* are assumed to be measured without error. As described in this report, however, mrY_o and *lmtau* are estimated from parameters simultaneously fit to a nonlinear equation from a limited set of baseflow recession events. The uncertainty associated with these variables will likely result in biased estimates of index flow. Additional analyses are needed to determine the error characteristics of the *Augm**rY**50* estimator before application to issues affecting water withdrawal.

Example calculations are provided (Appendix 2) to illustrate the application of the Discrete, Integrated, Site, and Augmented estimators to index streamgage 04112500 Red Cedar River at East Lansing, Mich. Based on 77 years of continuous streamflow data, the index flow (*Gage**Q**50*) is 38.0 ft^3/s and the root index water yield (*Gage**rY**50*) is 0.332 $\sqrt{ft^3/(s \cdot mi^2)}$. The site is in hydrologic subregion 2 and has a drainage area of 344 mi^2. The example calculation shows input data, results of intermediate and final computations. Matlab[4] code is provided to document the computational approach in Appendix 2.

Summary and Conclusions

In Michigan, index flow ***Q**50* is a streamflow characteristic defined as the minimum of median daily flows during the months of July, August, or September. Index flow is used by the state of Michigan to regulate large water withdrawals so as to prevent adverse effects on characteristic fish populations. At U.S. Geological Survey (USGS) streamgages, daily flow values for the period of gage operation can be used to compute an index flow estimate *Gage**Q**50* that is expected to converge to ***Q**50* with increasing length of record. Index flow commonly is expressed in units of cubic feet per second.

Estimates of index flow also are needed at sites where streamgage records are not available or are not sufficiently long to compute a reliable estimate. Hamilton and others (2008) developed an equation to estimate index flow *Indx**Q**50* as the product of the drainage area *DArea* contributing to flow upstream from a site, and the index (root) water yield across the contributing basin calculated by use of multiple linear regression. Water yield is expressed in units of cubic feet per second per square mile of drainage area.

For consistency with statistical assumptions underlying regression, the index regression equation estimates the square root of water yield, which is indicated by the bold "***r***" for square **r**oot preceding "***Y,***" which is used to suggest yield, as *Indx**rY**50.* Daily streamflow data from water years 1901 to 2005 for 147 selected streamgages in or near Michigan were used to compute *Gage**Q**50* and *Gage**rY**50* values for use in developing the index regression equation. Six explanatory variables were included in the equation, which included measures of aquifer transmissivity, land cover, annual precipitation, and hydrologic soil classification. Estimates from the index regression equation accounted for 70.8 percent of the variability in *Gage**rY**50,* and 94.0 percent of the variability in *Gage**Q**50*. Seven Michigan hydrologic subregions (*MHS*) were defined along watersheds to partition the state to more homogeneous units for analysis.

At some ungaged stream sites, more accurate estimates of index flow are needed than *Indx**Q**50*. In this report, information obtainable from instantaneous measurements of flow was evaluated to determine the potential for increasing the accuracy of index flow estimates. Instantaneous measurements, which are direct measurements of flow that typically require about 1 hour to complete, are made at miscellaneous sites once or more often on a random or discontinuous basis to provide better areal coverage for describing hydrologic conditions than that provided by streamgages alone. In Michigan, a total of 22,368 flow measurements have been documented at 3,692 miscellaneous sites (Marlio Lesmez, Michigan Department of Natural Resources and Environment, written commun., 2009). Although these measurements could not be used to develop the methods described in this report, many of them could be used to improve estimates of index flow at miscellaneous sites from those based on the index regression estimate.

Monte Carlo simulations were used to assess a sequence of estimators for improving the estimates of index flow with information obtainable from instantaneous flow measurements. The Monte Carlo method is a generalized numerical technique that involves defining a sampling frame for analysis, repeated random sampling of the frame, evaluating alternative methods of estimation for each sample, and assessing and comparing the results.

In this report, the sampling frame contained daily mean streamflow values for water years 1961 to 2008 for the 147 streamgages used to develop the index regression equation. Each daily flow value during the months of July, August, and September was considered a possible simulated instantaneous flow measurement for use in estimating index flow. Each streamgage, in turn, served as a target site. Each target site was evaluated for a set of 12 measurement intensities corresponding to the total number of simulated instantaneous measurements, from 1 to 100, available for analysis. Each target site J^o was evaluated for each measurement intensity by an average of 10,000 random samples of daily flow values; selected days were constrained to water yield conditions that were less than $GagerY50_{J^o} + 1.96 \cdot \hat{\sigma}_{IndxrY50,J^o}$. Otherwise, an alternate day was randomly selected to represent the simulated

[4] Matlab is a computer language for technical computation by the Mathworks, Inc.

instantaneous measurement. Other streamgages in the same hydrologic subregion, meeting the same site-specific flow criteria, were selected as auxiliary sites.

A simple linear regression was developed between the square roots of daily values of water yields, ***rY****dv*, and index water yields, ***rY****50,* at the auxiliary sites. This relation was used to estimate index water yield at the target site based on the corresponding daily water yield at the target site. The first index water yield estimator in the sequence is based on simulated discrete water yields, and symbolized as *Disc**rY**50* with corresponding index flow *Disc**Q**50*. For measurement intensities greater than 1, a weighted average of *Disc**rY***50 values was computed, where the weights are the reciprocals of their estimated variances. Although results varied among hydrologic subregions, *Disc**rY**50* estimates with 1 simulated instantaneous measurement generally had a 32.6-percent lower root mean square error (*RMSE*) than *Indx**rY**50* estimates, and *Disc**rY**50* estimates computed with 100 simulated measurements had a 66.5- percent lower *RMSE* than *Indx**rY**50* estimates. Thus, the expected discrete water yield returned by the discrete estimator is more precise than the index regression estimator for all measurement intensities investigated.

The second estimator in the sequence was the integrated discrete estimator, which is formed by combining estimates based on the index regression estimator and the discrete estimator. In particular, a weighted integrated estimate *Intg**rY**50* was formed as a weighted average of the *Indx**rY**50* estimate and *Disc**rY**50* estimates for all measurement intensities evaluated. Again, weights were the reciprocals of estimated variances. Results indicated that *Intg**rY**50* had a lower *RMSE* than *Disc**rY**50* for one or two simulated measurements. At greater measurement intensities, the *RMSE* of *Intg**rY**50* converged to the *Disc**rY**50* estimates.

Neither the *Disc**rY**50* nor the *Intg**rY**50* estimates necessarily converge to site-specific values of *Gage**rY**50* with increasing measurement intensities. Thus, even if all daily flow values used to compute *Gage**rY**50* at a particular streamgage were used with the first two estimators, the resulting estimates would not match the corresponding index water yield or flow statistic at the streamgage.

To facilitate this convergence, the third estimator in the sequence *Site**rY**50* was developed. *Site**rY**50* is a weighted average of *Intg**rY**50* and the median daily water yield $med(\boldsymbol{rY}dv_1, \boldsymbol{rY}dv_2, \ldots \boldsymbol{rY}dv_n)$ on sampled days in July, August, or September at the target site for all simulated instantaneous measurements. Again, the weights were reciprocals of corresponding estimated variances, where the variances of the median (square root) water yields were computed as a function of measurement intensities from resampling experiments. Results indicate that the *RMSE* of the *Site**rY**50* estimator with respect to *Intg**rY**50* decreases with increasing measurement intensity. At some sites, convergence of *Site**rY**50* estimates to site-specific *Gage**rY**50* values was evident. Convergence characteristics of *Site**rY**50* estimates may be improved by selecting months for instantaneous measurements that are expected to have minimum median flows more than other months. Also, convergence characteristics may improve with an improved flow-selection criterion. It may be helpful, for example, if a threshold percentage of index streamgages in the target hydrologic subregion are above and below their index flows, thus forming a mixed index-flow condition for the subregion. Real-time monitoring of daily flows at index streamgages might be used to select a day for instantaneous measurement that would be expected to meet the criterion. Further analysis would help identify the likelihood of these mixed index flow conditions for a range of threshold percentages. Additional Monte Carlo simulations may clarify how the threshold percentages affect the rate of convergence of the *Site**rY**50* estimates to ***rY****50* values.

The accuracy of the index regression can be augmented with two additional variables that characterize base-flow recession by use of a two-parameter recession model. In this report, 9,724 recession events were identified at 147 index streamgages by use of daily streamflow values. Recession parameters estimated from these events provided a basis for the augmented index regression variables that provided a basis for the *AugIndx**rY**50* estimates, which had a 66.3 percent lower *RMSE* than corresponding *Indx**rY**50* estimates. Monte Carlo simulations were used to develop the relation between the number of recession events sampled and the precision of the estimator *Augm**rY**50*, which is based only on sampled events. In general, the uncertainty of the *Augm**rY**50* estimates decreased monotonically with increased numbers of sampled recession events from slightly less than the *Indx**rY**50* estimates with 1 recession event to slightly more than the *AugIndx**rY**50* estimates with 100 events. Additional analysis of the Integrated estimator is needed before being used to estimate index flow to more fully assess the uncertainty of estimates where two of the critical explanatory variables are uncertain.

This report develops empirical estimates of variances and variance components for all estimators. These variances may be helpful in forming the weighted estimators *Disc**rY**50*, *Intg**rY**50*, and *Site**rY**50*. All variance estimates assume independence between variance components. The utility of the estimated variances was confirmed by forming nominal 95-percent confidence intervals that generally contained slightly more than 95 percent of the measurements.

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Appendix 1. Tables of streamgage attributes and flow characteristics used to assess the utility of instantaneous flow measurements for improving regression estimates of index flow in Michigan

Table 1–1. Streamgage and flow characteristics used to assess the utility of instantaneous flow measurements for improving index flow estimates.

U.S. Geological Survey streamgage number	Michigan hydrologic subregion	U.S. Geological Survey Strreamgage name	Latitude (decimal degrees)	Longitude (decimal degrees)	Drainage area (square miles) *(DArea)*	Minimum monthly median flow (cubic feet per second) (Q_{50})	Minimum monthly median yield (cubic feet per second per square mile) (Y_{50})	Month of minimum flow	Years of record	Real time flow data in 2010	Water years used to compute index flow
04001000	1	Washington Creek at Windigo, Mich.	47.92306	89.14500	13.2	2.3	0.17424	August	39	No	1965–2003
04031000	1	Black River near Bessemer, Mich.	46.51134	90.07462	200	40.0	0.20000	August	33	Yes	1955–82, 2001–05
04031500	1	Presque Isle River at Marenisco, Mich.	46.37217	89.69238	172	71.0	0.41183	August	38	No	1945–82
04032000	1	Presque Isle River near Tula, Mich.	46.54689	89.77738	264	90.0	0.34078	August	29	No	1945–73
04033000	1	Middle Branch Ontonagon River near Paulding, Mich.	46.35689	89.07736	162	100	0.61843	August	59	Yes	1943–95, 2001–05
04035000	1	East Br Ontonagon River near Mass, Mich.	46.68994	89.07347	273	134	0.49048	August	38	No	1942–79
04040000	1	Ontonagon River near Rockland, Mich	46.72077	89.20709	1,330	634	0.47530	August	64	Yes	1942–2005
04040500	1	Sturgeon River near Sidnaw, Mich.	46.58411	88.57597	169	44.0	0.26036	August	66	Yes	1913–15, 1943–2005
04041500	1	Sturgeon River near Alston, Mich.	46.72632	88.66208	343	184	0.53629	August	72	Yes	1932–40, 1943–2005
04043050	1	Trap Rock River near Lake Linden, Mich.	47.22854	88.38539	29.6	13.0	0.43919	August	39	Yes	1967–2005
04045500	1	Tahquamenon River near Paradise, Mich.	46.57501	85.26955	757	321	0.42410	August	52	Yes	1954–2005
04046000	1	Black River near Garnet, Mich.	46.11806	85.36537	33.5	10.0	0.29851	August	38	Yes	1952–78, 1995–2005
04049500	1	Manistique River at Germfask, Mich.	46.23331	85.92791	420	238	0.56721	August	33	No	1938–70
04055000	1	Manistique River near Blaney, Mich.	46.08609	86.05930	716	352	0.49183	August	33	No	1938–70
04056000	1	West Branch Manistique River near Manistique, Mich.	46.08886	86.16125	326	154	0.47312	August	19	No	1938–56
04056500	1	Manistique River near Manistique, Mich.	46.03053	86.16125	1,130	605	0.53716	August	68	Yes	1938–2005
04057510	1	Sturgeon River near Nahma Junction, Mich.	45.94302	86.70570	184	82.0	0.44662	August	39	Yes	1967–2005

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04057800	1	Middle Branch Escanaba River at Humboldt, Mich.	46.49910	87.88652	45.7	16.0	0.35011	August	47	Yes	1959–2005
04058000	1	M Br Escanaba River near Ishpeming, Mich.	46.39438	87.75847	128	43.0	0.33515	August	22	No	1954–75
04058400	1	Goose Lake Outlet near Sands Station, Mich.	46.39300	87.49375	36.3	12.0	0.33058	August	17	No	1966–82
04059000	1	Escanaba River at Cornell, Mich.	45.90857	87.21375	871	330	0.37892	August	55	Yes	1951–2005
04059500	1	Ford River near Hyde, Mich.	45.75552	87.20152	444	85.0	0.19140	August	51	No	1955–2005
04060993	1	Brule River near Florence, Wisc.	45.96079	88.31597	378	236	0.62269	August	62	Yes	1944–2005
04061500	1	Paint River at Crystal Falls, Mich.	46.10578	88.33486	600	288	0.47976	August	52	No	1945–96
04062200	1	Peshekee River near Champion, Mich.	46.55688	88.00263	132	28.0	0.21244	August	23	Yes	1961–78, 2001–05
04096015	2	Galien River near Sawyer, Mich.	41.87365	86.57502	80.8	21.0	0.25990	September	11	Yes	1995–2005
04096405	2	St. Joseph River at Burlington, Mich.	42.10282	85.04025	201	61.0	0.30348	September	43	Yes	1963–2005
04096515	2	South Branch Hog Creek near Allen, Mich.	41.94866	84.82774	48.7	7.9	0.16222	September	36	Yes	1970–2005
04096600	2	Coldwater River near Hodunk, Mich.	42.02921	85.10692	286	65.0	0.22759	September	27	No	1963–89
04096900	2	Nottawa Creek near Athens, Mich.	42.05560	85.30832	162	72.0	0.44444	September	31	No	1967–97
04097170	2	Portage River near Vicksburg, Mich.	42.11477	85.48555	68.2	30.0	0.43988	September	21	No	1946–51, 1965–79
04097540	2	Prairie River near Nottawa, Mich.	41.88838	85.40943	107	46.0	0.43152	September	43	Yes	1963–2005
04099000	2	St. Joseph River at Mottville, Mich.	41.80088	85.75610	1,880	850	0.45227	September	82	Yes	1924–2005
04101500	2	St. Joseph River at Niles, Mich.	41.82921	86.25973	3,670	1,850	0.50464	September	75	Yes	1931–2005

Table 1–1. Streamgage and flow characteristics used to assess the utility of instantaneous flow measurements for improving index flow estimates.

U.S. Geological Survey streamgage number	Michigan hydrologic subregion	U.S. Geological Survey Strreamgage name	Latitude (decimal degrees)	Longitude (decimal degrees)	Drainage area (square miles) *(DArea)*	Minimum monthly median flow (cubic feet per second) (Q_{50})	Minimum monthly median yield (cubic feet per second per square mile) (Y_{50})	Month of minimum flow	Years of record	Real time flow data in 2010	Water years used to compute index flow
04101800	2	Dowagiac River at Sumnerville, Mich.	41.91338	86.21307	252	177	0.70378	August	45	Yes	1961–2005
04102500	2	Paw Paw River at Riverside, Mich.	42.18615	86.36836	390	262	0.67197	August	54	Yes	1952–2005
04102700	2	South Branch Black River near Bangor, Mich.	42.35420	86.18753	83.5	35.0	0.41916	September	40	Yes	1966–2005
04103010	2	Kalamazoo River near Marengo, Mich.	42.26171	84.85581	270	147	0.54545	September	19	Yes	1987–2005
04104945	2	Wanadoga Creek near Battle Creek, Mich.	42.39643	85.13166	48.3	15.0	0.31056	August	11	Yes	1995–2005
04105000	2	Battle Creek at Battle Creek, Mich.	42.33199	85.15416	274	70.0	0.25547	September	72	Yes	1934–2005
04105700	2	Augusta Creek near Augusta, Mich.	42.35337	85.35389	36.8	31.0	0.84239	August	41	Yes	1965–2005
04108600	2	Rabbit River near Hopkins, Mich.	42.64225	85.72197	65.1	22.0	0.33794	September	40	Yes	1966–2005
04108801	2	Macatawa River near Zeeland, Mich.	42.77919	86.01837	66.9	4.3	0.06428	September	45	Yes	1961–2005
04110000	2	Orchard Creek at Munith, Mich.	42.39365	84.26496	47.3	5.3	0.11205	September	13	No	1944–56
04111500	2	Deer Creek near Dansville, Mich.	42.60837	84.32080	16.3	0.9	0.05521	September	52	Yes	1954–2005
04112000	2	Sloan Creek near Williamston, Mich.	42.67587	84.36386	10.4	0.2	0.02308	September	52	Yes	1954–2005
04112500	2	Red Cedar River at East Lansing, Mich.	42.72781	84.47775	344	38.0	0.11047	September	77	Yes	1902–03, 1931–2005
04114498	2	Looking Glass River near Eagle, Mich.	42.82809	84.75943	284	40.0	0.14080	September	57	Yes	1944–96, 2002–05
04115000	2	Maple River at Maple Rapids, Mich.	43.10975	84.69305	420	30.0	0.07141	September	62	Yes	1944–2005
04116500	2	Flat River at Smyrna, Mich.	43.05281	85.26474	516	222	0.42998	August	36	Yes	1951–86
04117000	2	Quaker Brook near Nashville, Mich.	42.56587	85.09361	7.8	2.8	0.35897	September	33	Yes	1954–75, 1995–2005

Table 1–1. Streamgage and flow characteristics used to assess the utility of instantaneous flow measurements for improving index flow estimates.

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04117500	2	Thornapple River near Hastings, Mich.	42.61587	85.23639	410	109	0.26553	September	61	Yes	1945–2005
04118000	2	Thornapple River near Caledonia, Mich.	42.81114	85.48335	795	281	0.35337	September	41	No	1952–81, 1984–94
04118500	2	Rogue River near Rockford, Mich.	43.08225	85.59086	257	127	0.49378	September	50	Yes	1952–82, 1988–2005
04121000	3	Muskegon River near Merritt, Mich.	44.33557	84.89003	352	115	0.32689	August	27	No	1947–73
04121300	3	Clam River at Vogel Center, Mich.	44.20057	85.05281	239	73.0	0.30506	August	40	Yes	1966–2005
04121900	3	Little Muskegon River near Morley, Mich.	43.50253	85.34254	136	72.0	0.53137	July	30	No	1967–96
04122100	3	Bear Creek near Muskegon, Mich.	43.28863	86.22284	16.7	5.0	0.29940	August	40	Yes	1966–2005
04122200	3	White River near Whitehall, Mich.	43.46418	86.23257	404	273	0.67491	August	49	Yes	1957–2005
04122500	3	Pere Marquette River at Scottville, Mich.	43.94501	86.27869	689	455	0.65999	August	67	Yes	1939–2005
04123000	3	Big Sable River near Freesoil, Mich.	44.12028	86.28008	115	101	0.87826	August	32	No	1942–73
04123500	3	Manistee River near Grayling, Mich.	44.69307	84.84726	132	170	1.28496	August	31	No	1943–73
04124000	3	Manistee River near Sherman, Mich.	44.43639	85.69868	865	856	0.98971	August	86	Yes	1903–16, 1934–2005
04124500	3	East Branch Pine River near Tustin, Mich.	44.10251	85.51728	58.9	9.9	0.16808	August	26	Yes	1952–63, 1992–2005
04125000	3	Pine River near Leroy, Mich.	44.06279	85.54894	130	51.0	0.39140	August	12	No	1952–63
04125500	3	Pine River near Hoxeyville, Mich.	44.20306	85.79951	254	230	0.90658	August	31	Yes	1952–82
04126200	3	Little Manistee River near Freesoil, Mich.	44.18362	86.16758	185	141	0.76381	August	19	No	1957–75
04127918	1	Pine River near Rudyard, Mich.	46.18585	84.59783	202	77.0	0.38138	August	34	No	1972–2005

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04127997	4	Sturgeon River at Wolverine, Mich.	45.29890	84.61114	181	170	0.93975	August	64	No	1942–2005
04128990	4	Pigeon River near Vanderbilt, Mich.	45.15668	84.46669	57.7	58.0	1.00520	July	55	Yes	1951–2005
04133501	4	Thunder Bay River at Herron Road near Bolton, Mich.	45.12446	83.64721	586	309	0.52739	September	40	Yes	1945–80, 2002–05
04135000	4	Thunder Bay River near Alpena, Mich.	45.09418	83.49970	1,240	474	0.38288	August	22	No	1901–08, 1980–93
04135500	4	Au Sable River at Grayling, Mich.	44.65974	84.71253	96.6	61.0	0.63147	August	51	No	1943–93
04135600	4	East Branch Au Sable River at Grayling, Mich.	44.66890	84.70558	71.2	34.0	0.47753	August	27	No	1958–84
04135700	4	South Branch Au Sable River near Luzerne, Mich.	44.61474	84.45557	391	134	0.34236	August	38	Yes	1967–89, 1991–2005
04136500	4	Au Sable River at Mio, Mich.	44.66001	84.13112	1,360	788	0.57894	August	54	Yes	1952–2005
04137500	4	Au Sable R near Au Sable, Mich.	44.43640	83.43386	1,740	1,120	0.64383	September	19	Yes	1987–2005
04138000	5	East Branch Au Gres River at McIvor, Mich.	44.23252	83.70082	89.9	38.0	0.42269	August	23	No	1951–73
04138500	5	Au Gres River near National City, Mich.	44.17613	83.74248	151	21.0	0.13871	August	31	No	1951–81
04139000	5	Houghton Creek near Lupton, Mich.	44.39585	84.04722	29.8	39.0	1.30872	August	23	No	1950–72
04139500	5	Rifle River at "The Ranch" near Lupton, Mich.	44.39335	84.03833	57.4	66.0	1.14983	August	22	No	1950–71
04140000	5	Prior Creek near Selkirk, Mich.	44.33502	84.06833	21.0	6.7	0.31905	August	22	No	1951–72
04140500	5	Rifle River at Selkirk, Mich.	44.31335	84.06944	116	87.0	0.74742	August	32	No	1951–82
04141000	5	South Branch Shepards Creek near Selkirk, Mich.	44.30780	84.08694	1.1	0.0	0.03636	July	27	No	1952–78
04141500	5	West Branch Rifle River near Selkirk, Mich.	44.26113	84.10916	65.4	32.0	0.48930	July	12	No	1952–63

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04142000	5	Rifle River near Sterling, Mich.	44.07252	84.01999	333	159	0.47791	August	69	Yes	1937–2005
04143900	5	Shiawassee River at Linden, Mich.	42.81586	83.80190	81.9	22.0	0.26862	August	30	No	1968–94, 2001–03
04144000	5	Shiawassee River at Byron, Mich.	42.82364	83.94579	363	71.0	0.19543	September	36	No	1948–83
04144500	5	Shiawassee River at Owosso, Mich.	43.01503	84.18108	530	86.0	0.16214	September	75	Yes	1931–2005
04145500	5	Bad River near Brant, Mich.	43.29669	84.22915	89.9	0.6	0.00667	August	11	No	1949–59
04146000	5	Farmers Creek near Lapeer, Mich.	43.04475	83.33717	51.1	5.1	0.09980	August	73	Yes	1933–2005
04146063	5	South Branch Flint River near Columbiaville, Mich.	43.15947	83.35078	211	48.5	0.23029	August	26	Yes	1980–2005
04147500	5	Flint River near Otisville, Mich.	43.11114	83.51940	526	109	0.20715	August	52	Yes	1953–89, 1991–2005
04147990	5	Butternut Creek near Genesee, Mich.	43.13586	83.59912	34.8	3.6	0.10345	August	14	No	1970–83
04148140	5	Kearsley Creek near Davison, Mich.	43.03364	83.58134	99.7	13.0	0.13039	August	40	Yes	1966–2005
04148160	5	Gilkey Creek near Flint, Mich.	43.02419	83.62551	6.9	0.2	0.02319	August	14	No	1970–83
04148200	5	Swartz Creek near Holly, Mich.	42.82753	83.62828	12.1	1.5	0.12397	September	20	No	1956–75
04148300	5	Swartz Creek at Flint, Mich.	42.98781	83.73246	114	5.6	0.04904	August	14	Yes	1970–83
04148440	5	Thread Creek near Flint, Mich.	42.97503	83.63579	54.4	4.7	0.08640	August	14	Yes	1970–83
04148500	5	Flint River near Flint, Mich.	43.03892	83.77163	960	171	0.17805	August	73	Yes	1933–2005
04150000	5	South Branch Cass River near Cass City, Mich.	43.56696	83.11189	239	5.0	0.02090	September	32	No	1949–80
04150500	5	Cass River at Cass City, Mich.	43.58419	83.17606	363	11.0	0.03034	September	55	Yes	1948–97, 2001–2005
04151500	5	Cass River at Frankenmuth, Mich.	43.32780	83.74802	842	64.0	0.07597	September	69	Yes	1935–36, 1939–2005

Table 1–1. Streamgage and flow characteristics used to assess the utility of instantaneous flow measurements for improving index flow estimates.

U.S. Geological Survey streamgage number	Michigan hydrologic subregion	U.S. Geological Survey Strreamgage name	Latitude (decimal degrees)	Longitude (decimal degrees)	Drainage area (square miles) *(DArea)*	Minimum monthly median flow (cubic feet per second) (Q_{50})	Minimum monthly median yield (cubic feet per second per square mile) (Y_{50})	Month of minimum flow	Years of record	Real time flow data in 2010	Water years used to compute index flow
04152238	5	South Branch Tobacco River near Beaverton, Mich.	43.86697	84.54529	152	63.0	0.41366	September	19	Yes	1987–2005
04153500	5	Salt River near North Bradley, Mich.	43.70281	84.47056	145	8.5	0.05874	August	38	No	1934–71
04154000	5	Chippewa River near Mount Pleasant, Mich.	43.62558	84.70779	409	150	0.36702	August	73	Yes	1933–2005
04155000	5	Pine River at Alma, Mich.	43.37948	84.65556	309	80.0	0.25882	August	75	Yes	1931–2005
04157500	5	State Drain near Sebewaing, Mich.	43.71196	83.42774	67.3	0	0.00000	August	15	No	1940–54
04158000	5	Columbia Drain near Sebewaing, Mich.	43.72724	83.39607	33.9	0	0.00000	August	18	No	1940–54, 1988–90
04158500	5	Pigeon River near Owendale, Mich.	43.76363	83.24606	53.3	3.0	0.05629	September	30	No	1953–82
04159492	6	Black River near Jeddo, Mich.	43.15253	82.62409	479	22.0	0.04589	September	62	Yes	1944–2005
04159900	6	Mill Creek near Avoca, Mich.	43.05447	82.73465	169	6.8	0.04033	September	31	Yes	1963–75, 1988–2005
04160000	6	Mill Creek near Abbottsford, Mich.	43.04503	82.61381	184	8.5	0.04620	August	18	No	1947–64
04160050	6	Black River near Port Huron, Mich.	42.99003	82.53770	683	19.0	0.02783	September	11	No	1933–43
04160570	6	North Branch Belle River at Imlay City, Mich.	43.03031	83.06716	16.1	2.3	0.14286	August	36	No	1966–2001
04160600	6	Belle River at Memphis, Mich.	42.90086	82.76909	151	13.0	0.08587	September	43	Yes	1963–2005
04160800	6	Sashabaw Creek near Drayton Plains, Mich.	42.72003	83.35355	21.0	2.2	0.10476	September	46	Yes	1960–2005
04160900	6	Clinton River near Drayton Plains, Mich.	42.66031	83.39022	78.5	14.0	0.17834	August	46	Yes	1960–2005
04161000	6	Clinton River at Auburn Hills, Mich.	42.63337	83.22438	123	44	0.35685	August	34	Yes	1935–38, 1940, 1957–82, 2001–02, 2004–05

Table 1–1. Streamgage and flow characteristics used to assess the utility of instantaneous flow measurements for improving index flow estimates.

U.S. Geological Survey streamgage number	Michigan hydrologic subregion	U.S. Geological Survey Strreamgage name	Latitude (decimal degrees)	Longitude (decimal degrees)	Drainage area (square miles) *(DArea)*	Minimum monthly median flow (cubic feet per second) (Q_{50})	Minimum monthly median yield (cubic feet per second per square mile) (Y_{50})	Month of minimum flow	Years of record	Real time flow data in 2010	Water years used to compute index flow
04161100	6	Galloway Creek near Auburn Heights, Mich.	42.66725	83.20049	17.4	1.6	0.09195	August	32	No	1960–91
04161500	6	Paint Creek near Lake Orion, Mich.	42.76753	83.21994	39.8	9.0	0.22613	August	23	No	1956–75, 1989–91
04161540	6	Paint Creek at Rochester, Mich.	42.68837	83.14299	71.8	21.0	0.29248	August	46	Yes	1960–2005
04161580	6	Stony Creek near Romeo, Mich.	42.80086	83.09021	23.8	3.9	0.16387	August	41	Yes	1965–2005
04161800	6	Stony Creek near Washington, Mich.	42.71531	83.09188	69.1	13.0	0.18813	August	48	Yes	1958–2005
04163400	6	Plum Brook at Utica, Mich.	42.60142	83.07409	16.6	2.8	0.16867	August	40	Yes	1965–98, 2000–05
04163500	6	Plum Brook near Utica, Mich.	42.58365	83.03048	23.8	0.7	0.02857	July	13	Yes	1954–66
04164100	6	East Pond Creek at Romeo, Mich.	42.82253	83.02021	20.8	4.9	0.23558	August	47	Yes	1959–2005
04164300	6	East Branch Coon Creek at Armada, Mich.	42.84586	82.88493	12.8	0.2	0.01641	August	47	Yes	1959–2005
04164500	6	North Branch Clinton R near Mount Clemens, Mich.	42.62920	82.88881	198	12.0	0.06073	September	59	Yes	1947–2005
04164800	6	Middle Branch Clinton River at Macomb, Mich.	42.70642	82.95909	41.2	3.5	0.08495	September	19	No	1963–68, 1970–82
04166000	6	River Rouge at Birmingham, Mich.	42.54587	83.22354	36.7	6.2	0.16894	September	56	Yes	1950–2005
04166200	6	Evans Ditch at Southfield, Mich.	42.45781	83.26743	10.2	1.9	0.18627	September	48	No	1958–2005
04166300	6	Upper River Rouge at Farmington, Mich.	42.46448	83.36966	17.6	4.1	0.23295	September	48	Yes	1958–2005
04169500	7	Huron River at Commerce, Mich.	42.59031	83.48466	49.9	12.0	0.24048	August	30	No	1946–75
04170000	7	Huron River at Milford, Mich.	42.57892	83.62661	139	46.0	0.33141	August	57	Yes	1949–2005
04170500	7	Huron River near New Hudson, Mich.	42.51253	83.67633	155	54.0	0.34771	August	57	Yes	1949–2005

Table 1–1. Streamgage and flow characteristics used to assess the utility of instantaneous flow measurements for improving index flow estimates.

U.S. Geological Survey streamgage number	Michigan hydrologic subregion	U.S. Geological Survey Strreamgage name	Latitude (decimal degrees)	Longitude (decimal degrees)	Drainage area (square miles) *(DArea)*	Minimum monthly median flow (cubic feet per second) (Q_{50})	Minimum monthly median yield (cubic feet per second per square mile) (Y_{50})	Month of minimum flow	Years of record	Real time flow data in 2010	Water years used to compute index flow
04171500	7	South Ore Creek near Brighton, Mich.	42.49781	83.80244	33.3	7.4	0.22222	September	18	No	1951–68
04172000	7	Huron River near Hamburg, Mich.	42.46531	83.79994	320	103	0.32177	September	54	Yes	1952–2005
04173000	7	Huron River near Dexter, Mich.	42.38615	83.91106	538	120	0.22326	August	29	No	1946–72, 1976–77
04173500	7	Mill Creek near Dexter, Mich.	42.30004	83.89856	131	23.0	0.17598	September	42	Yes	1952–82, 1995–2005
04174500	7	Huron River at Ann Arbor, Mich.	42.28615	83.73327	747	147	0.19684	August	91	Yes	1915–2005
04174800	7	Huron River at Ypsilanti, Mich.	42.24921	83.61244	817	235	0.28750	August	16	No	1974–84, 1990–94
04175600	7	River Raisin near Manchester, Mich.	42.16809	84.07606	128	32.0	0.25059	August	33	Yes	1970–81, 1985–2005
04175700	7	River Raisin near Tecumseh, Mich.	41.94310	83.94578	266	62.5	0.23532	August	24	No	1957–80
04176000	7	River Raisin near Adrian, Mich.	41.90421	83.98050	460	98.5	0.21422	September	46	Yes	1954–78, 1985–2005
04176605	7	Otter Creek at La Salle, Mich.	41.86699	83.45354	63.7	1.7	0.02669	September	18	Yes	1988–2005
04184500	7	Bean Creek at Powers, Ohio	41.67755	84.23217	205	19.0	0.09255	September	65	Yes	1941–2005

Table 1–2. Standard deviation of the median water yield as a function of the number of simulated instantaneous measurements by streamgage

Michigan hydro-logic subregion	U.S. Geological Survey Streamgage number	Number of eligible days	Number of simulated instantaneous measurements											
			1	2	3	4	5	6	7	10	15	20	50	100
1	04001000	1122	0.16103	0.11380	0.11810	0.09615	0.09782	0.08441	0.08600	0.07107	0.06384	0.05530	0.03802	0.02751
1	04031000	816	0.14543	0.10254	0.09959	0.08120	0.07943	0.06842	0.06775	0.05471	0.04714	0.04022	0.02673	0.01960
1	04031500	463	0.14527	0.10287	0.10414	0.08493	0.08459	0.07323	0.07288	0.05909	0.05037	0.04262	0.02720	0.01934
1	04032000	307	0.14462	0.10238	0.10373	0.08467	0.08453	0.07285	0.07292	0.05936	0.05155	0.04412	0.02904	0.02094
1	04033000	1,191	0.09455	0.06682	0.06508	0.05307	0.05244	0.04550	0.04531	0.03717	0.03262	0.02800	0.01883	0.01377
1	04035000	551	0.08708	0.06162	0.05797	0.04696	0.04541	0.03915	0.03839	0.03076	0.02606	0.02206	0.01441	0.01059
1	04040000	1,110	0.11433	0.08085	0.07763	0.06335	0.06191	0.05354	0.05309	0.04300	0.03707	0.03157	0.02043	0.01415
1	04040500	909	0.18019	0.12743	0.13190	0.10746	0.10905	0.09417	0.09470	0.07752	0.06716	0.05723	0.03687	0.02642
1	04041500	991	0.19973	0.14136	0.12856	0.10389	0.09573	0.08214	0.07850	0.06242	0.05372	0.04607	0.03056	0.02233
1	04043050	1,146	0.10181	0.07195	0.06745	0.05483	0.05416	0.04665	0.04683	0.03811	0.03398	0.02907	0.02045	0.01589
1	04045500	1,275	0.11789	0.08339	0.08142	0.06617	0.06459	0.05580	0.05457	0.04349	0.03608	0.02998	0.01774	0.01199
1	04046000	910	0.11472	0.08131	0.07706	0.06226	0.05891	0.05042	0.04785	0.03673	0.02841	0.02261	0.01192	0.00759
1	04049500	297	0.08485	0.05987	0.05703	0.04643	0.04507	0.03890	0.03829	0.03079	0.02611	0.02208	0.01420	0.01030
1	04055000	281	0.09901	0.06993	0.06632	0.05374	0.05119	0.04413	0.04271	0.03376	0.02793	0.02340	0.01452	0.01020
1	04056500	1,302	0.09808	0.06939	0.06684	0.05439	0.05337	0.04613	0.04588	0.03749	0.03268	0.02811	0.01882	0.01365
1	04057510	914	0.12556	0.08892	0.09097	0.07415	0.07486	0.06468	0.06514	0.05335	0.04681	0.04006	0.02635	0.01895
1	04057800	973	0.15033	0.10631	0.10739	0.08735	0.08628	0.07449	0.07337	0.05880	0.04946	0.04126	0.02555	0.01814
1	04058000	332	0.14386	0.10171	0.10734	0.08729	0.08942	0.07722	0.07824	0.06445	0.05653	0.04832	0.03147	0.02226
1	04058400	462	0.12003	0.08489	0.08443	0.06863	0.06878	0.05947	0.05988	0.04908	0.04352	0.03734	0.02536	0.01891
1	04059000	1,234	0.12208	0.08633	0.08646	0.07054	0.07051	0.06093	0.06118	0.05003	0.04391	0.03772	0.02538	0.01854
1	04059500	1,282	0.14992	0.10614	0.10418	0.08479	0.08374	0.07237	0.07196	0.05847	0.05067	0.04308	0.02761	0.01911
1	04060993	1,378	0.09325	0.06585	0.06578	0.05368	0.05334	0.04613	0.04587	0.03737	0.03193	0.02698	0.01686	0.01179
1	04061500	997	0.11179	0.07910	0.07954	0.06476	0.06529	0.05640	0.05702	0.04697	0.04196	0.03641	0.02543	0.01889
1	04062200	571	0.18298	0.12927	0.13093	0.10661	0.10679	0.09235	0.09265	0.07578	0.06622	0.05661	0.03764	0.02726
2	04096015	396	0.09951	0.07031	0.06656	0.05417	0.05349	0.04621	0.04619	0.03777	0.03302	0.02793	0.01721	0.01145
2	04096405	1,055	0.14676	0.10377	0.09900	0.08041	0.07750	0.06697	0.06542	0.05264	0.04501	0.03833	0.02548	0.01874
2	04096515	1,020	0.15132	0.10711	0.10442	0.08479	0.08156	0.07008	0.06763	0.05317	0.04272	0.03485	0.01973	0.01308
2	04096600	653	0.15051	0.10642	0.10092	0.08153	0.07725	0.06625	0.06304	0.04840	0.03798	0.03104	0.01913	0.01401
2	04096900	770	0.11789	0.08339	0.08227	0.06709	0.06627	0.05735	0.05685	0.04606	0.03967	0.03366	0.02129	0.01465
2	04097170	379	0.08428	0.05955	0.05795	0.04714	0.04860	0.04195	0.04336	0.03619	0.03288	0.02853	0.02023	0.01580
2	04097540	1,079	0.12654	0.08958	0.08338	0.06774	0.06500	0.05610	0.05485	0.04402	0.03750	0.03177	0.02062	0.01497

Table 1–2. Standard deviation of the median water yield as a function of the number of simulated instantaneous measurements by streamgage

Michigan hydro-logic subregion	U.S. Geological Survey Streamgage number	Number of eligible days	Number of simulated instantaneous measurements											
			1	2	3	4	5	6	7	10	15	20	50	100
2	04099000	1,145	0.12197	0.08631	0.08240	0.06734	0.06556	0.05671	0.05574	0.04474	0.03752	0.03152	0.01944	0.01352
2	04101500	1,218	0.11560	0.08154	0.07918	0.06440	0.06292	0.05446	0.05357	0.04300	0.03645	0.03069	0.01912	0.01304
2	04101800	1,292	0.09871	0.06985	0.06865	0.05608	0.05501	0.04770	0.04701	0.03800	0.03234	0.02746	0.01759	0.01247
2	04102500	1,341	0.09024	0.06381	0.06094	0.04975	0.04841	0.04187	0.04123	0.03325	0.02801	0.02360	0.01442	0.00972
2	04102700	1,116	0.10376	0.07337	0.06785	0.05500	0.05218	0.04489	0.04348	0.03429	0.02847	0.02367	0.01493	0.01097
2	04103010	622	0.08884	0.06284	0.06157	0.05007	0.04891	0.04244	0.04171	0.03366	0.02887	0.02459	0.01657	0.01237
2	04104945	397	0.10355	0.07337	0.07222	0.05880	0.05871	0.05077	0.05097	0.04143	0.03627	0.03083	0.02065	0.01543
2	04105000	1,309	0.12277	0.08664	0.08411	0.06845	0.06627	0.05706	0.05578	0.04415	0.03642	0.03018	0.01802	0.01249
2	04105700	1,109	0.10968	0.07759	0.08089	0.06585	0.06720	0.05823	0.05905	0.04878	0.04316	0.03710	0.02508	0.01837
2	04108600	1,009	0.12366	0.08744	0.08472	0.06901	0.06734	0.05811	0.05705	0.04581	0.03873	0.03270	0.02144	0.01605
2	04108801	1,292	0.12244	0.08667	0.07468	0.05956	0.05337	0.04542	0.04234	0.03245	0.02632	0.02206	0.01419	0.01021
2	04111500	1,341	0.13989	0.09893	0.09334	0.07548	0.07210	0.06205	0.06016	0.04744	0.03936	0.03285	0.02018	0.01418
2	04112000	1,351	0.12837	0.09076	0.08382	0.06763	0.06380	0.05447	0.05233	0.04068	0.03279	0.02696	0.01607	0.01119
2	04112500	1,350	0.12967	0.09161	0.08857	0.07199	0.06981	0.06021	0.05915	0.04707	0.03983	0.03367	0.02174	0.01546
2	04114498	1,209	0.11465	0.08127	0.07478	0.06084	0.05741	0.04944	0.04765	0.03740	0.03065	0.02550	0.01575	0.01127
2	04115000	1,307	0.11841	0.08374	0.07521	0.06068	0.05802	0.05000	0.04893	0.03905	0.03305	0.02779	0.01718	0.01184
2	04116500	748	0.09718	0.06864	0.06542	0.05325	0.05217	0.04490	0.04445	0.03600	0.03086	0.02620	0.01698	0.01228
2	04117000	742	0.11578	0.08189	0.07841	0.06377	0.06211	0.05363	0.05288	0.04251	0.03632	0.03080	0.02033	0.01465
2	04117500	1,302	0.11602	0.08179	0.07868	0.06402	0.06238	0.05377	0.05289	0.04241	0.03568	0.02989	0.01823	0.01243
2	04118000	836	0.12052	0.08539	0.08415	0.06854	0.06899	0.05973	0.06026	0.04956	0.04365	0.03747	0.02442	0.01729
2	04118500	1,099	0.10066	0.07130	0.06946	0.05662	0.05526	0.04783	0.04697	0.03768	0.03175	0.02665	0.01633	0.01129
3	04121000	337	0.11171	0.07914	0.06802	0.05488	0.04837	0.04129	0.03759	0.02803	0.02153	0.01776	0.01129	0.00805
3	04121300	1,324	0.08149	0.05746	0.04975	0.04026	0.03691	0.03164	0.03009	0.02337	0.01896	0.01581	0.00999	0.00725
3	04121900	817	0.09505	0.06714	0.06365	0.05180	0.04984	0.04301	0.04185	0.03331	0.02791	0.02343	0.01505	0.01078
3	04122100	1,126	0.11879	0.08405	0.07623	0.06194	0.05816	0.05021	0.04838	0.03830	0.03179	0.02667	0.01639	0.01149
3	04122200	1,413	0.08082	0.05718	0.05586	0.04551	0.04426	0.03832	0.03769	0.03031	0.02546	0.02146	0.01329	0.00913
3	04122500	1,432	0.07383	0.05219	0.05283	0.04304	0.04304	0.03721	0.03720	0.03031	0.02629	0.02254	0.01492	0.01067
3	04123000	396	0.04437	0.03142	0.03046	0.02478	0.02446	0.02114	0.02097	0.01693	0.01436	0.01204	0.00725	0.00504
3	04123500	378	0.03323	0.02350	0.02401	0.01958	0.01982	0.01713	0.01732	0.01423	0.01252	0.01070	0.00700	0.00513
3	04124000	1,451	0.04759	0.03368	0.03135	0.02560	0.02440	0.02112	0.02052	0.01645	0.01390	0.01179	0.00764	0.00544
3	04124500	584	0.12003	0.08485	0.08478	0.06864	0.06762	0.05798	0.05740	0.04570	0.03873	0.03247	0.02110	0.01582

Table 1–2. Standard deviation of the median water yield as a function of the number of simulated instantaneous measurements by streamgage

Michigan hydro-logic subregion	U.S. Geological Survey Streamgage number	Number of eligible days	Number of simulated instantaneous measurements											
			1	2	3	4	5	6	7	10	15	20	50	100
3	04125000	93	0.04566	0.03225	0.02324	0.01856	0.01558	0.01323	0.01214	0.00923	0.00764	0.00640	0.00451	0.00361
3	04125500	659	0.04542	0.03210	0.03084	0.02507	0.02451	0.02117	0.02086	0.01681	0.01427	0.01203	0.00763	0.00546
3	04126200	452	0.05010	0.03541	0.03451	0.02806	0.02764	0.02384	0.02369	0.01901	0.01609	0.01347	0.00817	0.00574
1	04127918	1,031	0.10449	0.07386	0.06980	0.05638	0.05414	0.04670	0.04551	0.03630	0.03041	0.02560	0.01636	0.01177
4	04127997	1,349	0.05926	0.04195	0.04198	0.03432	0.03430	0.02970	0.02971	0.02426	0.02099	0.01788	0.01140	0.00795
4	04128990	1,313	0.06907	0.04892	0.04912	0.04001	0.03980	0.03443	0.03422	0.02774	0.02364	0.02002	0.01290	0.00949
4	04133501	675	0.10989	0.07757	0.07566	0.06161	0.06072	0.05256	0.05187	0.04176	0.03515	0.02956	0.01883	0.01354
4	04135000	356	0.15431	0.10933	0.09876	0.07983	0.07457	0.06402	0.06194	0.04919	0.04170	0.03511	0.02110	0.01360
4	04135500	1,011	0.06888	0.04860	0.04407	0.03588	0.03419	0.02943	0.02865	0.02283	0.01904	0.01590	0.00973	0.00680
4	04135600	690	0.08536	0.06032	0.05415	0.04385	0.04122	0.03559	0.03438	0.02726	0.02299	0.01942	0.01289	0.00972
4	04135700	1,270	0.07035	0.04978	0.04107	0.03331	0.03090	0.02663	0.02570	0.02035	0.01698	0.01429	0.00884	0.00619
4	04136500	1,477	0.06465	0.04572	0.03820	0.03102	0.02907	0.02512	0.02430	0.01936	0.01622	0.01369	0.00877	0.00627
4	04137500	644	0.08005	0.05666	0.05496	0.04461	0.04376	0.03781	0.03720	0.03008	0.02557	0.02163	0.01409	0.01042
5	04138000	387	0.08192	0.05785	0.05045	0.04085	0.03745	0.03212	0.03052	0.02376	0.01959	0.01638	0.01060	0.00799
5	04138500	624	0.09124	0.06454	0.05456	0.04410	0.04097	0.03514	0.03406	0.02714	0.02315	0.01960	0.01263	0.00878
5	04139000	340	0.03282	0.02325	0.02218	0.01801	0.01772	0.01514	0.01532	0.01234	0.01134	0.00964	0.00756	0.00696
5	04139500	323	0.04205	0.02982	0.02906	0.02369	0.02328	0.02007	0.01992	0.01618	0.01429	0.01230	0.00909	0.00753
5	04140000	367	0.07156	0.05060	0.04385	0.03533	0.03264	0.02800	0.02674	0.02106	0.01722	0.01438	0.00882	0.00622
5	04140500	657	0.06574	0.04647	0.04427	0.03606	0.03483	0.03013	0.02946	0.02356	0.01996	0.01680	0.01095	0.00810
5	04141000	550	0.13429	0.09513	0.08820	0.07146	0.06709	0.05751	0.05492	0.04246	0.03438	0.02805	0.01675	0.01182
5	04141500	92	0.06246	0.04409	0.04104	0.03312	0.03237	0.02763	0.02728	0.02164	0.01833	0.01499	0.00905	0.00657
5	04142000	1,436	0.07367	0.05205	0.04711	0.03814	0.03639	0.03141	0.03065	0.02457	0.02102	0.01788	0.01176	0.00843
5	04143900	701	0.14420	0.10202	0.09542	0.07761	0.07482	0.06448	0.06312	0.05033	0.04216	0.03480	0.02014	0.01420
5	04144000	657	0.12899	0.09133	0.08959	0.07287	0.07208	0.06246	0.06222	0.05055	0.04370	0.03718	0.02402	0.01664
5	04144500	1,337	0.13964	0.09856	0.10060	0.08208	0.08235	0.07100	0.07139	0.05823	0.05072	0.04337	0.02793	0.01941
5	04146000	1,435	0.13954	0.09845	0.09693	0.07869	0.07759	0.06712	0.06637	0.05388	0.04655	0.03969	0.02609	0.01883
5	04146063	813	0.12602	0.08925	0.08801	0.07151	0.07075	0.06112	0.06069	0.04910	0.04203	0.03563	0.02246	0.01541
5	04147500	1,367	0.10391	0.07361	0.06042	0.04903	0.04548	0.03923	0.03805	0.03057	0.02626	0.02241	0.01447	0.00987
5	04147990	421	0.08399	0.05917	0.04745	0.03844	0.03577	0.03095	0.03030	0.02449	0.02116	0.01799	0.01151	0.00798
5	04148140	1,283	0.12471	0.08824	0.08269	0.06708	0.06457	0.05575	0.05460	0.04367	0.03701	0.03111	0.01946	0.01358
5	04148160	400	0.12332	0.08732	0.07962	0.06443	0.06116	0.05263	0.05077	0.04025	0.03324	0.02766	0.01715	0.01235

Table 1–2. Standard deviation of the median water yield as a function of the number of simulated instantaneous measurements by streamgage

Michigan hydro-logic subregion	U.S. Geological Survey Streamgage number	Number of eligible days	Number of simulated instantaneous measurements											
			1	2	3	4	5	6	7	10	15	20	50	100
5	04148200	432	0.14684	0.10370	0.09800	0.07954	0.07667	0.06610	0.06459	0.05203	0.04442	0.03751	0.02416	0.01646
5	04148300	418	0.11106	0.07826	0.06966	0.05667	0.05302	0.04563	0.04356	0.03408	0.02733	0.02249	0.01340	0.00921
5	04148440	422	0.11493	0.08129	0.07299	0.05949	0.05744	0.04972	0.04889	0.03945	0.03345	0.02820	0.01770	0.01245
5	04148500	1,411	0.10972	0.07752	0.07078	0.05758	0.05483	0.04715	0.04582	0.03683	0.03105	0.02617	0.01655	0.01163
5	04150000	577	0.08923	0.06317	0.04660	0.03708	0.03178	0.02719	0.02517	0.01942	0.01575	0.01320	0.00858	0.00617
5	04150500	1,225	0.11610	0.08215	0.07344	0.05927	0.05532	0.04754	0.04558	0.03586	0.02983	0.02485	0.01527	0.01083
5	04151500	1,326	0.11699	0.08258	0.07634	0.06200	0.05960	0.05114	0.04993	0.03999	0.03385	0.02859	0.01827	0.01292
5	04152238	592	0.09628	0.06802	0.06270	0.05091	0.04841	0.04180	0.04034	0.03226	0.02676	0.02232	0.01351	0.00925
5	04153500	341	0.07742	0.05480	0.04690	0.03805	0.03628	0.03128	0.03078	0.02474	0.02134	0.01811	0.01217	0.00910
5	04154000	1,411	0.09315	0.06571	0.06416	0.05238	0.05213	0.04509	0.04521	0.03711	0.03294	0.02850	0.01993	0.01495
5	04155000	1,387	0.11288	0.07988	0.07826	0.06372	0.06350	0.05488	0.05487	0.04481	0.03914	0.03338	0.02189	0.01563
5	04158000	80	0.15104	0.10694	0.12140	0.09870	0.10641	0.09148	0.09699	0.08153	0.07708	0.06725	0.04604	0.03029
5	04158500	621	0.11331	0.08015	0.06709	0.05414	0.04890	0.04179	0.03919	0.03010	0.02396	0.01968	0.01126	0.00735
6	04159492	1,342	0.10442	0.07380	0.06039	0.04858	0.04321	0.03683	0.03454	0.02681	0.02220	0.01870	0.01183	0.00823
6	04159900	991	0.09949	0.07036	0.06298	0.05101	0.04886	0.04192	0.04099	0.03276	0.02775	0.02345	0.01522	0.01084
6	04160000	124	0.10015	0.07079	0.06577	0.05292	0.04952	0.04211	0.03969	0.02962	0.02260	0.01777	0.00923	0.00510
6	04160570	1,078	0.14839	0.10480	0.09942	0.08099	0.07829	0.06762	0.06614	0.05331	0.04534	0.03820	0.02397	0.01643
6	04160600	1,319	0.12102	0.08554	0.07770	0.06282	0.05947	0.05102	0.04951	0.03922	0.03296	0.02773	0.01762	0.01245
6	04160800	1,273	0.15895	0.11250	0.10854	0.08832	0.08482	0.07326	0.07086	0.05582	0.04534	0.03756	0.02246	0.01567
6	04160900	1,355	0.15597	0.11038	0.11143	0.09041	0.08864	0.07611	0.07444	0.05849	0.04703	0.03827	0.02138	0.01463
6	04161000	664	0.13654	0.09673	0.09811	0.08015	0.07944	0.06894	0.06836	0.05548	0.04794	0.04087	0.02724	0.01973
6	04161100	886	0.16272	0.11509	0.11060	0.08995	0.08590	0.07387	0.07133	0.05591	0.04568	0.03775	0.02317	0.01664
6	04161500	493	0.13445	0.09513	0.09716	0.07918	0.07981	0.06912	0.06932	0.05688	0.04921	0.04188	0.02716	0.01941
6	04161540	1,313	0.11398	0.08056	0.07762	0.06333	0.06249	0.05403	0.05390	0.04373	0.03784	0.03218	0.02146	0.01592
6	04161580	1,313	0.12246	0.08637	0.08375	0.06816	0.06696	0.05783	0.05720	0.04601	0.03918	0.03303	0.02083	0.01464
6	04161800	1,367	0.14468	0.10251	0.10306	0.08403	0.08412	0.07278	0.07318	0.05970	0.05216	0.04465	0.02945	0.02122
6	04163400	1,054	0.17489	0.12383	0.12752	0.10406	0.10441	0.09029	0.09052	0.07399	0.06448	0.05515	0.03558	0.02470
6	04163500	185	0.09439	0.06652	0.05872	0.04771	0.04520	0.03908	0.03807	0.03030	0.02541	0.02120	0.01338	0.00988
6	04164100	1,304	0.14173	0.10000	0.09927	0.08069	0.07980	0.06911	0.06862	0.05561	0.04754	0.04014	0.02499	0.01701
6	04164300	1,436	0.09868	0.06986	0.05591	0.04492	0.04033	0.03457	0.03250	0.02511	0.02029	0.01674	0.00995	0.00687
6	04164500	1,354	0.13897	0.09846	0.09182	0.07449	0.07144	0.06163	0.06010	0.04781	0.03999	0.03358	0.02090	0.01454
6	04164800	550	0.13096	0.09256	0.08511	0.06862	0.06562	0.05632	0.05493	0.04398	0.03737	0.03152	0.02058	0.01473

Table 1–2. Standard deviation of the median water yield as a function of the number of simulated instantaneous measurements by streamgage

Michigan hydro-logic subregion	U.S. Geological Survey Streamgage number	Number of eligible days	Number of simulated instantaneous measurements											
			1	2	3	4	5	6	7	10	15	20	50	100
6	04166000	1,242	0.15535	0.10994	0.10792	0.08769	0.08588	0.07401	0.07284	0.05829	0.04886	0.04125	0.02607	0.01837
6	04166200	1,188	0.13727	0.09685	0.09078	0.07380	0.07037	0.06072	0.05900	0.04671	0.03876	0.03241	0.02017	0.01447
6	04166300	1,047	0.15222	0.10784	0.11054	0.09015	0.09053	0.07831	0.07834	0.06408	0.05555	0.04737	0.03097	0.02217
7	04169500	409	0.12833	0.09086	0.09667	0.07878	0.08157	0.07051	0.07207	0.05960	0.05314	0.04569	0.03039	0.02244
7	04170000	1,244	0.12186	0.08612	0.08383	0.06846	0.06725	0.05803	0.05760	0.04669	0.04019	0.03422	0.02194	0.01526
7	04170500	1,189	0.12820	0.09065	0.09007	0.07362	0.07321	0.06330	0.06301	0.05120	0.04375	0.03703	0.02357	0.01665
7	04171500	222	0.15222	0.10742	0.10992	0.08928	0.08815	0.07579	0.07461	0.05920	0.05034	0.04297	0.02994	0.02286
7	04172000	1,166	0.13002	0.09176	0.09100	0.07405	0.07191	0.06201	0.06030	0.04810	0.03996	0.03356	0.02113	0.01514
7	04173000	377	0.10684	0.07545	0.06759	0.05489	0.05228	0.04517	0.04417	0.03555	0.03047	0.02599	0.01714	0.01226
7	04173500	1,037	0.10647	0.07552	0.06906	0.05598	0.05360	0.04621	0.04527	0.03634	0.03138	0.02677	0.01801	0.01358
7	04174500	1,395	0.12975	0.09177	0.08781	0.07138	0.06912	0.05961	0.05853	0.04698	0.03975	0.03354	0.02097	0.01463
7	04174800	449	0.12043	0.08508	0.08191	0.06657	0.06538	0.05652	0.05587	0.04536	0.03918	0.03339	0.02184	0.01541
7	04175600	923	0.15094	0.10662	0.10823	0.08798	0.08790	0.07610	0.07608	0.06189	0.05367	0.04565	0.02924	0.02072
7	04175700	537	0.12321	0.08703	0.07996	0.06483	0.06156	0.05304	0.05111	0.04055	0.03340	0.02788	0.01736	0.01217
7	04176000	1,202	0.12788	0.09052	0.09068	0.07384	0.07325	0.06307	0.06253	0.05035	0.04228	0.03533	0.02163	0.01511
7	04176605	614	0.14687	0.10403	0.10490	0.08538	0.08578	0.07418	0.07464	0.06132	0.05447	0.04714	0.03223	0.02300
7	04184500	810	0.11771	0.08319	0.07346	0.05958	0.05535	0.04782	0.04592	0.03621	0.02989	0.02491	0.01470	0.00989

Table 1- 3. Average standard deviations of median water yields as a function of measurement intensity by Michigan hydrologic subregion.

Number of simulated instantaneous measurements	Michigan Hydrologic Subregion							Average over all subregions
	1	2	3	4	5	6	7	
1	0.131385	0.117701	0.078764	0.089280	0.108090	0.135032	0.128636	0.117690
2	0.092913	0.083238	0.055697	0.063159	0.076425	0.095503	0.090942	0.083225
3	0.092024	0.079558	0.052060	0.058399	0.071773	0.092317	0.089178	0.080202
4	0.074860	0.064618	0.042246	0.047398	0.058270	0.075045	0.072544	0.065179
5	0.074068	0.062688	0.040332	0.045451	0.056708	0.073181	0.071434	0.063682
6	0.063932	0.054061	0.034719	0.039203	0.048862	0.063133	0.061662	0.054929
7	0.063627	0.053040	0.033758	0.038324	0.048296	0.062120	0.061076	0.054219
10	0.051684	0.042453	0.026778	0.030695	0.038970	0.049836	0.049355	0.043675
15	0.044771	0.035885	0.022411	0.025970	0.033650	0.042169	0.042388	0.037355
20	0.038164	0.030233	0.018839	0.021899	0.028545	0.035540	0.036042	0.031641
50	0.025028	0.019240	0.012000	0.013786	0.018419	0.022519	0.023479	0.020384
100	0.018034	0.013741	0.008639	0.009710	0.012859	0.015910	0.016870	0.014515

Table 1- 4. Base-flow recession parameters for index streamgages.

U.S. Geological Survey streamgage number	Base-flow events	Attained significance	Lower confidence limit of rY_o	Median rY_o	Upper confidence interval rY_o	Lower confidence limit of *tau*	Median *tau*	Upper confidence interval *tau*
04001000	98	3.34	0.483002	0.546629	0.629298	18.6389	20.6585	24.3119
04031000	62	3.00	0.448095	0.522489	0.562420	17.5324	22.2514	27.5401
04031500	51	4.89	0.679478	0.733736	0.816257	24.0804	32.7301	34.9878
04032000	24	2.27	0.570470	0.658792	0.783379	18.6917	27.6530	37.4554
04033000	108	4.28	0.820272	0.838981	0.857654	57.3214	64.4034	78.6423
04035000	38	3.36	0.681058	0.711272	0.750226	46.5927	56.8473	73.2682
04040000	22	1.69	0.651279	0.818127	0.918836	15.9055	29.5217	49.3306
04040500	124	3.84	0.598121	0.634689	0.699791	16.8858	18.5635	21.0777
04041500	33	3.51	0.743946	0.862618	0.934153	16.3910	26.2351	48.3138
04043050	117	4.15	0.690063	0.695454	0.717553	55.9373	64.9492	77.8613
04045500	109	3.46	0.701353	0.738398	0.797655	38.1566	42.0212	48.5561
04046000	75	3.70	0.576702	0.618992	0.661284	49.5608	61.3142	84.9628
04049500	19	1.92	0.726628	0.797301	0.956832	43.9437	74.6480	121.0388
04055000	19	1.92	0.701841	0.770897	0.947378	42.1947	69.4234	107.9107
04056000	51	4.89	0.743125	0.779994	0.839601	59.2561	71.0280	77.5253
04056500	106	4.09	0.768538	0.802955	0.845176	63.2556	71.0039	79.3304
04057510	94	4.95	0.671898	0.699816	0.766339	39.1462	42.0183	44.3452
04057800	105	3.13	0.611122	0.650636	0.715771	21.9841	24.9776	28.4630
04058000	33	3.51	0.568197	0.692847	0.726962	28.8438	35.8778	42.5677
04058400	45	3.57	0.647821	0.696660	0.761067	37.7426	51.7492	57.9199
04059000	67	4.98	0.686220	0.712366	0.781436	23.9468	29.4356	32.2099
04059500	116	3.23	0.521148	0.562709	0.622614	21.2654	23.3582	25.8288
04060993	95	3.96	0.799921	0.838379	0.855140	66.8279	76.9984	90.3910
04061500	58	4.79	0.803269	0.843825	0.870507	31.8663	36.7301	41.8837
04062200	70	4.14	0.529393	0.629267	0.680828	15.4421	16.2633	18.9393
04096015	45	3.57	0.562585	0.588737	0.627385	47.2838	59.9665	71.7735
04096405	125	4.87	0.649902	0.687439	0.727149	35.2120	37.5957	41.3043
04096515	112	4.67	0.524752	0.554696	0.620815	23.1516	24.8803	28.8979
04096600	59	3.63	0.527473	0.616143	0.734279	30.9879	37.3271	54.4367
04096900	79	4.22	0.753160	0.781469	0.810631	42.6499	50.5221	59.1391
04097170	27	1.92	0.730409	0.778235	0.866843	71.5850	95.9012	124.9975

Table 1- 4. Base-flow recession parameters for index streamgages.

U.S. Geological Survey streamgage number	Base-flow events	Attained significance	Lower confidence limit of rY_o	Median rY_o	Upper confidence interval rY_o	Lower confidence limit of *tau*	Median *tau*	Upper confidence interval *tau*
04097540	128	4.16	0.726704	0.763332	0.807841	39.2234	42.3626	46.7371
04099000	31	2.94	0.763038	0.892894	1.035445	20.0465	26.6289	36.3106
04101500	25	4.33	0.839802	0.924211	1.038887	27.1450	32.3344	34.5922
04101800	109	3.46	0.859170	0.880146	0.908345	57.9519	63.8285	69.3578
04102500	94	4.95	0.866953	0.892286	0.916344	64.0849	69.2473	82.9090
04102700	117	4.15	0.673646	0.697698	0.721626	52.2417	60.9805	68.9778
04103010	61	3.96	0.785379	0.803085	0.854193	70.8714	80.4017	97.7969
04104945	38	3.36	0.538382	0.568339	0.590319	32.8699	38.5286	59.9742
04105000	100	3.52	0.557284	0.598272	0.620611	33.0650	40.3533	46.3442
04105700	97	4.17	0.925024	0.947272	0.979495	45.7398	52.4309	57.9881
04108600	112	4.67	0.594491	0.621802	0.644489	47.3755	50.2165	58.2848
04108801	91	3.54	0.278517	0.297660	0.316535	26.4110	30.0517	35.5390
04110000	39	2.37	0.369166	0.406994	0.452727	44.2779	56.7189	90.6982
04111500	111	3.63	0.257629	0.281818	0.332400	21.5977	24.3187	27.9612
04112000	108	4.28	0.211287	0.235244	0.261780	20.1172	21.7884	23.5522
04112500	129	3.42	0.377884	0.397825	0.425668	30.6599	37.4381	41.7066
04114498	111	3.63	0.433943	0.461787	0.485582	36.3960	41.1107	45.5222
04115000	125	4.87	0.343672	0.363153	0.394382	24.2164	27.2011	31.1758
04116500	25	4.33	0.633943	0.694560	0.861086	32.6507	41.4245	56.0440
04117000	58	4.79	0.601151	0.626004	0.658613	60.6439	67.8450	75.8928
04117500	125	4.87	0.567363	0.590223	0.608695	45.8925	51.0001	60.5820
04118000	27	1.92	0.658775	0.754606	0.839950	19.7161	24.7076	42.0717
04118500	105	3.13	0.716853	0.740981	0.769832	57.7796	64.1379	73.2124
04121000	18	3.09	0.590125	0.646933	0.882204	61.1240	76.3426	99.5998
04121300	100	3.52	0.560966	0.582136	0.601953	89.5641	105.6101	130.4688
04121900	63	4.30	0.728472	0.752692	0.789414	49.8366	64.9913	74.9612
04122100	62	3.00	0.564544	0.622980	0.675024	34.1220	38.7786	44.6460
04122200	106	4.09	0.828181	0.843891	0.869350	98.6235	111.5726	120.8918
04122500	103	4.82	0.856050	0.883647	0.892394	100.7904	123.5606	134.5439
04123000	73	3.44	0.940891	0.955533	0.977004	166.4333	197.7745	260.0036
04123500	59	3.63	1.133718	1.146177	1.159290	334.9381	352.3355	487.0174
04124000	91	3.54	0.993975	1.003933	1.021019	162.3519	210.5108	239.3060

Table 1- 4. Base-flow recession parameters for index streamgages.

U.S. Geological Survey streamgage number	Base-flow events	Attained significance	Lower confidence limit of rY_o	Median rY_o	Upper confidence interval rY_o	Lower confidence limit of *tau*	Median *tau*	Upper confidence interval *tau*
04124500	50	3.28	0.426252	0.433725	0.510510	33.8514	49.8196	69.6154
04125000	24	2.27	0.620494	0.640645	0.672955	63.5432	117.9455	169.9590
04125500	35	4.10	0.951765	0.966906	0.996644	176.8408	231.1826	287.4295
04126200	35	4.10	0.874727	0.888904	0.914592	144.0475	249.0355	351.4802
04127918	78	3.08	0.614226	0.652666	0.686708	47.4254	55.9419	68.5399
04127997	67	4.98	0.962550	0.985580	1.015928	95.3136	112.4758	147.4575
04128990	26	2.90	0.982259	0.997077	1.018910	89.9831	140.9850	196.4983
04133501	52	3.65	0.715664	0.740176	0.805400	65.4767	84.1466	97.9386
04135000	1	100.00	1.002797	1.002797	1.002797	6.4630	6.4630	6.4630
04135500	78	3.08	0.800509	0.812771	0.832210	96.7878	131.7077	156.8697
04135600	63	4.30	0.702798	0.719072	0.740230	86.0927	108.3531	126.8406
04135700	98	3.34	0.593227	0.602980	0.615988	100.7027	118.6502	132.7899
04136500	43	3.15	0.786930	0.806003	0.842991	66.5729	88.1422	109.3318
04137500	12	3.86	0.758783	0.813092	0.907695	22.0867	43.8408	59.0355
04138000	40	3.85	0.650186	0.670375	0.685881	74.7462	93.6872	124.8575
04138500	39	2.37	0.353523	0.393135	0.437380	31.4091	42.6136	63.3794
04139000	32	2.01	1.129300	1.140576	1.165089	157.5767	346.8487	1002.1563
04139500	33	3.51	1.061818	1.074644	1.096418	135.7355	197.4808	299.1878
04140000	22	1.69	0.538125	0.554761	0.617382	46.3023	59.3169	98.3149
04140500	47	4.00	0.868647	0.876180	0.897961	60.6363	80.8578	92.7693
04141000	41	2.75	0.169938	0.201503	0.234199	15.8593	21.1600	29.1354
04141500	13	2.25	0.704042	0.718135	0.772268	43.9723	61.2213	151.2955
04142000	89	3.34	0.692363	0.713763	0.734539	64.6641	75.0414	104.2144
04143900	56	4.40	0.554992	0.612372	0.681827	21.1937	33.9247	64.5810
04144000	34	2.43	0.505303	0.577050	0.698170	24.4487	33.1772	37.8643
04144500	89	3.34	0.478131	0.507430	0.559247	26.7874	32.7474	37.1567
04145500	33	3.51	0.102605	0.150521	0.223135	5.0849	8.6671	11.2126
04146000	127	3.28	0.419966	0.450410	0.480807	19.0131	21.5966	24.2275
04146063	63	4.30	0.533099	0.608216	0.660450	21.2629	25.4957	29.9562
04147500	104	3.90	0.523054	0.544147	0.571029	28.5675	37.6635	57.0494
04147990	35	4.10	0.335628	0.357206	0.374552	34.9159	49.4544	68.7736

Table 1- 4. Base-flow recession parameters for index streamgages.

U.S. Geological Survey streamgage number	Base-flow events	Attained significance	Lower confidence limit of rY_o	Median rY_o	Upper confidence interval rY_o	Lower confidence limit of *tau*	Median *tau*	Upper confidence interval *tau*
04148140	88	4.22	0.374868	0.415290	0.465033	21.4115	23.9547	25.9229
04148160	27	1.92	0.067678	0.119109	0.169963	5.5394	10.5667	20.0000
04148200	34	2.43	0.372348	0.411545	0.543801	18.6920	23.3498	28.0031
04148300	24	2.27	0.206442	0.321323	0.451109	16.5152	20.4521	25.2211
04148440	29	2.41	0.270691	0.359427	0.449336	11.0525	15.1952	19.2393
04148500	53	2.70	0.505167	0.571274	0.645807	20.4988	25.3080	31.7318
04150000	44	4.88	0.169150	0.192955	0.213191	26.7781	36.1641	46.0767
04150500	119	4.33	0.219755	0.239991	0.272774	17.9604	21.1901	25.7867
04151500	119	4.33	0.337913	0.361777	0.400801	26.0523	28.4006	30.2868
04152238	34	2.43	0.642240	0.657580	0.680344	51.1028	66.9182	103.3117
04153500	86	3.99	0.264423	0.279119	0.289738	39.8810	47.2435	53.2327
04154000	114	4.87	0.661305	0.674792	0.702158	57.0459	62.6003	70.1240
04155000	47	4.00	0.548834	0.581255	0.701086	22.4104	28.0769	33.3428
04157500	46	2.59	0.038547	0.071546	0.129938	4.7286	6.9118	20.0000
04158000	50	3.28	0.054313	0.073249	0.131280	4.5969	7.5590	15.9529
04158500	41	2.75	0.236818	0.264524	0.308933	14.9540	21.6309	33.6393
04159492	118	3.38	0.254759	0.270237	0.291754	22.2436	26.6894	31.9695
04159900	81	4.48	0.263603	0.277679	0.315992	23.9942	26.8720	32.3498
04160000	44	4.88	0.230021	0.258253	0.283705	25.5116	31.2106	37.4337
04160050	13	2.25	0.155340	0.199311	0.287650	11.7208	14.5183	35.8570
04160570	70	4.14	0.367134	0.406298	0.456220	16.8720	20.3360	23.7839
04160600	101	4.60	0.321488	0.345863	0.360782	26.9577	31.8084	34.0496
04160800	108	4.28	0.385942	0.423073	0.452196	24.1915	27.6115	29.2796
04160900	80	3.30	0.480556	0.527135	0.651762	21.6157	25.6273	35.0132
04161000	23	3.47	0.564640	0.656908	0.843072	15.3164	20.3012	32.9656
04161100	45	3.57	0.336247	0.385676	0.467745	18.6941	24.1681	29.9744
04161500	41	2.75	0.506681	0.614673	0.734003	23.5259	27.6929	39.9829
04161540	83	4.75	0.538669	0.584676	0.632497	32.2311	39.1186	47.9305
04161580	58	4.79	0.427571	0.474596	0.562940	20.3628	31.6945	47.2380
04161800	86	3.99	0.518170	0.551143	0.617837	18.2987	20.7397	23.7154
04163400	53	2.70	0.376712	0.434570	0.519663	11.4057	15.3426	19.8948

Table 1- 4. Base-flow recession parameters for index streamgages.

U.S. Geological Survey streamgage number	Base-flow events	Attained significance	Lower confidence limit of rY_o	Median rY_o	Upper confidence interval rY_o	Lower confidence limit of *tau*	Median *tau*	Upper confidence interval *tau*
04163500	23	3.47	0.096705	0.163732	0.204890	8.1369	14.3396	17.1998
04164100	103	4.82	0.526775	0.577908	0.655446	24.1752	28.3403	32.4928
04164300	71	3.19	0.174865	0.192491	0.263539	11.5435	13.7107	15.7927
04164500	106	4.09	0.274002	0.303158	0.325461	16.6803	18.2998	19.9073
04164800	30	4.28	0.255525	0.318423	0.388076	20.2289	26.5707	32.6305
04166000	80	3.30	0.443979	0.474117	0.521379	26.1733	31.7530	39.1724
04166200	47	4.00	0.369327	0.395604	0.445477	27.3076	36.9693	49.9576
04166300	62	3.00	0.497637	0.556876	0.600234	24.3636	28.7248	32.0810
04169500	28	3.57	0.524439	0.561612	0.684833	28.8345	40.4468	49.9953
04170000	83	4.75	0.655854	0.690095	0.752802	31.4510	35.4578	39.2989
04170500	88	4.22	0.671783	0.713902	0.789242	29.9405	35.0958	40.3490
04171500	37	4.70	0.481984	0.557099	0.640337	25.0973	30.7579	41.9178
04172000	119	4.33	0.631822	0.674905	0.721384	37.6269	41.7419	45.3362
04173000	17	4.90	0.460651	0.499871	0.698957	32.3792	49.7533	70.2636
04173500	100	3.52	0.446603	0.478743	0.505545	59.3894	73.4198	81.4147
04174500	28	3.57	0.507609	0.611135	0.717864	18.4807	24.7191	33.1891
04174800	2	50.00	0.504461	0.577293	0.650124	16.3412	59.1641	101.9870
04175600	85	2.95	0.589416	0.647665	0.692521	25.4977	29.2724	34.7240
04175700	35	4.10	0.520763	0.575401	0.726165	34.3525	39.3302	49.3163
04176000	96	3.15	0.518386	0.573385	0.608250	33.4850	36.9313	42.6374
04176605	55	3.00	0.189307	0.221861	0.270466	14.0255	15.9844	18.7360
04184500	84	3.75	0.349484	0.400590	0.455913	25.4209	31.2190	36.8208

Appendix 2. Example computations of index flow estimates with simulated instantaneous measurements

IndexFlow_MiscQ
User required input
Parameters in index regression equation
Parameters in augmented index regression equation
Specify run parameters and labels
Load data for index flow simulation
Analysis of instantaneous flow measurements
Compute water yields and flows with the Discrete Estimator
Compute water yields and flows with the Integrated Estimator
Compute water yields and flows with the Site Estimator
List results for all estimators to console

Matlab Code 2- 1. IndexFlow_MiscQ.

Index Flow Estimation with Index Regression and Instantaneous Flows. Index flow is a streamflow characteristic defined as the median flow during the summer month (July, August, September) of minimum flow in Michigan. Index flows are used to help regulate new or increased water withdrawals exceeding 100,000 gallons per day in any consecutive 30-day period from causing adverse impacts on characteristic fish populations in Michigan.

The techniques used in this code are based on the Index regression equation developed by Hamilon, D.A., Sorrell, R.C., and Holtschlag, D.J., 2008, A regression model for computing index flows describing the median flow for the summer month of lowest flow in Michigan: U.S. Geological Survey Scientific Investigations Report 2008-5096, 43 p. In addition, estimation using instantaneous flow measurements is based on techniques developed by Holtschlag, D.J., 2010, Improving regression estimates of index flow, describing the median streamflows for the summer month of lowest flow of streams in Michigan, by use of instantaneous streamflow measurements: U.S. Geological Survey Scientific Investigations Report 2010-5236, xx p. The following code is considered experimental and may not return correct results. User accepts responsibility for accuracy, applicability, and utility of all results as the terms of use.

```
clear; clc;
```

Matlab Code 2–2. User required input

Enter site specific value for regression variables

```
Mhs      =         2;                        % Michigan hydrologic subregion.
SiteNo   = '04112500';                                % Given streamgage number.
SiteName = 'Red Cedar River at East Lansing, Mich.'; % Given streamgage name.
DArea0   =  344.0 ;
L_Trans  =    0.0 ;
H_Trans  =   13.7 ;
Forest   =   14.16626 ;
Precip   =   32.6 ;
A_Soils  =   11.0 ;
D_Soils  =   14.0 ;
%
% Enter instantaneous flow for Discrete, Integrated, and Site Estimators
nMeasQ     =    3;
% Preallocate variables
DateNo = NaN(nMeasQ,1); InstQmea = NaN(nMeasQ,1);
% Enter nMeasQ measurements of date and flow magnitude
DateNo(1)  = datenum('08-Sep-2001','dd-mmm-yyyy'); InstQmea(1) =   37.0;
DateNo(2)  = datenum('25-Sep-1979','dd-mmm-yyyy'); InstQmea(2) =   18.0;
DateNo(3)  = datenum('20-Sep-2001','dd-mmm-yyyy'); InstQmea(3) =   70.0;
%
% Augmented Index regression explanatory variables
% For each baseflow recession period 1 through nRecess analyzed
% enter the corresponding parameter estimates of the rY0vec is a vector...
% of one or more initial root water yields
```

START SAMPLE DATA (Expected format shown with sample data.).

```
rY0vec =  [0.38275 0.596658 0.39783]; % Vector of initial yields during baseflow
recession.
tauvec =  [25.2610 64.5668  37.4378]; % Vector of time constants during baseflow
recession.
%   END SAMPLE DATA.
```

Matlab Code 2–3. Parameters in index regression equation

Explanatory vector

```
x0 = [1 L_Trans H_Trans Forest Precip A_Soils D_Soils]';
% Parameter estimates
bIntcp   = -0.541982;
bL_Trans = -0.00136258;
bH_Trans =  0.00204796;
bForest  =  0.00402190;
bPrecip  =  0.0236424;
bA_Soils =  0.00225536;
bD_Soils =  0.00162107;
% Parameter vector
bvec = [bIntcp bL_Trans bH_Trans bForest bPrecip bA_Soils bD_Soils];
%
% Inverse of the X'X matrx needed to compute prediction uncertainty
InvXpX = [ 2.38035e+0 -8.42611e-4  1.43560e-3  7.23794e-4 -7.11625e-2 -3.54982e-3
-3.50692e-3; ...
          -8.42611e-4  1.90162e-5  9.13024e-6 -1.04607e-6  8.79202e-6 -2.67491e-6
-9.25476e-7; ...
           1.43560e-3  9.13204e-6  1.70298e-5  4.69128e-6 -6.03695e-5 -1.60371e-5
-8.67804e-6; ...
           7.23794e-4 -1.04607e-6  4.69128e-6  1.93996e-5 -2.49764e-5 -1.74926e-5
-2.33578e-5; ...
          -7.11625e-2  8.79202e-6 -6.03695e-5 -2.49764e-5  2.17903e-3  1.09528e-4
8.13390e-5; ...
          -3.54982e-3 -2.67491e-6 -1.60371e-5 -1.74926e-5  1.09528e-4  3.85354e-5
2.70690e-5; ...
          -3.50692e-3 -9.25476e-7 -8.67801e-6 -2.33578e-5  8.13390e-5  2.70690e-5
8.42089e-5];
%
IndxrY50_mse  = 0.015320;
IndxrY50_rmse = sqrt(IndxrY50_mse);
% Index root yields
IndxrY50_hat  = bvec * x0;
IndxrY50_var  = IndxrY50_mse * (1 + x0'*InvXpX*x0);
IndxrY50_sep  = sqrt(IndxrY50_var);
IndxrY50_l95  = IndxrY50_hat + tinv(.025,147-7)*IndxrY50_sep;
IndxrY50_u95  = IndxrY50_hat + tinv(.975,147-7)*IndxrY50_sep;
% Index flows
IndxQ50_hat   = IndxrY50_hat^2 * DArea0;
IndxQ50_l95   = IndxrY50_l95^2 * DArea0;
IndxQ50_u95   = IndxrY50_u95^2 * DArea0;
%
% Index regression residuals
IndxrY50_res = [-0.007576813 -0.193160541 0.000288932 -0.076300339 0.084963459 ...
0.002973825 0.046814793 -0.124385304 0.061876491 0.201011933 -0.003608545 ...
-0.023682698 0.097397392 0.107360353 0.019729621 0.135140746 -0.169003059 ...
-0.031989724 -0.008403485 -0.096016247 -0.093484089 -0.203537087 0.212716133 ...
-0.021016441 -0.057917802 0.055962715 0.00068285 -0.070625215 -0.045399968 ...
0.008741081 0.049519021 0.050241785 0.082269415 0.104325843 0.137093468 ...
0.115758648 0.000997975 0.247519925 -0.009711837 -0.011725471 0.243007414 ...
-0.001314207 -0.180446343 -0.141835772 -0.116096304 -0.14654114 -0.028932584 ...
-0.016960894 -0.036515331 -0.007714564 0.123701013 0.065270498 0.109738346 ...
```

```
0.113171529 -0.194725929 -0.171603729 -0.041337003 0.057331834 -0.004784252 ...
-0.116128492 0.001748437 0.200130288 0.082559296 -0.195076717 -0.080462014 ...
0.164080737 -0.120009651 -0.017182485 0.094138057 0.184926391 0.070738178 ...
0.00068102 -0.103002066 -0.19917193 -0.309125987 -0.127259386 -0.047153008 ...
0.078919278 -0.038588799 0.508387742 0.436465315 0.006212689 0.241435973 ...
-0.051198964 0.09215994 0.099394298 0.107441485 0.041780081 0.026585722 ...
-0.123052071 -0.02621801 0.122248708 0.081317958 -0.065194777 -0.029654524 ...
0.001412433 -0.126592344 -0.093203984 -0.083599741 0.067769137 -0.184076643 ...
-0.177106108 -0.099015733 0.008299128 -0.035193265 -0.038619136 -0.071096618 ...
-0.11227454 -0.092503758 0.00888174 -0.023826084 -0.08363744 -0.068947764 ...
-0.089168557 0.142600675 0.094726854 -0.265190342 -0.116936969 0.103301189 ...
-0.122586974 -0.024504426 0.103048046 0.012930496 -0.000667989 0.173564612 ...
-0.073877409 0.113991476 -0.045810706 0.044647904 0.066259253 0.05151493 ...
0.255807281 0.131717401 -0.106459947 0.014539439 0.028374572 -0.047578852 ...
0.0303895 -0.063527979 -0.039972854 -0.064027292 0.036581922 -0.077301413 ...
-0.034568378 -0.013906751 -0.128537752 -0.078167275 ];
%
% Compute the Smearing estimate of flow
IndxQ50_sl95 = 0;
IndxQ50_shat = 0;
IndxQ50_su95 = 0;
for i = 1:147,
    IndxQ50_sl95 = IndxQ50_sl95 + (IndxrY50_l95 + IndxrY50_res(i)).^2;
    IndxQ50_shat = IndxQ50_shat + (IndxrY50_hat + IndxrY50_res(i)).^2;
    IndxQ50_su95 = IndxQ50_su95 + (IndxrY50_u95 + IndxrY50_res(i)).^2;
end
IndxQ50_shat = IndxQ50_shat/147 * DArea0;
IndxQ50_sl95 = IndxQ50_sl95/147 * DArea0;
IndxQ50_su95 = IndxQ50_su95/147 * DArea0;
 %
ndash = 82;
fprintf(1,'Index water yield and flow for site %s %s \n',SiteNo ,SiteName);
fprintf(1,[repmat('-',1,ndash) '\n']);
fprintf(1,'                                        Lower                      Upper
StdErr  \n');
fprintf(1,'ESTIMATOR                          95 Percent     Expected    95 Percent
Predict \n');
fprintf(1,[repmat('-',1,ndash) '\n']);
fprintf(1,'Index Water Yield (CFSM)         %10.3f     %10.3f    %10.3f %8.5f\n',...
    IndxrY50_l95,IndxrY50_hat,IndxrY50_u95,IndxrY50_sep);
fprintf(1,'Conventional Index Flow (CFS)  %10.1f     %10.1f    %10.1f \n',IndxQ50_l95,
IndxQ50_hat, IndxQ50_u95);
fprintf(1,'Smearing Index Flow (CFS)      %10.1f     %10.1f    %10.1f \n',IndxQ50_
sl95,IndxQ50_shat,IndxQ50_su95);
fprintf(1,[repmat('-',1,ndash) '\n']);
%
Index water yield and flow for site 04112500 Red Cedar River at East Lansing, Mich.
--------------------------------------------------------------------------------
                                        Lower                      Upper    StdErr
ESTIMATOR                           95 Percent     Expected    95 Percent  Predict
--------------------------------------------------------------------------------
Index Water Yield (CFSM)              0.113          0.361         0.610  0.12569
Conventional Index Flow (CFS)         4.4           44.9         127.9
Smearing Index Flow (CFS)             9.4           49.9         132.9
--------------------------------------------------------------------------------
```

Matlab Code 2–4. Parameters in augmented index regression equation

Explanatory vector QA sample recession parameter to see if they differ from the default.

```
QARecess = 0;  % QA recession parameter on by default
if median(rY0vec) == median([0.38275 0.596658 0.39783]) && ...
   log10(median(tauvec)) == log10(median([25.2610 64.5668  37.4378])) && ...
   QARecess == 1
    fprintf(1,'ERROR! Enter site specific recession parameters. \n');
    nRecess = 0;
    mrY0    = []; lmtau = [];
else
    nRecess = length(rY0vec);
    mrY0 = median(rY0vec); lmtau = log10(median(tauvec));
end
%
AugmX0 = [1 IndxrY50_hat mrY0 lmtau]';
% Parameter estimates
bAugmIntcp    = -0.22994;
bAugmIndxrY50 =  0.01082;
bAugmrY0      =  0.81126;
bAugmlmtau    =  0.16590;
% Parameter vector
bAugmvec = [bAugmIntcp bAugmIndxrY50 bAugmrY0 bAugmlmtau];
%
% Inverse of the X'X matrx needed to compute prediction uncertainty
AugInvXpX = [ 0.202449  0.054157  0.023014 -0.147576;...
              0.054157  0.694465 -0.380471 -0.119923;...
              0.023014 -0.380471  0.459334 -0.059837;...
             -0.147576 -0.119923 -0.059837  0.152903];
%
AugmrY50_mse  = 0.001424;
AugmrY50_rmse = sqrt(AugmrY50_mse);
%
if nRecess >= 1
    AugmrY50_hat  = bAugmvec * AugmX0;
    AugmrY50_var  = AugmrY50_mse * (1 + AugmX0'*AugInvXpX*AugmX0);
    AugmrY50_sep  = sqrt(AugmrY50_var);
    AugmrY50_l95  = AugmrY50_hat + tinv(.025,147-7)*AugmrY50_sep;
    AugmrY50_u95  = AugmrY50_hat + tinv(.975,147-7)*AugmrY50_sep;
    % Index flows
    AugmQ50_hat   = AugmrY50_hat^2 * DArea0;
    AugmQ50_l95   = AugmrY50_l95^2 * DArea0;
    AugmQ50_u95   = AugmrY50_u95^2 * DArea0;
end
%
```

Matlab Code 2–5. Specify run parameters and labels

```
MinStaEst =      5;  % Minimum number of active streamgages required for estimation
DV_cv     =   0.01;  % Coefficient of variation for approximating daily flows
%                         by a short-interval flow measurement
% Indices of months to use in estimation, Jan = 1;
MonthLab  = {'Jan','Feb','Mar','Apr','May','Jun','Jul','Aug','Sep','Oct','Nov','Dec'};
MhsName = {'Upper Peninsula','Mhs0405','Mhs0406','Mhs0407','Mhs0408',...
    'Mhs 04090001-0004','Mhs 04090005 0410'}; % Michigan subregion names
nmJndxP = NaN(nMeasQ,1);  NumObs  = NaN(nMeasQ,1);
```

Matlab Code 2–6. Load data for index flow simulation

Spreadsheet Qdv1960_2009.xlsx!Numeric has the daily values MLDate = XLDate + 693960;
save('Qdv1960_2009','MLDate','Qdv');

whos -file Qdv1960_2009;

```
load Qdv1960_2009  % Matlab version of spreadsheet
% whos -file StdMedrYdv;
% StdMedrYdv was computed w/o lower bound and 1.96s upper bound
% REMEMBER: StdMedrYdv has 15 fields:
%   StaSeq Mhs StaNum Stdev(1:7,10,15,20,50,100)
load StdMedrYdv
% Eliminate streamgages w/o daily values
MeanQdv = nanmean(Qdv);
xBarNdx = find(~isnan(MeanQdv));
% [~, mo, ~, ~, ~, ~] = datevec(MLDate);
% %GMo is short for Good Month
%  GMo = find(ismember(mo,MonthInd)==1 & ...
%                 MLDate<datenum([2008 10 1 0 0 0]));
% MLGMoDate = MLDate(GMo);
% nGMo      = length(GMo);
% QdvGMo    = Qdv(GMo,:);
% StaNumber is the streamgage number
% StaName is the streamgage name
% Q50 is the Index flow
% DArea is the drainage area at each of the streamgages
% rY50    is the sqrt of the index yield
% IndxrY50_h is the regression estimate of rY50
% IndxrY50_r is the residual rY50 - IndxrY50_h
% IndxrY50_s is the standard error of prediction from the index regression
% YrsDV      is the number of years of daily values at the streamgage
% whos -file IndexReg147
load IndexReg147
Q50147   = Q50;
DArea147 = DArea;
 MhsJndx      = intersect(xBarNdx,find(MiSubRegion == Mhs))';
nMhsJndx      = length(MhsJndx);
MhsQdv        = Qdv(:,MhsJndx);
% Eliminate streamgages w/o daily values or outside Mhs
MhsStaNum     = StaNumber(MhsJndx);
MhsStaNam     = StaName(MhsJndx);
MhsQ50        = Q50(       MhsJndx); % Index flow
```

```
MhsDArea      = DArea(     MhsJndx); % Drainage area
MhsrY50       = SqrtY50(   MhsJndx); % Square root of index water yield
MhsIndxrY50_h = SqrtY50hat(MhsJndx); % Index regression estimate of water yield
MhsIndxrY50_r = SqrtY50res(MhsJndx); % Index regression residual of water yield
MhsIndxrY50_s = SqrtY50SEP(MhsJndx); % Standard error of prediction
MhsYrsDV      = YrsDV(     MhsJndx);
% Specify flow max and min to be considered an Index flow estimator
MhsIndxQ50_h  = MhsIndxrY50_h.^2.*MhsDArea;
%
```

Find if site (SiteNo) is an index streamgage

```
 JndxO = strmatch(SiteNo ,MhsStaNum);
% if JndxO is a valid index, then drop the target site from the aux
%  JndxP = setdiff(MhsJndx,MhsJndx(JndxO));
% nJndxP = length(JndxP);
% MhsQdv = Qdv(:,JndxP);
%
ry50m_h = NaN(nMeasQ,1);
ry50m_v = NaN(nMeasQ,1);
```

Matlab Code 2–7. Analysis of instantaneous flow measurements

```
for m = 1:nMeasQ
    % Find the index of the measurement date
    IndxO  = find(DateNo(m)==MLDate);
    % Find days with valid flow data at Mhs streamgages
    mJndx  = find(~isnan(Qdv(IndxO,MhsJndx))==1);
    mJndxP = setdiff(mJndx,JndxO);
    mMhsrY50 =                 MhsrY50( mJndxP);
    mMhsQdv  = Qdv(IndxO,      MhsJndx( mJndxP))';
    mMhsrYdv = sqrt(mMhsQdv ./ MhsDArea(mJndxP));
    %
    if(~isempty(JndxO))
        mInstQmea  = Qdv(IndxO,      MhsJndx(  JndxO));
        mInstrYmea = sqrt(mInstQmea ./ MhsDArea( JndxO));
    else
        mInstQmea  = InstQmea(m);
        mInstrYmea = sqrt(mInstQmea ./ DAreaO);
    end
    %
    % List daily flows at auxiliary streamgages
    ndash = 72;
    fprintf(1,['\n' repmat('-',1,ndash) '\n']);
    fprintf(1,'Relation between daily mean and index flow on %s in Mhs %u \n',...
        datestr(DateNo(m)),Mhs);
    fprintf(1,' at %s %s for measurement%3u. \n',SiteNo ,SiteName,m);
    fprintf(1,'                           DArea         Q50       rY50      Qdv
rYdv\n');
    fprintf(1,' jSta iJndxP  Streamgage     (mi^2)       (cfs)     (cfsm)    (cfs)
(cfsm)\n');
```

```
    fprintf(1,[repmat('-',1,ndash) '\n']);
    for jSta = 1:length(mJndxP)
        fprintf(1,'%4u  %4u   %s %10.1f %10.1f %10.4f %8.1f %8.4f \n',...
            jSta,     mJndxP(jSta),...
            MhsStaNum{mJndxP(jSta)},...
            MhsDArea( mJndxP(jSta)), ...
            MhsDArea( mJndxP(jSta)) * mMhsrY50(jSta)^2, ...
            mMhsrY50(        jSta )^2,...
            mMhsQdv(         jSta ), ...
            mMhsrYdv(        jSta ));
    end
    fprintf(1,[repmat('-',1,ndash) '\n\n']);
    % Compute Index regression estimates
    nmJndxP(m) = length(mJndxP);
    NumObs(m) = nmJndxP(m);
    xbar  =  mean(mMhsrYdv);
    X = [ones(size(mMhsrYdv)),mMhsrYdv];
    [b,bint,r,rint,stats] = regress(mMhsrY50,X);
    fprintf(1,'Regression results for measurement %u for %s \n',...
        m,datestr(DateNo(m)));
    fprintf(1,' nReg  R2-value  F-Stat  p-value      RMSE    b_Intercept  b_Slope \n');
    fprintf(1,' %3u    %6.4f    %6.2f   %6.5f   %7.4f   %8.4f   %8.4f \n',...
        mJndxP(m),stats(1:3),sqrt(stats(4)),b);
    Sxx   = sum((mMhsrYdv - xbar).^2);
    x2    =     (mInstrYmea - xbar).^2 ;
    ry50m_h(m) = b(1)+b(2)*mInstrYmea;
    rYRes       = mMhsrY50 - (b(1)+b(2)*mMhsrYdv);
    ry50m_v(m) = var(rYRes)*(1 + 1/length(mMhsrYdv)+x2/Sxx)+ ...
        (DV_cv*mInstrYmea)^2;
    %
    % Plot daily and index flows for each instantaneous flow measurement
    figure(5); clf(5);
    plot(mMhsrYdv,mMhsrY50,'bs',...
        'MarkerFaceColor','b','MarkerSize',4);
    hold on
    minx = min(mMhsrYdv);
    maxx = max(mMhsrYdv);
    plot([minx,maxx],b(1)+b(2)*[minx,maxx],'k-');
    xlabel('ROOT DAILY WATER YIELD, IN CFSM');
    ylabel('ROOT INDEX WATER YIELD, IN CFSM');
    title([SiteNo ,' ', SiteName,...
        ' on ',datestr(DateNo(m))]);
    text(minx+0.05*(maxx-minx),minx+.4,['R^2= ',num2str(stats(1),'%6.4f')]);
    xmin = min(get(gca,'XLim')); ymin = min(get(gca,'YLim'));
    plot([mInstrYmea,mInstrYmea],[ymin,ry50m_h(m)],'r-');
    plot([mInstrYmea,xmin],[ry50m_h(m),ry50m_h(m)],'r-');
    plot(mInstrYmea,ry50m_h(m),'r*','MarkerSize',5);
    legend('Auxilary streamgages','Daily and Index flow regression',...
        [SiteNo ' instantaneous flow'],'Location','NorthWest');
    hold off
    pause
end
```

Relation between daily mean and index flow on 08-Sep-2001 in Mhs 2
at 04112500 Red Cedar River at East Lansing, Mich. for measurement 1.

jSta	iJndxP	Streamgage	DArea (mi^2)	Q50 (cfs)	rY50 (cfsm)	Qdv (cfs)	rYdv (cfsm)
1	1	04096015	80.8	21.0	0.2599	20.0	0.4975
2	2	04096405	201.0	61.0	0.3035	56.0	0.5278
3	3	04096515	48.7	7.9	0.1622	6.5	0.3653
4	7	04097540	106.6	46.0	0.4315	50.0	0.6849
5	8	04099000	1879.4	850.0	0.4523	966.0	0.7169
6	9	04101500	3666.0	1850.0	0.5046	1980.0	0.7349
7	10	04101800	251.5	177.0	0.7038	157.0	0.7901
8	11	04102500	389.9	262.0	0.6720	229.0	0.7664
9	12	04102700	83.5	35.0	0.4192	34.0	0.6381
10	13	04103010	269.5	147.0	0.5455	159.0	0.7681
11	14	04104945	48.3	15.0	0.3106	16.0	0.5756
12	15	04105000	274.0	70.0	0.2555	70.0	0.5054
13	16	04105700	36.8	31.0	0.8424	27.0	0.8566
14	17	04108600	65.1	22.0	0.3379	33.0	0.7120
15	18	04108801	66.9	4.3	0.0643	26.0	0.6234
16	19	04111500	16.3	0.9	0.0552	0.6	0.1837
17	20	04112000	10.4	0.2	0.0231	0.1	0.1201
18	23	04115000	420.1	30.0	0.0714	31.0	0.2716
19	25	04117000	7.8	2.8	0.3590	3.5	0.6699
20	26	04117500	410.5	109.0	0.2655	123.0	0.5474
21	28	04118500	257.2	127.0	0.4938	185.0	0.8481

Regression results for measurement 1 for 08-Sep-2001

nReg	R2-value	F-Stat	p-value	RMSE	b_Intercept	b_Slope
1	0.7980	75.08	0.00000	0.0958	0.0380	0.8898

Relation between daily mean and index flow on 25-Sep-1979 in Mhs 2
at 04112500 Red Cedar River at East Lansing, Mich. for measurement 2.

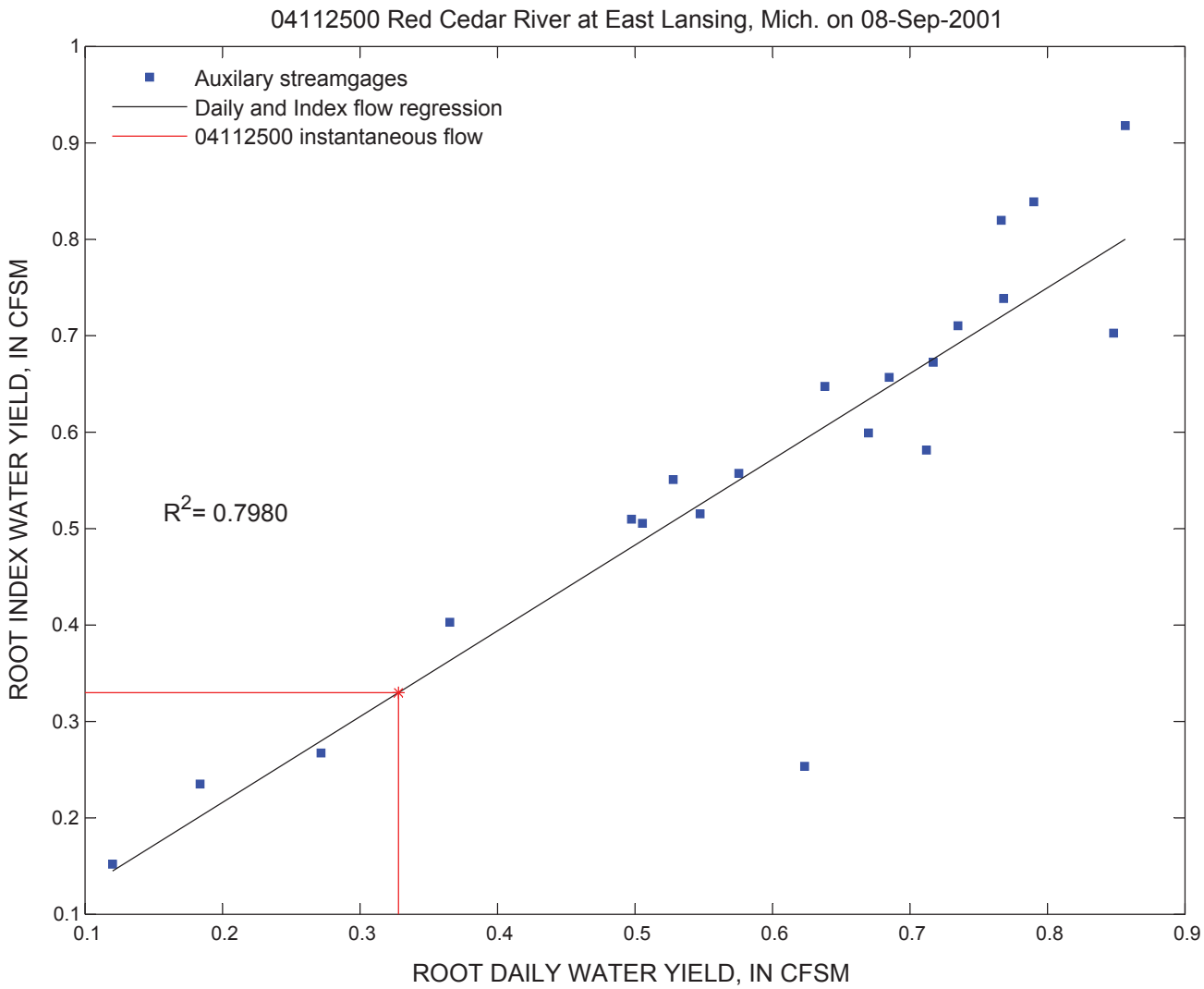
jSta	iJndxP	Streamgage	DArea (mi^2)	Q50 (cfs)	rY50 (cfsm)	Qdv (cfs)	rYdv (cfsm)
1	2	04096405	201.0	61.0	0.3035	45.0	0.4732
2	3	04096515	48.7	7.9	0.1622	4.3	0.2971
3	4	04096600	285.6	65.0	0.2276	48.0	0.4100
4	5	04096900	162.0	72.0	0.4444	42.0	0.5092
5	6	04097170	68.2	30.0	0.4399	30.0	0.6632
6	7	04097540	106.6	46.0	0.4315	32.0	0.5479
7	8	04099000	1879.4	850.0	0.4523	815.0	0.6585
8	9	04101500	3666.0	1850.0	0.5046	1650.0	0.6709
9	10	04101800	251.5	177.0	0.7038	173.0	0.8294
10	11	04102500	389.9	262.0	0.6720	295.0	0.8698
11	12	04102700	83.5	35.0	0.4192	30.0	0.5994
12	15	04105000	274.0	70.0	0.2555	54.0	0.4439
13	16	04105700	36.8	31.0	0.8424	26.0	0.8405
14	17	04108600	65.1	22.0	0.3379	20.0	0.5543
15	18	04108801	66.9	4.3	0.0643	5.6	0.2893
16	19	04111500	16.3	0.9	0.0552	0.1	0.0893
17	20	04112000	10.4	0.2	0.0231	0.1	0.0760

```
  18    22   04114498      284.1       40.0     0.1408     25.0   0.2966
  19    23   04115000      420.1       30.0     0.0714      5.3   0.1123
  20    24   04116500      516.3      222.0     0.4300    148.0   0.5354
  21    26   04117500      410.5      109.0     0.2655     86.0   0.4577
  22    27   04118000      795.2      281.0     0.3534    178.0   0.4731
  23    28   04118500      257.2      127.0     0.4938     90.0   0.5915
-----------------------------------------------------------------------

Regression results for measurement 2 for 25-Sep-1979
 nReg  R2-value  F-Stat  p-value      RMSE    b_Intercept  b_Slope
   3    0.9400    328.94   0.00000    0.0508     0.1295     0.8756

-----------------------------------------------------------------------
Relation between daily mean and index flow on 20-Sep-2001 in Mhs 2
 at 04112500 Red Cedar River at East Lansing, Mich. for measurement  3.
                          DArea         Q50       rY50      Qdv    rYdv
 jSta iJndxP  Streamgage     (mi^2)       (cfs)     (cfsm)    (cfs)   (cfsm)
-----------------------------------------------------------------------
   1     1   04096015       80.8       21.0     0.2599     28.0   0.5887
   2     2   04096405      201.0       61.0     0.3035     71.0   0.5943
   3     3   04096515       48.7        7.9     0.1622     13.0   0.5167
   4     7   04097540      106.6       46.0     0.4315     63.0   0.7688
   5     8   04099000     1879.4      850.0     0.4523   1200.0   0.7991
   6     9   04101500     3666.0     1850.0     0.5046   2520.0   0.8291
   7    10   04101800      251.5      177.0     0.7038    209.0   0.9116
   8    11   04102500      389.9      262.0     0.6720    261.0   0.8182
   9    12   04102700       83.5       35.0     0.4192     49.0   0.7660
  10    13   04103010      269.5      147.0     0.5455    168.0   0.7895
  11    14   04104945       48.3       15.0     0.3106     25.0   0.7194
  12    15   04105000      274.0       70.0     0.2555     92.0   0.5795
  13    16   04105700       36.8       31.0     0.8424     34.0   0.9612
  14    17   04108600       65.1       22.0     0.3379     33.0   0.7120
  15    18   04108801       66.9        4.3     0.0643     41.0   0.7829
  16    19   04111500       16.3        0.9     0.0552      3.3   0.4499
  17    20   04112000       10.4        0.2     0.0231      0.6   0.2402
  18    23   04115000      420.1       30.0     0.0714     35.0   0.2886
  19    25   04117000        7.8        2.8     0.3590      4.8   0.7845
  20    26   04117500      410.5      109.0     0.2655    130.0   0.5627
  21    28   04118500      257.2      127.0     0.4938    212.0   0.9079
-----------------------------------------------------------------------

Regression results for measurement 3 for 20-Sep-2001
 nReg  R2-value  F-Stat  p-value      RMSE    b_Intercept  b_Slope
   3    0.7253     50.16   0.00000    0.1117    -0.0590     0.9098
```



Matlab Code 2–8. Compute water yields and flows with the Discrete Estimator

Standard definition of the weighted estimate

```
DiscrY50_hat = (sum(ry50m_h./ry50m_v))/sum(1./ry50m_v);
DiscrY50_vw  =      1/sum(1./ry50m_v);
% Effective number of observations in the regression equation.
nReg         = min(NumObs);
% Correcting for under dispersion of a weighted variance
DiscrY50_vb = 0;
if nMeasQ > 1
    cf = (1/(nMeasQ-1))*sum(((ry50m_h-mean(ry50m_h)).^2)./ry50m_v);
    DiscrY50_vw  = DiscrY50_vw * max(1,cf);
    wstd         = (1./ry50m_v)./sum(1./ry50m_v);
    V2           = sum(wstd.^2);
    DiscrY50_vb  = 1/(1-V2)*sum(wstd.*(ry50m_h-DiscrY50_hat).^2);
end
%
DiscrY50_l95 = DiscrY50_hat-tinv(0.975,nReg+nMeasQ-1)*sqrt(DiscrY50_vb+DiscrY50_vw);
DiscrY50_u95 = DiscrY50_hat+tinv(0.975,nReg+nMeasQ-1)*sqrt(DiscrY50_vb+DiscrY50_vw);
% Test whether the index flow is within the CI
%
```

Matlab Code 2–9. Compute water yields and flows with the Integrated Estimator

```
IntgrY50_hat = ( IndxrY50_hat/IndxrY50_var + DiscrY50_hat/DiscrY50_vw )/...
               (1/IndxrY50_var + 1/(DiscrY50_vw));
% Weighted variance with correction for underdispersion
IntgrY50_vw = 1/(1/IndxrY50_var + 1/(DiscrY50_vw))* ...
    ( ((IndxrY50_hat - mean([IndxrY50_hat,IntgrY50_hat])).^2) ...
    ./  IndxrY50_var + ...
      ((DiscrY50_hat - mean([IndxrY50_hat,IntgrY50_hat])).^2) ...
    ./ (DiscrY50_vw));
%
wstd = [1/IndxrY50_var,1/DiscrY50_vw]./(1/IndxrY50_var + 1/DiscrY50_vw);
V2   = sum(wstd.^2);
IntgrY50_vb   = 1/(1-V2)*(wstd(1)*(IndxrY50_hat-IntgrY50_hat)^2+...
                          wstd(2)*(DiscrY50_hat-IntgrY50_hat)^2);
%
IntgrY50_hat  = (IndxrY50_hat/IndxrY50_var +
DiscrY50_hat/(DiscrY50_vb+DiscrY50_vw))/...
                           (1/IndxrY50_var +
1/(DiscrY50_vb+DiscrY50_vw));
IntgrY50_var  =       1 /  (1/IndxrY50_var +
1/(DiscrY50_vb+DiscrY50_vw));
%
IntgrY50_l95 = IntgrY50_hat+tinv(0.025,nReg+nMeasQ-1)*sqrt(IntgrY50_var);
IntgrY50_u95 = IntgrY50_hat+tinv(0.975,nReg+nMeasQ-1)*sqrt(IntgrY50_var);
%
```

Matlab Code 2–10. Compute water yields and flows with the Site Estimator

Compute root water yields from instantaneous flow measurements

```
InstrYmea = sqrt(InstQmea ./ DArea0) ;
if length(InstrYmea)>1
    % whos -file MhsStdMedrYdv
    load MhsStdMedrYdv
    % Average standard deviation of median rY50s as a function of nMeasQ
    %   as a function of measurement intensity [1,...,100] in rows and
    %   Mhs in corresponding columns.
    % Interpolate SdMednMeasQMhs from standard intensities
    StdnMeasQ = [1:7 10 15 20 50 100];
    MhsStdMedrYdv_hat = interp1(StdnMeasQ,MhsStdMedrYdv(:,Mhs)',nMeasQ);
    % Look up the standard deviation of the median rY50 values for the
    %   number of measurements and the Mhs.
    SiterY50_hat =(median(InstrYmea) / MhsStdMedrYdv_hat^2 + ...
                        IntgrY50_hat /IntgrY50_var) ./ ...
           (1/(MhsStdMedrYdv_hat^2)+1/(IntgrY50_var));
    %
    SiterY50_var = 1 /( 1/MhsStdMedrYdv_hat^2 + 1/ IntgrY50_var);
    SiterY50_l95 = SiterY50_hat+tinv(0.025,nReg-1)*sqrt(SiterY50_var);
    SiterY50_u95 = SiterY50_hat+tinv(0.975,nReg-1)*sqrt(SiterY50_var);
    %
end
```

Matlab Code 2–11. List results for all estimators to console

```
ndash = 77;
fprintf(1,[‘\n’,repmat(‘-’,1,ndash),’\n’ ]);
fprintf(1,’Results for all Estimators at %s %s \n’,SiteNo,SiteName);
fprintf(1,[repmat(‘-’,1,ndash),’\n’ ]);
```

```
-----------------------------------------------------------------------------
Results for all Estimators at 04112500 Red Cedar River at East Lansing, Mich.
-----------------------------------------------------------------------------
```

```
fprintf(1,’INDEX REGRESSION ESTIMATOR: \n’);
fprintf(1,’Charact-  Number of    Index     Standard_Deviation  Confidence_Limits\n’);
fprintf(1,’teristic  measurements  estimate     mean   prediction  95-lower  95-upper
\n’);
fprintf(1,[repmat(‘-’,1,ndash),’\n’ ]);
fprintf(1,’rYield (rcfsm):%3u  %10.4f   %9.6f %9.6f  %8.4f %8.4f \n’,...
    0,IndxrY50_hat,IndxrY50_rmse,IndxrY50_sep,IndxrY50_l95,IndxrY50_u95);
fprintf(1,’Flow (cfs):       %10.2f       -----     -----  %8.2f %8.2f \n’,...
    IndxQ50_hat,IndxQ50_l95,IndxQ50_u95);
fprintf(1,[repmat(‘-’,1,ndash),’\n\n’ ]);
```

```
INDEX REGRESSION ESTIMATOR:
Charact-  Number of    Index     Standard_Deviation  Confidence_Limits
teristic  measurements  estimate     mean   prediction  95-lower  95-upper
-----------------------------------------------------------------------------
rYield (rcfsm):  0      0.3613    0.123774  0.125692    0.1128   0.6098
Flow (cfs):            44.90       -----     -----      4.38   127.92
-----------------------------------------------------------------------------
```

```
fprintf(1,’DISCRETE ESTIMATOR: \n’);
fprintf(1,[repmat(‘-’,1,ndash),’\n’ ]);
fprintf(1,’Charact-  Number of    Discrete   Standard_Deviation  Confidence_Limits\n’);
fprintf(1,’teristic  measurements  estimate    within    between  95-lower  95-upper
\n’);
fprintf(1,[repmat(‘-’,1,ndash),’\n’ ]);
fprintf(1,’rYield (rcfsm):%3u  %10.4f   %9.6f %9.6f  %8.4f %8.4f \n’,...

nMeasQ,DiscrY50_hat,sqrt(DiscrY50_vw),sqrt(DiscrY50_vb),DiscrY50_l95,DiscrY50_u95);
fprintf(1,’Flow (cfs):       %10.2f       -----     -----  %8.2f %8.2f \n’,...
    [DiscrY50_hat^2,DiscrY50_l95^2,DiscrY50_u95^2].*DArea0);
fprintf(1,[repmat(‘-’,1,ndash),’\n\n’ ]);
```

```
DISCRETE ESTIMATOR:
-----------------------------------------------------------------------------
Charact-  Number of    Discrete   Standard_Deviation  Confidence_Limits
teristic  measurements  estimate    within    between  95-lower  95-upper
-----------------------------------------------------------------------------
rYield (rcfsm):  3      0.3328    0.042876  0.010639    0.2414   0.4241
Flow (cfs):            38.09       -----     -----     20.04    61.89
-----------------------------------------------------------------------------
```

```
fprintf(1,’INTEGRATED ESTIMATOR: \n’);
fprintf(1,[repmat(‘-’,1,ndash),’\n’ ]);
fprintf(1,’Charact-  Number of    Discrete   Standard_Deviation  Confidence_Limits\n’);
fprintf(1,’teristic  measurements  estimate   IndxrY50 DiscrY50   95-lower  95-upper
\n’);
```

```
fprintf(1,[repmat('-',1,ndash),'\n' ]);
fprintf(1,'rYield (rcfsm):%3u  %10.4f   %9.6f %9.6f  %8.4f %8.4f \n',...

nMeasQ,IntgrY50_hat,sqrt(IndxrY50_var),sqrt(DiscrY50_vb+DiscrY50_vw),IntgrY50_
l95,IntgrY50_u95);
fprintf(1,'Flow (cfs):       %10.2f       -----     -----  %8.2f %8.2f \n',...
    [IntgrY50_hat^2,IntgrY50_l95^2,IntgrY50_u95^2].*DArea0);
fprintf(1,[repmat('-',1,ndash),'\n\n' ]);
```

```
INTEGRATED ESTIMATOR:
-------------------------------------------------------------------------------
Charact-  Number of    Discrete   Standard_Deviation  Confidence_Limits
teristic  measurements  estimate   IndxrY50 DiscrY50   95-lower  95-upper
-------------------------------------------------------------------------------
rYield (rcfsm):  3      0.3359    0.125692  0.044176    0.2497   0.4221
Flow (cfs):            38.81       -----     -----     21.45    61.29
-------------------------------------------------------------------------------
```

```
if nMeasQ >= 3
    fprintf(1,'SITE ESTIMATOR: \n');
    fprintf(1,[repmat('-',1,ndash),'\n' ]);
    fprintf(1,'Charact-  Number of      Site   Standard_Deviation   Confidence_
Limits\n');
    fprintf(1,'teristic  measurements  estimate IntgrY50 MhsStdMedrY  95-lower
95-upper \n');
    fprintf(1,[repmat('-',1,ndash),'\n' ]);
    fprintf(1,'rYield (rcfsm):%3u  %10.4f   %9.6f %9.6f  %8.4f %8.4f \n',...

    nMeasQ,SiterY50_hat,sqrt(SiterY50_var),MhsStdMedrYdv_hat,SiterY50_l95,SiterY50_u95);
    fprintf(1,'Flow (cfs):       %10.2f       -----     -----  %8.2f %8.2f \n',...
        [SiterY50_hat^2,SiterY50_l95^2,SiterY50_u95^2].*DArea0);
    fprintf(1,[repmat('-',1,ndash),'\n\n' ]);
else
    fprintf(1,'Too few instantaneous measurements to compute the site estimator.\n');
end
%
if nRecess >= 1
    fprintf(1,'AUGMENTED ESTIMATOR with user data mrY0= %8.6f and lmtau=
%8.6f\n',mrY0,lmtau);
    fprintf(1,'*** Inaccurate results are expected if mrY0 or lmtau are poorly
estimated. *** \n');
    fprintf(1,'Charact-  Number of    Augmented   Standard_Deviation  Confidence_
Limits\n');
    fprintf(1,'teristic  measurements  estimate     mean   prediction  95-lower
95-upper \n');
    fprintf(1,[repmat('-',1,ndash),'\n' ]);
    fprintf(1,'rYield (rcfsm):%3u  %10.4f   %9.6f %9.6f  %8.4f %8.4f \n',...
        nRecess,AugmrY50_hat,AugmrY50_rmse,AugmrY50_sep,AugmrY50_l95,AugmrY50_u95);
    fprintf(1,'Flow (cfs):       %10.2f       -----     -----  %8.2f %8.2f \n',...
        AugmQ50_hat,AugmQ50_l95,AugmQ50_u95);
    fprintf(1,[repmat('-',1,ndash),'\n\n' ]);
else
    fprintf(1,'Baseflow recession variables not defined.\n');
end
%
```

```
SITE ESTIMATOR:
-------------------------------------------------------------------------------
Charact-  Number of      Site   Standard_Deviation   Confidence_Limits
teristic  measurements  estimate IntgrY50 MhsStdMedrY  95-lower  95-upper
-------------------------------------------------------------------------------
rYield (rcfsm):  3      0.3342    0.036918  0.079558    0.2572   0.4112
Flow (cfs):            38.42       -----     -----     22.75    58.17
-------------------------------------------------------------------------------

AUGMENTED ESTIMATOR with user data mrY0= 0.397830 and lmtau= 1.573310
*** Inaccurate results are expected if mrY0 or lmtau are poorly estimated. ***
Charact-  Number of    Augmented   Standard_Deviation  Confidence_Limits
teristic  measurements  estimate     mean   prediction  95-lower  95-upper
-------------------------------------------------------------------------------
rYield (rcfsm):  3      0.3577    0.037736  0.038051    0.2825   0.4330
Flow (cfs):            44.02       -----     -----     27.45    64.48
-------------------------------------------------------------------------------
```

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