SIMPLIFIED TECHNIQUES
For Fitting Frequency Distributions To Hydrologic Data

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CONTENTS

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SIMPLIFIED TECHNIQUES For Fitting Frequency Distributions To Hydrologic Data

By J. L. McGuinness, *analytical statistician*, and D. L. Brakensiek, *hydraulic engineer, Soil and Water Conservation Research Division, Agricultural Research Service*

INTRODUCTION

Within the past decade, the technique of fitting frequency distributions to hydrologic data for small areas has become generally accepted (*17*, *24*).[1] For a frequency analysis, the hydrologic data are treated as statistical variables associated with an assumed statistical distribution. The empirical frequency distribution of the data is then used to determine the magnitude of the variable to be expected in a given recurrence interval or return period. As an illustration, from a set of 19 annual-maximum peak-runoff events, the hydrologist may require an estimate of the magnitude of the peak that will be equaled or exceeded once in 25 years on the average. Procedures for fitting frequency distributions to data are available but are spread through various technical publications and textbooks. Thus, it may be inconvenient for the hydrologic analyst to find the fitting procedures for the distribution he chooses.

This handbook brings together simplified procedures for fitting the most commonly used distributions in hydrologic frequency analysis—the normal, extreme-value, log-normal, and modified log-normal distributions. The modified log-normal distribution is well suited for the case where a specific statistical distribution is not or cannot be postulated. In addition to fitting procedures, a simple test of the hydrologic representativeness of a set of hydrologic data is outlined.

The theories behind the several frequency distributions, including their advantages and limitations, are not discussed in this report. The references contained herein will serve to introduce the interested reader to the voluminous and sometimes contradictory writings on these subjects.

The question of the appropriate formula to use in computing the frequency plotting positions for data is one illustration of the disagreement that generally exists. The authors prefer the formula $m/(n+1)$, where m is the order number of the point and n is the number of points to be fitted. This formula was derived by Gumbel (*14*) and Chow (*8*) and is distribution-free. Yet, an earlier formula developed empirically by Hazen (*16*), $m/(2n-1)$, is still favored by some workers (*19*). After investigating the plotting-position formulas, E. L. Neff, of the Soil and Water Conservation Research Division, in an unpublished report, concluded that the quality and representativeness of the record are much more important than the choice of a plotting-position formula in the development of meaningful frequency lines.

There has also been some debate in the past as to whether only the maximum of annual values or all values above a certain base value should be selected for a frequency analysis. Chow (*6*) has shown that for recurrence intervals of about 10 years or longer there is essentially no difference between the two methods of selecting data. Since the series of maximum values is simpler and theoretically more suited for frequency analysis, the authors prefer it to the partial-duration series.

[1] Italic numbers in parentheses refer to Selected References, p. 8.

The hydrologist using frequency distributions to describe his data should recognize the inherent limitations of the method. The data constitute a sample only. Samples, particularly small samples, are noted for the way in which they may vary from their parent populations. To obtain the true 50-year peak rate estimate for a watershed with 20 years of record, it would be necessary to show—

(1) That the 20-year record is completely representative of the long-term (but unknown) peak rate statistical distribution; and
(2) That the statistical distribution used to describe the 20-year record is the true distribution that exactly describes the long-term (unknown) peak rate frequency distribution for the watershed under study.

The hydrologist may take steps as described below to provide a satisfactory answer to the first of these requirements. The second must rest on assumptions that generally cannot be proved. Despite the limitation, however, frequency analyses are a valuable tool in hydrologic studies of small areas.

TESTING FOR REPRESENTATIVENESS OF THE SAMPLE

For a frequency analysis of a short set of data to have meaning, it must be shown that the years of record are representative of what might be expected on a long-term basis. In practice, this usually means a check on the representativeness of some observed climatic element, such as precipitation, as these records are usually long enough to define a representative period. If a long-term runoff record is available, it can be used instead of a precipitation record in the test proposed in this section.

The U-test (*18*) is suggested as a simple, distribution-free method of checking representativeness. The following example uses the annual precipitation amounts for the Coshocton, Ohio, Weather Bureau station for the period 1909-59. The U-test is here used to determine whether the frequency distribution of a given period—for instance, 1939–59—is identical with that of the longer period of rainfall record. If it is the same, then conclusions drawn from the 1939–59 record are based on a weather sample that is representative of a much longer period of time.

To perform the U-test, the 51 annual precipitation totals for the 1909–59 period at Coshocton are ranked in decreasing order. (See table 1, Appendix.) The sum of the rank numbers (T)[2] for the 1939–59 period is found to be 556. If the number of years in the 1939–59 period is denoted as $a=21$, and the remaining years as $b=30$, then the statistic U may be computed as

$$U=a\,b+[a(a+1)/2]-T. \tag{1}$$

When the hypothesis of the two distributions being identical is true (for a and b both larger than 8), the random variable (U) will have approximately a normal distribution with the mean

$$\overline{U}=a\,b/2 \tag{2}$$

and variance

$$\sigma_U{}^2=a\,b(b+a+1)/12. \tag{3}$$

For the city of Coshocton data, $\overline{U}=315$, $\sigma_U=52.2$, and

$$t=(U-\overline{U})/\sigma_U=-0.19. \tag{4}$$

The probability of a value of t greater than ± 0.19 is approximately 85 percent (*3, 22*).

The conclusion is that the 1939–59 period at the city of Coshocton has the same frequency function as the remaining period. Therefore, this period is an entirely representative period at Coshocton station with respect to the annual precipitation amounts examined, since precipitation totals at the station are highly correlated with the city of Coshocton totals. Thus, a frequency analysis estimate of the annual precipitation that would be equaled or exceeded once in 25 years at the Coshocton station should have a real meaning, because it is based on a 1939–59 record that is representative of the long-term climate of the area. Since annual precipitation and annual runoff are also correlated in the Coshocton area, the extension to frequency analysis of annual runoff amounts is fairly safe.

The problem of judging representativeness is more complex for such variates as peak runoff rates. Concerning such rates, it would be unusual to have a long-term runoff record available for a comparable stream in the area. However, correlation analysis may show that the short-term record in question is significantly related to some precipitation parameter, such as maximum 30-minute intensity. In this case, it may be

[2] The symbols are given on p. 9.

possible to obtain a long-term record of maximum 30-minute intensities from a nearby Weather Bureau station. If there is significant correlation between the study record and the U.S. Weather Bureau intensity record for the short-time period, the long-term intensity record could be used for the representativeness test. The reliability of the representativeness of this test as an inference to peak rates will, in this case, depend upon the degree of correlation found between the peak rates and the short-term intensities.

In the event that the test shows a short-term record to be nonrepresentative, it would be idle to fit a frequency curve to the data. The resulting curve would have little predictive value for future events. The test might be of use, however, in selecting parts of a record for a specific characteristic such as low flow periods. Here, the analyst would be gaining some idea of how certain parts of a record act in comparison with normal flow expectations.

After a given record is found to be representative of long-term expectations for the area, the next step is to fit a frequency distribution to the data. As mentioned earlier, some assumption must be made about the true frequency distribution that would describe the record. A wide range of distributions is available, but there is no way to prove that one is generally superior to another.

MODIFIED LOG-NORMAL DISTRIBUTION

Chow (*9*) has shown that the modified log-normal distribution has the widest application in hydrologic analysis. The distribution is based on estimation of a transformation constant to obtain data described by a log-normal distribution. Whereas the log-normal distribution will plot as a straight line on log-normal graph paper, the modified log-normal distribution may plot as a concave curve or as a convex curve.

The generality of the modified log-normal distribution is emphasized by the fact that it includes the normal, log-normal, and approximate extreme-value distributions as special cases. Perhaps the main reason it has not been more widely adopted by hydrologists is the difficulty of computing the parameters needed to obtain an exact fit. Development of the mathematical concepts that lead to an exact fit of the modified log-normal distribution is presented in a later section.

A technique of adding a constant to rectify skewed data was recommended by a Subcommittee of the Joint Division Committee on Floods, American Society of Civil Engineers (*2*) which reviewed flood-frequency methods. Procedures given herein make possible direct determination of this rectifying constant, thus eliminating the need for a trial-and-error process.

The graphical fitting procedure, presented in this section, is exact enough for the usual hydrologic analysis. One requirement of the graphical procedure is a plotting of the data. This plotting should always be made prior to any fitting. From the plotting, a tentative idea can be obtained of whether a normal, extreme-value, log-normal, or modified log-normal fit would be more appropriate.

Thus, the graphical procedure as outlined in this handbook is useful for any or all of the following:

(1) To obtain estimates of the magnitude of a variate at selected frequencies of occurrence.

(2) To evaluate directly the constant that will rectify a skewed set of data.

(3) To provide a means of determining which of several distributions will best fit the data and obtain estimates of the parameters of this distribution.

To fit a modified log-normal distribution, the first step of the graphical procedure is to list the original data (x) in decreasing order, compute the mean ($\bar{x}$) of the series, and divide each original variate by the mean to form new variates ($x/\bar{x}$). (See table 2.) Plotting positions are obtained from table 3. The $x/\bar{x}$ values are plotted at their computed plotting positions on transparent log-normal probability paper (fig. 1, Appendix).

The sheet with the plotted data is then superimposed on the graphs of the C_v curve set (figs. 2 to 11). It will be noted that as C_v increases, the curves become steeper. From these 10 figures, it will not be too difficult to find a graphical estimate of a value of C_v that most nearly fits the data. At the same time, interpolation among the curves on the C_v sheet will provide an initial estimate of the value of CV.

To refine the initial estimate of CV, the data sheet is superimposed on figures 12 to 22, a set of CV curve sheets. Whereas the C_v sheets differentiate as to slope, the CV sheets differentiate as to curvature. Superimposing the data sheet on the CV graphs and following the curves for the particular C_v value previously obtained will enable the user to obtain a graphical estimate of the value of CV.

After the graphical estimates of both C_v and CV are obtained, the theoretical modified log-normal frequency curve is calculated from the formula

$$x/\bar{x}=1+C_vK. \qquad (5)$$

(See Chow, 7.)

Table 4 gives the K values corresponding to nine different probabilities for a range of values of CV. The graphical estimate of CV is used to enter a specific line of table 4. The K values on this line are substituted in turn in equation 5 together with the graphical estimate of C_v to calculate the theoretical $x/\bar{x}$ values (see table 2). These nine values of $x/\bar{x}$ define the theoretical probability curve that fits the data. The observed data and theoretical curve points can be reconverted to the original units of measurement by multiplication by $\bar{x}$, as shown in the last column of the lower part of table 2.

As with any graphical method, increased familiarity with the procedure will result in more exact fitting. The following points may be helpful in deciding on a final fit. Increasing the value of C_v will tend to make the curve steeper. Thus, if the fit developed graphically tends to underestimate the high points and overestimate the lower values, it would indicate that a higher value of C_v should be used. An increase in the value of CV, for a fixed C_v, will tend to make the theoretical curve more concave upward. Decreasing the C_v and CV values has the reverse effect.

It is possible that in some problems the user would want to give more weight to a part of the data. In a minimum water-yield study, the lower values might be considered as more important. It would be possible to fit the lower tail of the theoretical curve more closely by the use of the C_v and CV properties discussed above while giving less weight to the fit of the higher values.

Experience has shown that it is more important to obtain a good first graphical estimate of C_v than of CV. If the correct C_v is selected, the slope of the theoretical curve will be correct. An incorrect CV value would result in either too much or too little curvature. This situation can be corrected by making the appropriate change in CV only and recomputing the curve. If C_v has been incorrectly estimated, it will mean first that the appropriate change in C_v be made to correct the slope and then CV reestimated to obtain the proper curvature.

There is another feature of this procedure that is of practical use. It is possible to compute a constant that will transform a curved frequency line to a straight line. The exception is the case of $CV=0$, where the curves will plot as straight lines on arithmetic normal probability paper without the use of a transformation. The constant is computed from the equation

$$c=\bar{x}[(C_v/CV)-1]. \qquad (6)$$

The constant relative to and derived from the data of table 2 is computed as follows:

$$c=5.5992\ [(0.70/0.15)-1]=20.53.$$

Figure 23 shows a plotting in terms of original measurement, also the fitted curve as determined in table 2. The constant ($c=20.53$) is added to each observation and to the points of the fitted curve. Also shown in figure 23 are the transformed data with a fitted straight line drawn through them. Thus, it follows that the graphical fitting procedure presented herein is based on the idea of transforming data to make them more nearly log-normally distributed, fitting the pure log-normal distribution to the transformed data, and then retransforming back to the initial values of measurement. The graphical procedure arrives at the net result without the necessity of calculating the transforming function.

FITTING EQUATIONS FOR KNOWN STATISTICAL DISTRIBUTIONS

In some cases it may be found appropriate to use known statistical distributions for fitting sets of hydrologic data. Thus, the hydrologic analyst also needs procedures for fitting the more commonly used statistical distributions.

NORMAL DISTRIBUTION

The best known of the frequency distributions is the normal. The normal frequency curve is bell shaped and symmetrical about the mean. Special graph paper has been developed on which the normal distribution curve will plot as a straight line. Figure 24 shows annual runoff amounts plotted on this arithmetic probability paper.

For the normal distribution, the mean of the set of data ($\bar{x}$) is computed as usual from $\Sigma x/n$. The standard deviation (s) is estimated from

$$s=\Sigma(x-\bar{x})^2/\Sigma xK_y, \tag{7}$$

where the values of K_y are read from a table of areas of the normal distribution (*3*, *22*); such values are also listed in table 3 for a range of sample sizes. This fitting equation is used so as to be consistent with the extreme-value fitting procedure.

The step-by-step procedure for fitting a normal distribution is given below. A worked example is given in table 5 and shown in figure 24.

1. Rank the observed data in decreasing order.
2. Refer to table 3 for values of K_y corresponding to the size of sample or the number of years of data (n).
3. Calculate the corrected sum of squares of the data, $\Sigma(x-\bar{x})^2=\Sigma x^2-(\Sigma x)^2/n$.
4. Calculate the sum of cross products of the ranked x and their respective K_y values, ΣxK_y.
5. Calculate s by equation 7 and the mean ($\bar{x}$), $\bar{x}=\Sigma x/n$.
6. Multiply equation 5 through by $\bar{x}$ to obtain

$$x=\bar{x}+\bar{x}C_vK, \tag{8}$$

or, since C_v is defined as $s/\bar{x}$,

$$x=\bar{x}+sK. \tag{9}$$

The calculated $\bar{x}$ and s, together with the K values from the $CV=0$ row of table 4, are substituted into equation 9. The data can be plotted on normal probability paper and the fitted straight line drawn in. (See fig. 24.) Plotting positions are given in table 3.

EXTREME-VALUE DISTRIBUTION

The extreme-value theory was first investigated by Fisher and Tippett (*11*) and applied to hydrologic data by Gumbel (*13*). The theory states that the annual maximum values of n years of record approach a definite frequency distribution when the number of observations within each year becomes large. A description of the theory is given by Gumbel (*14*). Figure 25 shows special graph paper on which the extreme-value distribution plots as a straight line.

Chow (*9*) has shown that the extreme-value distribution is approximately the same as a log-normal distribution with $CV=0.364$. This specifies the K values to be used from table 4. The mean of the set of data ($\bar{x}$) is computed as usual. The standard deviation of this distribution (s) is calculated from

$$s=1.067\Sigma(x-\bar{x})^2/\Sigma x\,K_y, \tag{10}$$

where the terms are the same as previously defined. This simplified formulation of the fitting procedure was developed by Brakensiek (*5*).

The step-by-step procedures for fitting the extreme-value distribution are given below. A worked example is given in table 6 and shown in figure 25.

1. Rank the observed data in decreasing order.
2. Refer to table 3 for values of K_y corresponding to the size of sample (n).
3. Calculate the corrected sum of squares of the data,

$$\Sigma(x-\bar{x})^2=\Sigma x^2-(\Sigma x)^2/n.$$

4. Calculate the sum of cross products of the ranked x and their respective K_y values, $\Sigma x\ K_y$.
5. Calculate s by equation 10 and the mean ($\bar{x}$) by $\bar{x}=\Sigma x/n$.
6. Values of the frequency factor (K) for the extreme-value distribution correspond to the $CV=0.364$ row of table 4. These values together with the calculated $\bar{x}$ and s values allow several points on the fitted line to be calculated. If the original data are plotted on extreme-value probability paper and utilize the plotting positions of table 3, draw the fitted straight line.

The fit obtained by this simplified procedure was found by Brakensiek (*5*) to be close to the fit obtained by the method developed by Gumbel (*14*).

LOG-NORMAL DISTRIBUTION

The procedure for fitting the log-normal distribution is identical with that for the normal distribution with one exception: log x is used instead of x. Therefore, a worked example is not given to illustrate this case. The special paper designed to plot log-normal distributions as a straight line has the same horizontal and vertical scale divisions as figures 2 to 22.

DISCUSSION

Previous fitting procedures associated with the modified log-normal distribution have had the disadvantage of requiring the calculation of a coefficient of skew. As is well known (*21*), the estimation of this statistic from small samples is unreliable. The procedure as presented in this report for fitting modified log-normal distributions to data does not depend on an explicit estimation of a coefficient of skew. An advantage that may be derived from the use of the graphical procedure is that approximate fits may be obtained much more easily and quickly than with analytical methods.

The concept of rectifying skewed data by adding a constant to observed data is included in the procedures presented. A system for rectifying skewed data was recommended by the subcommittee reviewing flood-frequency methods, American Society of Civil Engineers (*2*). Determination of the constant, however, was accomplished by a trial-and-error process. The procedures presented in this report make possible the direct determination of the rectifying constant.

The flexibility of utilizing the modified log-normal distribution is quite apparent and was pointed out by Hazen (*16*). The range of skewness that can be fitted by the modified log-normal distribution covers most of the cases usually encountered in hydrologic analysis. The graphical procedure permits more weight to be given to selected parts of the data, an important consideration in some analyses where engineering judgment must substitute for more theoretical statistical considerations.

Recent work by Chow (*9*) has shown that the log-normal distribution for $CV=0.364$ corresponds quite closely to the extreme-value distribution. This fact adds considerable theoretical justification for using modified log-normal distributions in hydrologic analysis. Since modified log-normal distributions thus include the normal, log-normal, and extreme-value distributions as special cases, the use of the modified distribution would save the hydrologist from having to be familiar with the theory of a number of statistical distributions.

An analytical fitting procedure for the modified log-normal distribution has been given by Brakensiek (*5*). This technique may be used to refine the graphical fit if desired. The procedure is not illustrated with an example in this report.

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SYMBOLS

C_v —Coefficient of variability of a set of x variates.
CV—Coefficient of variability of a set of transformed variates.
F_n —Plotting position.
K —Frequency factor.
K_v—Standardized normal deviate; equals K when $CV=0$.
m —Order number of variate when set is ranked in decreasing order.
n —Number of observed variates in set.
P —Precipitation.
$\overline{P}$ —Mean precipitation.
Q —Runoff.
$\overline{Q}$ —Mean runoff.
s —Standard deviation; the sample estimate of σ.
t —"Students" t statistic.
T —Sum of order numbers of a subset of variates.
U —Nonparametric statistic for testing whether or not two samples are drawn from the same population.
$\overline{U}$ —Mean of the U statistic.
x —Observed variate.
$\overline{x}$ —Mean of a set of variates.
σ —Standard deviation of the population; square root of the population variance.

APPENDIX

TABLES

TABLE 1.—*Test of representativeness of annual precipitation between 1939 and 1959 at Coshocton, Ohio*

RANKED YEARLY PRECIPITATION FOR 1909–59 PERIOD

Rank No. (m)	Year	Annual precipitation	Rank No. (m)	Year	Annual precipitation	Rank No. (m)	Year	Annual precipitation
		Inches			*Inches*			*Inches*
10	1909	46.96	1	1926	52.60	30	1943	39.22
34	1910	38.42	14	1927	46.20	50	1944	30.28
9	1911	47.62	46	1928	34.49	7	1945	48.08
29	1912	39.99	12	1929	46.77	20	1946	42.56
4	1913	49.36	51	1930	22.03	32	1947	38.98
22	1914	42.45	38	1931	37.94	26	1948	40.02
18	1915	45.32	42	1932	36.30	23	1949	42.32
27	1916	40.00	31	1933	39.13	15	1950	46.18
28	1917	40.00	49	1934	30.48	16	1951	46.10
40	1918	37.32	11	1935	46.86	41	1952	36.51
13	1919	46.29	25	1936	40.55	48	1953	30.74
24	1920	40.98	3	1937	49.43	43	1954	36.16
2	1921	49.46	21	1938	42.53	33	1955	38.89
44	1922	36.05	17	1939	45.53	6	1956	49.01
36	1923	38.24	5	1940	49.23	19	1957	43.23
39	1924	37.78	37	1941	38.21	45	1958	35.26
47	1925	34.48	35	1942	38.31	8	1959	47.82

CALCULATION OF U AND TEST OF REPRESENTATIVENESS BY t

$T=556$ $a=21$ $b=30$

$U=21(30)+21(21+1)/2-556=305$

$\bar{U}=21(30)/2=315$

$\sigma_U{}^2=21(30)(30+21+1)/12=2730;\ \sigma_U=52.2$

$t=(305-315)/52.2=-0.19$

The probability of a t-value equal to or greater than ± 0.19 is approximately 85 percent.

TABLE 2.—*Fitting a frequency curve to annual runoff amounts (Q) by the modified log-normal method*

Rank No. (m)	Ranked runoff (Q $(=x)$)	Plotting position F_a[1]	$x/\bar{x}$	Computations
	Inches	*Percent*		
1	11.76	7.7	2.100	$\bar{Q}=67.19/12$ $=5.5992$
2	9.82	15.4	1.754	
3	8.82	23.1	1.575	
4	7.23	30.8	1.291	
5	5.83	38.5	1.041	
6	5.79	46.2	1.034	Graphical estimates:
7	4.74	53.8	.847	$C_v=0.7$
8	4.35	61.5	.777	$CV=0.15$
9	3.72	69.2	.664	
10	3.20	76.9	.572	
11	.97	84.6	.173	
12	.96	92.3	.171	
Total	67.19			

If the value of C_v is substituted into equation 5, the fitted frequency equation becomes

$$x/\bar{x}=1+0.7K$$

Since the frequency line is curved, more than two points are needed to orient the line. Several points lying on the fitted frequency line are found as follows:

Probability	K value[2]	$x/\bar{x}=1+C_v K$	$Q=\bar{Q}(1+C_v K)$
Percent			
99.0	−2.01		
95.0	−1.51		
80.0	−.86	0.398	2.23
50.0	−.08	.944	5.29
20.0	.81	1.567	8.77
5.0	1.76	2.232	12.50
1.0	2.66	2.862	16.02
.1	3.80	3.660	20.49
.01	4.81	4.367	24.45

[1] Read from the $n=12$ column of table 3.
[2] For $CV=0.15$, see table 4.

TABLE 3.—*Plotting positions (F_n) and standardized normal deviates (K_y) for a range of sample sizes or number of years of record (n)*

Rank No. (m)	n=9 F_n	n=9 K_y	n=10 F_n	n=10 K_y	n=11 F_n	n=11 K_y	n=12 F_n	n=12 K_y	n=13 F_n	n=13 K_y
	Percent		*Percent*		*Percent*		*Percent*		*Percent*	
1	10.0	1.28	9.1	1.34	8.3	1.38	7.7	1.43	7.1	1.46
2	20.0	.84	18.2	.91	16.7	.97	15.4	1.02	14.3	1.07
3	30.0	.52	27.3	.60	25.0	.68	23.1	.74	21.4	.79
4	40.0	.25	36.4	.35	33.3	.43	30.8	.50	28.6	.57
5	50.0	.00	45.5	.11	41.7	.21	38.5	.29	35.7	.37
6	60.0	−.25	54.5	−.11	50.0	.00	46.2	.10	42.9	.18
7	70.0	−.52	63.6	−.35	58.3	−.21	53.8	−.10	50.0	.00
8	80.0	−.84	72.7	−.60	66.7	−.43	61.5	−.29	57.1	−.18
9	90.0	−1.28	81.8	−.91	75.0	−.68	69.2	−.50	64.3	−.37
10			90.9	−1.34	83.3	−.97	76.9	−.74	71.4	−.57
11					91.7	−1.38	84.6	−1.02	78.6	−.79
12							92.3	−1.43	85.7	−1.07
13									92.9	−1.46

TABLE 3.—*Plotting positions (F_n) and standardized normal deviates (K_y) for a range of sample sizes or number of years of record (n)*—Continued

Rank No. (m)	n=14 F_n	n=14 K_y	n=15 F_n	n=15 K_y	n=16 F_n	n=16 K_y	n=17 F_n	n=17 K_y	n=18 F_n	n=18 K_y
	Percent		*Percent*		*Percent*		*Percent*		*Percent*	
1	6.7	1.50	6.2	1.53	5.9	1.57	5.6	1.60	5.3	1.62
2	13.3	1.11	12.5	1.15	11.8	1.19	11.1	1.22	10.5	1.25
3	20.0	.84	18.8	.89	17.6	.93	16.7	.97	15.8	1.00
4	26.7	.62	25.0	.67	23.5	.72	22.2	.76	21.1	.80
5	33.3	.43	31.2	.49	29.4	.54	27.8	.59	26.3	.63
6	40.0	.25	37.5	.32	35.3	.38	33.3	.43	31.6	.48
7	46.7	.08	43.8	.16	41.2	.22	38.9	.29	36.8	.33
8	53.3	−.08	50.0	.00	47.1	.07	44.4	.14	42.1	.20
9	60.0	−.25	56.2	−.16	52.9	−.07	50.0	.00	47.4	.07
10	66.7	−.43	62.5	−.32	58.8	−.22	55.6	−.14	52.6	−.07
11	73.3	−.62	68.8	−.49	64.7	−.38	61.1	−.29	57.9	−.20
12	80.0	−.84	75.0	−.67	70.6	−.54	66.7	−.43	63.2	−.33
13	86.7	−1.11	81.2	−.89	76.5	−.72	72.2	−.59	68.4	−.48
14	93.3	−1.50	87.5	−1.15	82.4	−.93	77.8	−.76	73.7	−.63
15			93.8	−1.53	88.2	−1.19	83.3	−.97	79.0	−.80
16					94.1	−1.57	88.9	−1.22	84.2	−1.00
17							94.4	−1.60	89.5	−1.25
18									94.7	−1.62

Table 3.—*Plotting positions (F_n) and standardized normal deviates (K_y) for a range of sample sizes or number of years of record (n)*—Continued

Rank No. (m)	n=19		n=20		n=21		n=22		n=23	
	F_n	K_y	F_n	K_y	F_n	K_y	F_n	K_y	F_n	K_y
	Percent		*Percent*		*Percent*		*Percent*		*Percent*	
1	5.0	1.64	4.8	1.67	4.5	1.69	4.3	1.71	4.2	1.73
2	10.0	1.28	9.5	1.31	9.1	1.34	8.7	1.36	8.3	1.38
3	15.0	1.04	14.3	1.07	13.6	1.10	13.0	1.12	12.5	1.15
4	20.0	.84	19.0	.87	18.2	.91	17.4	.94	16.7	.97
5	25.0	.67	23.8	.71	22.7	.75	21.7	.78	20.8	.81
6	30.0	.52	28.6	.57	27.3	.61	26.1	.64	25.0	.67
7	35.0	.39	33.3	.43	31.8	.47	30.4	.51	29.2	.55
8	40.0	.25	38.1	.30	36.4	.35	34.8	.39	33.3	.43
9	45.0	.13	42.9	.18	40.9	.23	39.1	.27	37.5	.32
10	50.0	.00	47.6	.06	45.5	.12	43.5	.16	41.7	.21
11	55.0	−.13	52.4	−.06	50.0	.00	47.8	.05	45.8	.10
12	60.0	−.25	57.1	−.18	54.5	−.12	52.2	−.05	50.0	.00
13	65.0	−.39	61.9	−.30	59.1	−.23	56.5	−.16	54.2	−.10
14	70.0	−.52	66.7	−.43	63.6	−.35	60.9	−.27	58.3	−.21
15	75.0	−.67	71.4	−.57	68.2	−.47	65.2	−.39	62.5	−.32
16	80.0	−.84	76.2	−.71	72.7	−.61	69.6	−.51	66.7	−.43
17	85.0	−1.04	81.0	−.87	77.3	−.75	73.9	−.64	70.8	−.55
18	90.0	−1.28	85.7	−1.07	81.8	−.91	78.3	−.78	75.0	−.67
19	95.0	−1.64	90.5	−1.31	86.4	−1.10	82.6	−.94	79.2	−.81
20	------	------	95.2	−1.67	90.9	−1.34	87.0	−1.12	83.3	−.97
21	------	------	------	------	95.5	−1.69	91.3	−1.36	87.5	−1.15
22	------	------	------	------	------	------	95.7	−1.71	91.7	−1.38
23	------	------	------	------	------	------	------	------	95.8	−1.73

Table 3.—*Plotting positions (F_n) and standardized normal deviates (K_y) for a range of sample sizes or number of years of record (n)*—Continued

Rank (m)	n=24		n=25		n=26		n=27		n=28	
	F_n	K_y	F_n	K_y	F_n	K_y	F_n	K_y	F_n	K_y
	Percent		*Percent*		*Percent*		*Percent*		*Percent*	
1	4.0	1.75	3.8	1.77	3.7	1.79	3.8	1.80	3.4	1.82
2	8.0	1.40	7.7	1.43	7.4	1.45	7.1	1.47	6.9	1.48
3	12.0	1.18	11.5	1.20	11.1	1.23	10.7	1.24	10.3	1.26
4	16.0	.99	15.4	1.02	14.8	1.04	14.3	1.07	13.8	1.09
5	20.0	.84	19.2	.87	18.5	.90	17.9	.92	17.2	.95
6	24.0	.71	23.1	.74	22.2	.77	21.4	.79	20.7	.82
7	28.0	.58	26.9	.61	25.9	.65	25.0	.67	24.1	.70
8	32.0	.47	30.8	.50	29.6	.54	28.6	.56	27.6	.59
9	36.0	.36	34.6	.40	33.3	.43	32.1	.46	31.0	.50
10	40.0	.25	38.5	.29	37.0	.33	35.7	.37	34.5	.40
11	44.0	.15	42.3	.19	40.7	.24	39.3	.27	37.9	.31
12	48.0	.05	46.2	.10	44.4	.14	42.9	.18	41.4	.22
13	52.0	−.05	50.0	.00	48.1	.05	46.4	.09	44.8	.13
14	56.0	−.15	53.8	−.10	51.9	−.05	50.0	.00	48.3	.04
15	60.0	−.25	57.7	−.19	55.6	−.14	53.6	−.09	51.7	−.04
16	64.0	−.36	61.5	−.29	59.3	−.24	57.1	−.18	55.2	−.13
17	68.0	−.47	65.4	−.40	63.0	−.33	60.7	−.27	58.6	−.22
18	72.0	−.58	69.2	−.50	66.7	−.43	64.3	−.37	62.1	−.31
19	76.0	−.71	73.1	−.61	70.4	−.54	67.9	−.46	65.5	−.40
20	80.0	−.84	76.9	−.74	74.1	−.65	71.4	−.56	69.0	−.50
21	84.0	−.99	80.8	−.87	77.8	−.77	75.0	−.67	72.4	−.59
22	88.0	−1.18	84.6	−1.02	81.5	−.90	78.6	−.79	75.9	−.70
23	92.0	−1.40	88.5	−1.20	85.1	−1.04	82.1	−.92	79.3	−.82
24	96.0	−1.75	92.3	−1.43	88.9	−1.23	85.7	−1.07	82.8	−.95
25	------	------	96.2	−1.77	92.6	−1.45	89.3	−1.24	86.2	−1.09
26	------	------	------	------	96.3	−1.79	92.9	−1.47	89.7	−1.26
27	------	------	------	------	------	------	96.4	−1.80	93.1	−1.48
28	------	------	------	------	------	------	------	------	96.6	−1.82

TABLE 3.—*Plotting positions (F_n) and standardized normal deviates (K_y) for a range of sample sizes or number of years of record (n)*—Continued

Rank No. (m)	n=29		n=30		n=31		n=32		n=33	
	F_n	K_y	F_n	K_y	F_n	K_y	F_n	K_y	F_n	K_y
	Percent		*Percent*		*Percent*		*Percent*		*Percent*	
1	3.3	1.84	3.2	1.85	3.1	1.87	3.0	1.88	2.9	1.90
2	6.7	1.50	6.5	1.51	6.2	1.54	6.1	1.55	5.9	1.56
3	10.0	1.28	9.7	1.30	9.4	1.32	9.1	1.33	8.8	1.35
4	13.3	1.11	12.9	1.13	12.5	1.15	12.1	1.17	11.8	1.18
5	16.7	.97	16.1	.99	15.6	1.01	15.2	1.03	14.7	1.05
6	20.0	.84	19.4	.86	18.8	.88	18.2	.91	17.6	.93
7	23.3	.73	22.6	.75	21.9	.78	21.2	.80	20.6	.82
8	26.7	.62	25.8	.65	25.0	.67	24.2	.70	23.5	.72
9	30.0	.52	29.0	.55	28.1	.58	27.3	.60	26.5	.63
10	33.3	.43	32.3	.46	31.2	.49	30.3	.52	29.4	.54
11	36.7	.34	35.5	.37	34.4	.40	33.3	.43	32.4	.46
12	40.0	.25	38.7	.29	37.5	.32	36.4	.35	35.3	.38
13	43.3	.17	41.9	.20	40.6	.24	39.4	.27	38.2	.30
14	46.7	.08	45.2	.12	43.8	.16	42.4	.19	41.2	.22
15	50.0	.00	48.4	.04	46.9	.08	45.5	.11	44.1	.15
16	53.3	-.08	51.6	-.04	50.0	.00	48.5	.04	47.1	.07
17	56.7	-.17	54.8	-.12	53.1	-.08	51.5	-.04	50.0	.00
18	60.0	-.25	58.1	-.20	56.2	-.16	54.5	-.11	53.0	-.07
19	63.3	-.34	61.3	-.29	59.4	-.24	57.6	-.19	55.9	-.15
20	66.7	-.43	64.5	-.37	62.5	-.32	60.6	-.27	58.8	-.22
21	70.0	-.52	67.7	-.46	65.6	-.40	63.6	-.35	61.8	-.30
22	73.3	-.62	71.0	-.55	68.8	-.49	66.7	-.43	64.7	-.38
23	76.7	-.73	74.2	-.65	71.9	-.58	69.7	-.52	67.6	-.46
24	80.0	-.84	77.4	-.75	75.0	-.67	72.7	-.60	70.6	-.54
25	83.3	-.97	80.6	-.86	78.1	-.78	75.8	-.70	73.5	-.63
26	86.7	-1.11	83.9	-.99	81.2	-.88	78.8	-.80	76.5	-.72
27	90.0	-1.28	87.1	-1.13	84.4	-1.01	81.8	-.91	79.4	-.82
28	93.3	-1.50	90.3	-1.30	87.5	-1.15	84.8	-1.03	82.4	-.93
29	96.7	-1.84	93.5	-1.51	90.6	-1.32	87.9	-1.17	85.3	-1.05
30	------	------	96.8	-1.85	93.8	-1.54	90.9	-1.33	88.2	-1.18
31	------	------	------	------	96.9	-1.87	93.9	-1.55	91.2	-1.35
32	------	------	------	------	------	------	97.0	-1.88	94.1	-1.56
33	------	------	------	------	------	------	------	------	97.1	-1.90

TABLE 3.—*Plotting positions (F_n) and standardized normal deviates (K_y) for a range of sample sizes or number of years of record (n)*—Continued

Rank No. (m)	n=34		n=35		n=36		n=37		n=38	
	F_n	K_y	F_n	K_y	F_n	K_y	F_n	K_y	F_n	K_y
	Percent		*Percent*		*Percent*		*Percent*		*Percent*	
1	2.9	1.90	2.8	1.91	2.7	1.93	2.6	1.94	2.6	1.94
2	5.7	1.58	5.6	1.59	5.4	1.61	5.3	1.62	5.1	1.64
3	8.6	1.37	8.3	1.39	8.1	1.40	7.9	1.41	7.7	1.43
4	11.4	1.21	11.1	1.22	10.8	1.24	10.5	1.25	10.3	1.26
5	14.3	1.07	13.9	1.08	13.5	1.10	13.2	1.12	12.8	1.13
6	17.1	.95	16.7	.97	16.2	.99	15.8	1.00	15.4	1.02
7	20.0	.84	19.4	.86	18.9	.88	18.4	.90	17.9	.92
8	22.9	.74	22.2	.77	21.6	.79	21.1	.80	20.5	.82
9	25.7	.65	25.0	.67	24.3	.70	23.7	.72	23.1	.74
10	28.6	.56	27.8	.59	27.0	.61	26.3	.63	25.6	.66
11	31.4	.48	30.6	.51	29.7	.53	28.9	.56	28.2	.58
12	34.3	.40	33.3	.43	32.4	.46	31.6	.48	30.8	.50
13	37.1	.33	36.1	.36	35.1	.38	34.2	.41	33.3	.43
14	40.0	.25	38.9	.28	37.8	.31	36.8	.34	35.9	.36
15	42.9	.18	41.7	.21	40.5	.24	39.5	.27	38.5	.29
16	45.7	.11	44.4	.14	43.2	.17	42.1	.20	41.0	.23
17	48.6	.04	47.2	.07	45.9	.10	44.7	.13	43.6	.16
18	51.4	-.04	50.0	.00	48.6	.04	47.4	.07	46.2	.10
19	54.3	-.11	52.8	-.07	51.4	-.04	50.0	.00	48.7	.03
20	57.1	-.18	55.6	-.14	54.1	-.10	52.6	-.07	51.3	-.03
21	60.0	-.25	58.3	-.21	56.8	-.17	55.3	-.13	53.8	-.10
22	62.9	-.33	61.1	-.28	59.5	-.24	57.9	-.20	56.4	-.16
23	65.7	-.40	63.9	-.36	62.2	-.31	60.5	-.27	59.0	-.23
24	68.6	-.48	66.7	-.43	64.9	-.38	63.2	-.34	61.5	-.29
25	71.4	-.56	69.4	-.51	67.6	-.46	65.8	-.41	64.1	-.36
26	74.3	-.65	72.2	-.59	70.3	-.53	68.4	-.48	66.7	-.43
27	77.1	-.74	75.0	-.67	73.0	-.61	71.1	-.56	69.2	-.50
28	80.0	-.84	77.8	-.77	75.7	-.70	73.7	-.63	71.8	-.58
29	82.9	-.95	80.6	-.86	78.4	-.79	76.3	-.72	74.4	-.66
30	85.7	-1.07	83.3	-.97	81.1	-.88	78.9	-.80	76.9	-.74
31	88.6	-1.21	86.1	-1.08	83.8	-.99	81.6	-.90	79.5	-.82
32	91.4	-1.37	88.9	-1.22	86.5	-1.10	84.2	-1.00	82.1	-.92
33	94.3	-1.58	91.7	-1.39	89.2	-1.24	86.8	-1.12	84.6	-1.02
34	97.1	-1.90	94.4	-1.59	91.9	-1.40	89.5	-1.25	87.2	-1.13
35	------	------	97.2	-1.91	94.6	-1.61	92.1	-1.41	89.7	-1.26
36	------	------	------	------	97.3	-1.93	94.7	-1.62	92.3	-1.43
37	------	------	------	------	------	------	97.4	-1.94	94.9	-1.64
38	------	------	------	------	------	------	------	------	97.4	-1.94

TABLE 3.—*Plotting positions (F_n) and standardized normal deviates (K_y) for a range of sample sizes or number of years of record (n)*—Continued

Rank No. (m)	n=39 F_n	n=39 K_y	n=40 F_n	n=40 K_y	n=41 F_n	n=41 K_y	n=42 F_n	n=42 K_y	n=43 F_n	n=43 K_y
	Percent		*Percent*		*Percent*		*Percent*		*Percent*	
1	2. 5	1. 96	2. 4	1. 98	2. 4	1. 98	2. 3	1. 99	2. 3	2. 00
2	5. 0	1. 64	4. 9	1. 66	4. 8	1. 67	4. 7	1. 68	4. 5	1. 69
3	7. 5	1. 44	7. 3	1. 45	7. 1	1. 47	7. 0	1. 48	6. 8	1. 49
4	10. 0	1. 28	9. 8	1. 29	9. 5	1. 31	9. 3	1. 32	9. 1	1. 34
5	12. 5	1. 15	12. 2	1. 16	11. 9	1. 18	11. 6	1. 19	11. 4	1. 21
6	15. 0	1. 04	14. 6	1. 05	14. 3	1. 07	14. 0	1. 08	13. 6	1. 10
7	17. 5	. 93	17. 1	. 95	16. 7	. 97	16. 3	. 98	15. 9	1. 00
8	20. 0	. 84	19. 5	. 86	19. 0	. 88	18. 6	. 89	18. 2	. 91
9	22. 5	. 76	22. 0	. 77	21. 4	. 79	20. 9	. 81	20. 5	. 83
10	25. 0	. 67	24. 4	. 69	23. 8	. 71	23. 3	. 73	22. 7	. 75
11	27. 5	. 60	26. 8	. 62	26. 2	. 64	25. 6	. 66	25. 0	. 67
12	30. 0	. 52	29. 3	. 54	28. 6	. 57	27. 9	. 59	27. 3	. 60
13	32. 5	. 45	31. 7	. 48	31. 0	. 50	30. 2	. 52	29. 5	. 54
14	35. 0	. 39	34. 1	. 41	33. 3	. 43	32. 6	. 45	31. 8	. 47
15	37. 5	. 32	36. 6	. 34	35. 7	. 37	34. 9	. 39	34. 1	. 41
16	40. 0	. 25	39. 0	. 28	38. 1	. 30	37. 2	. 33	36. 4	. 35
17	42. 5	. 19	41. 5	. 21	40. 5	. 24	39. 5	. 27	38. 6	. 29
18	45. 0	. 13	43. 9	. 15	42. 9	. 18	41. 9	. 21	40. 9	. 23
19	47. 5	. 06	46. 3	. 09	45. 2	. 12	44. 2	. 15	43. 2	. 17
20	50. 0	. 00	48. 8	. 03	47. 6	. 06	46. 5	. 09	45. 5	. 11
21	52. 5	−. 06	51. 2	−. 03	50. 0	. 00	48. 8	. 03	47. 7	. 06
22	55. 0	−. 13	53. 7	−. 09	52. 4	−. 06	51. 1	−. 03	50. 0	. 00
23	57. 5	−. 19	56. 1	−. 15	54. 8	−. 12	53. 5	−. 09	52. 3	−. 06
24	60. 0	−. 25	58. 5	−. 21	57. 1	−. 18	55. 8	−. 15	54. 5	−. 11
25	62. 5	−. 32	61. 0	−. 28	59. 5	−. 24	58. 1	−. 21	56. 8	−. 17
26	65. 0	−. 39	53. 4	−. 34	61. 9	−. 30	60. 5	−. 27	59. 1	−. 23
27	67. 5	−. 45	65. 9	−. 41	64. 3	−. 37	62. 8	−. 33	61. 4	−. 29
28	70. 0	−. 52	68. 3	−. 48	66. 7	−. 43	65. 1	−. 39	63. 6	−. 35
29	72. 5	−. 60	70. 7	−. 54	69. 0	−. 50	67. 4	−. 45	65. 9	−. 41
30	75. 0	−. 67	73. 2	−. 62	71. 4	−. 57	69. 8	−. 52	68. 2	−. 47
31	77. 5	−. 76	75. 6	−. 69	73. 8	−. 64	72. 1	−. 59	70. 5	−. 54
32	80. 0	−. 84	78. 0	−. 77	76. 2	−. 71	74. 4	−. 66	72. 7	−. 60
33	82. 5	−. 93	80. 5	−. 86	78. 6	−. 79	76. 7	−. 73	75. 0	−. 67
34	85. 0	−1. 04	82. 9	−. 95	81. 0	−. 88	79. 1	−. 81	77. 3	−. 75
35	87. 5	−1. 15	85. 4	−1. 05	83. 3	−. 97	81. 4	−. 89	79. 5	−. 83
36	90. 0	−1. 28	87. 8	−1. 16	85. 7	−1. 07	83. 7	−. 98	81. 8	−. 91
37	92. 5	−1. 44	90. 2	−1. 29	88. 1	−1. 18	86. 0	−1. 08	84. 1	−1. 00
38	95. 0	−1. 64	92. 7	−1. 45	90. 5	−1. 31	88. 4	−1. 19	86. 4	−1. 10
39	97. 5	−1. 96	95. 1	−1. 66	92. 9	−1. 47	90. 7	−1. 32	88. 6	−1. 21
40	------	------	97. 6	−1. 98	95. 2	−1. 67	93. 0	−1. 48	90. 9	−1. 34
41	------	------	------	------	97. 6	−1. 98	95. 3	−1. 68	93. 2	−1. 49
42	------	------	------	------	------	------	97. 7	−1. 99	95. 5	−1. 69
43	------	------	------	------	------	------	------	------	97. 7	−2. 00

TABLE 4.—*Theoretical log-probability frequency factors* [1]

CV	K values for probability in percentage greater than the given variate: 99 −	95 −	80 −	50 −	20 +	5 +	1 +	0.1 +	0.01 +
0. 000	2. 33	1. 64	0. 84	0. 00	0. 84	1. 64	2. 33	3. 09	3. 72
. 010	2. 31	1. 64	. 85	. 01	. 84	1. 65	2. 35	3. 13	3. 79
. 020	2. 29	1. 63	. 85	. 02	. 84	1. 66	2. 38	3. 17	3. 85
. 030	2. 27	1. 63	. 85	. 02	. 84	1. 67	2. 40	3. 22	3. 91
. 040	2. 24	1. 62	. 85	. 03	. 84	1. 68	2. 42	3. 26	3. 98
. 050	2. 22	1. 61	. 85	. 03	. 83	1. 69	2. 44	3. 30	4. 05
. 060	2. 20	1. 60	. 85	. 04	. 83	1. 69	2. 46	3. 35	4. 12
. 070	2. 18	1. 59	. 85	. 04	. 83	1. 70	2. 49	3. 40	4. 19
. 080	2. 16	1. 58	. 85	. 05	. 82	1. 71	2. 51	3. 45	4. 26
. 090	2. 13	1. 57	. 85	. 05	. 82	1. 72	2. 53	3. 50	4. 34
. 100	2. 11	1. 56	. 85	. 06	. 82	1. 72	2. 55	3. 56	4. 42
. 125	2. 06	1. 54	. 85	. 07	. 81	1. 74	2. 61	3. 67	4. 61
. 150	2. 01	1. 51	. 86	. 08	. 81	1. 76	2. 66	3. 80	4. 81
. 175	1. 96	1. 49	. 85	. 09	. 80	1. 78	2. 72	3. 92	5. 03
. 200	1. 91	1. 46	. 85	. 10	. 79	1. 79	2. 78	4. 05	5. 24
. 225	1. 86	1. 44	. 85	. 11	. 78	1. 81	2. 83	4. 18	5. 48
. 250	1. 81	1. 41	. 84	. 12	. 77	1. 82	2. 88	4. 32	5. 71
. 275	1. 77	1. 39	. 84	. 13	. 76	1. 83	2. 94	4. 46	5. 96
. 300	1. 72	1. 36	. 84	. 14	. 75	1. 84	2. 99	4. 60	6. 19
. 325	1. 68	1. 34	. 83	. 15	. 74	1. 85	3. 04	4. 74	6. 44
. 350	1. 63	1. 32	. 83	. 16	. 73	1. 86	3. 10	4. 88	6. 70
. 364	1. 61	1. 30	. 82	. 16	. 73	1. 87	3. 12	4. 94	6. 82
. 375	1. 59	1. 29	. 82	. 17	. 72	1. 87	3. 15	5. 02	6. 96
. 400	1. 55	1. 27	. 82	. 18	. 71	1. 87	3. 20	5. 16	7. 22
. 425	1. 51	1. 24	. 81	. 19	. 70	1. 88	3. 25	5. 30	7. 49
. 450	1. 47	1. 22	. 81	. 20	. 69	1. 88	3. 30	5. 44	7. 78
. 475	1. 43	1. 20	. 80	. 20	. 68	1. 89	3. 34	5. 58	8. 06
. 500	1. 40	1. 17	. 80	. 21	. 66	1. 89	3. 38	5. 71	8. 35
. 525	1. 37	1. 15	. 79	. 22	. 65	1. 89	3. 42	5. 86	8. 65
. 550	1. 34	1. 13	. 78	. 22	. 64	1. 89	3. 46	6. 00	8. 96
. 575	1. 31	1. 11	. 78	. 23	. 63	1. 89	3. 50	6. 14	9. 27
. 600	1. 28	1. 09	. 77	. 24	. 61	1. 89	3. 53	6. 27	9. 57
. 625	1. 24	1. 07	. 76	. 24	. 60	1. 89	3. 57	6. 41	9. 88
. 650	1. 22	1. 05	. 76	. 25	. 59	1. 89	3. 60	6. 55	10. 20
. 675	1. 19	1. 03	. 75	. 26	. 58	1. 89	3. 63	6. 70	10. 51
. 700	1. 16	1. 01	. 74	. 26	. 57	1. 88	3. 66	6. 83	10. 83
. 725	1. 13	. 99	. 73	. 27	. 56	1. 88	3. 69	6. 97	11. 15
. 750	1. 11	. 98	. 73	. 27	. 54	1. 87	3. 71	7. 10	11. 48
. 775	1. 08	. 96	. 72	. 27	. 53	1. 87	3. 74	7. 24	11. 80
. 800	1. 06	. 94	. 71	. 28	. 52	1. 86	3. 76	7. 37	12. 12
. 825	1. 03	. 92	. 70	. 28	. 51	1. 85	3. 78	7. 50	12. 45
. 850	1. 01	. 91	. 70	. 28	. 50	1. 85	3. 80	7. 62	12. 79
. 875	. 99	. 89	. 69	. 29	. 48	1. 84	3. 82	7. 74	13. 11
. 900	. 97	. 88	. 68	. 29	. 47	1. 83	3. 84	7. 86	13. 44
. 925	. 95	. 86	. 67	. 29	. 46	1. 82	3. 86	7. 98	13. 77
. 950	. 93	. 85	. 67	. 29	. 45	1. 81	3. 88	8. 09	14. 08
. 975	. 92	. 83	. 66	. 29	. 43	1. 79	3. 90	8. 20	14. 39
1. 000	. 90	. 82	. 65	. 29	. 42	1. 78	3. 91	8. 30	14. 70

[1] Adapted from Chow (*9*). Each column of Chow's table 2 was plotted against *CV*. Values in this table were then read from these graphs. Additional values may be found by a graphical method described by Chow (*10*).

TABLE 5.—*Fitting a normal distribution to annual runoff amounts* (Q)

Rank No. (m)	Ranked runoff (Q $(=x)$)	Plotting position (F_n)[1]	Standard normal deviate K_y)[1]	Computations
	In./Yr.	*Percent*		
1	23. 34	5. 0	1. 64	$\Sigma Q^2=3514.7377$
2	19. 37	10. 0	1. 28	
3	18. 46	15. 0	1. 04	$(\Sigma Q)^2/19=3089.7076$
4	17. 65	20. 0	. 84	
5	16. 72	25. 0	. 67	$\Sigma(Q-\bar{Q})^2=425.0301$
6	14. 10	30. 0	. 52	$\Sigma Q\ K_y=76.5454$
7	13. 75	35. 0	. 39	
8	12. 92	40. 0	. 25	$\bar{Q}=242.29/19$
9	12. 71	45. 0	. 13	
10	12. 70	50. 0	. 00	$=12.7521$
11	12. 43	55. 0	—. 13	By equation 7:
12	11. 52	60. 0	—. 25	
13	11. 24	65 0	—. 39	
14	10. 39	70. 0	—. 52	$s=425.0301/76.5454$
15	9. 93	75. 0	—. 67	
16	8. 28	80. 0	—. 84	$=5.5527$
17	6. 72	85. 0	−1. 04	
18	5. 69	90. 0	−1. 28	
19	4. 37	95. 0	−1. 64	
Total	242. 29	--------	--------	

[1] Read from the $n=19$ column of table 3.

If the values of s and $\bar{Q}$ are substituted into equation 9, the fitted frequency equation becomes

$$Q=12.75+5.55K.$$

Two points lying on the fitted frequency curve are found as follows:

(1) From the $CV=0$ row of table 4, read $K=1.64$ corresponding to a probability of 5 percent. Then the value of Q for this probability level is calculated to be

$$Q=12.75+5.55(1.64)=21.9.$$

(2) From the $CV=0$ row of table 4, read $K=-1.64$ corresponding to a probability of 95 percent. Then the value of Q for this probability level is calculated to be

$$Q=12.75+5.55(-1.64)=3.6.$$

Columns 2 and 3 of the above data are plotted on normal probability paper, figure 24. The fitted line is oriented by drawing it through the above calculated points, i.e., 5 percent=21.9 and 95 percent=3.6.

TABLE 6.—*Fitting an extreme value distribution to 24-hour maximum precipitation amounts* (P)

Rank No. (m)	Ranked precipitation ($P(=x)$)	Plotting position (Fn)[1]	Standard normal deviate (K_y)[1]	Computations
	Inches	*Percent*		
1	3. 49	5. 6	1. 60	$\Sigma P^2=103.3595$
2	3. 37	11. 1	1. 22	
3	3. 14	16. 7	. 97	$(\Sigma P)^2/17=97.4884$
4	2. 97	22. 2	. 76	
5	2. 70	27. 8	. 59	$\Sigma(P-\bar{P})^2=5.8711$
6	2. 63	33. 3	. 43	$\Sigma P\ K_y=8.2862$
7	2. 59	38. 9	. 29	
8	2. 54	44. 4	. 14	$\bar{P}=40.71/17$
9	2. 47	50. 0	. 00	
10	2. 21	55. 6	—. 14	$=2.3947$
11	2. 06	61. 1	—. 29	
12	1. 90	66. 7	—. 43	By equation 10:
13	1. 77	72. 2	—. 59	
14	1. 75	77. 8	—. 76	$s=1.067(5.8711)/8.2862$
15	1. 75	83. 3	—. 97	
16	1. 73	88. 9	−1. 22	$=0.7560$
17	1. 66	94. 4	−1. 60	
Total	40. 71	--------	--------	

[1] Read from the $n=17$ column of table 3.

If the values of s and $\bar{P}$ are substituted into equation 9, the fitted frequency equation becomes

$$P=2.39+0.76K.$$

Two points lying on the fitted frequency curve are found as follows:

(1) From the $CV=0.364$ row of table 4, read $K=1.87$ corresponding to a probability of 5 percent. Then the value of P for this probability level is calculated to be

$$P=2.39+0.76(1.87)=3.81.$$

(2) From the $CV=0.364$ row of table 4, read $K=-1.30$ corresponding to a probability of 95 percent. Then the value of P for this probability level is calculated to be

$$P=2.39+0.76(-1.30)=1.40.$$

Columns 2 and 3 of the above data are plotted on extreme-value probability paper, figure 25. The fitted line is oriented by drawing it through the above calculated points, i.e., 5 percent=3.81 and 95 percent=1.40.

FIGURES

473-008 O - 72 - 3

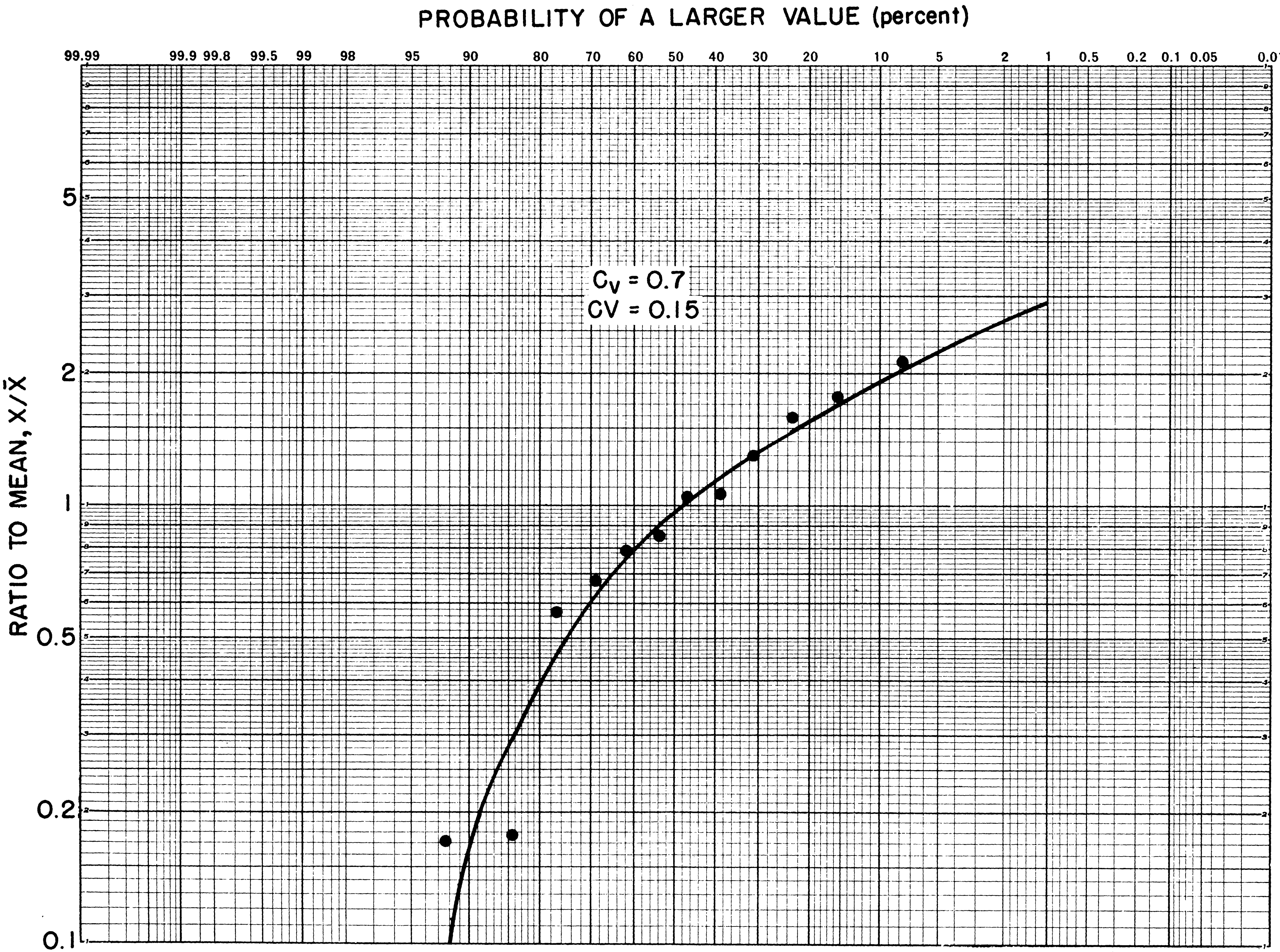

FIGURE 1.—Modified log-normal frequency curve fitted graphically to data of table 2.

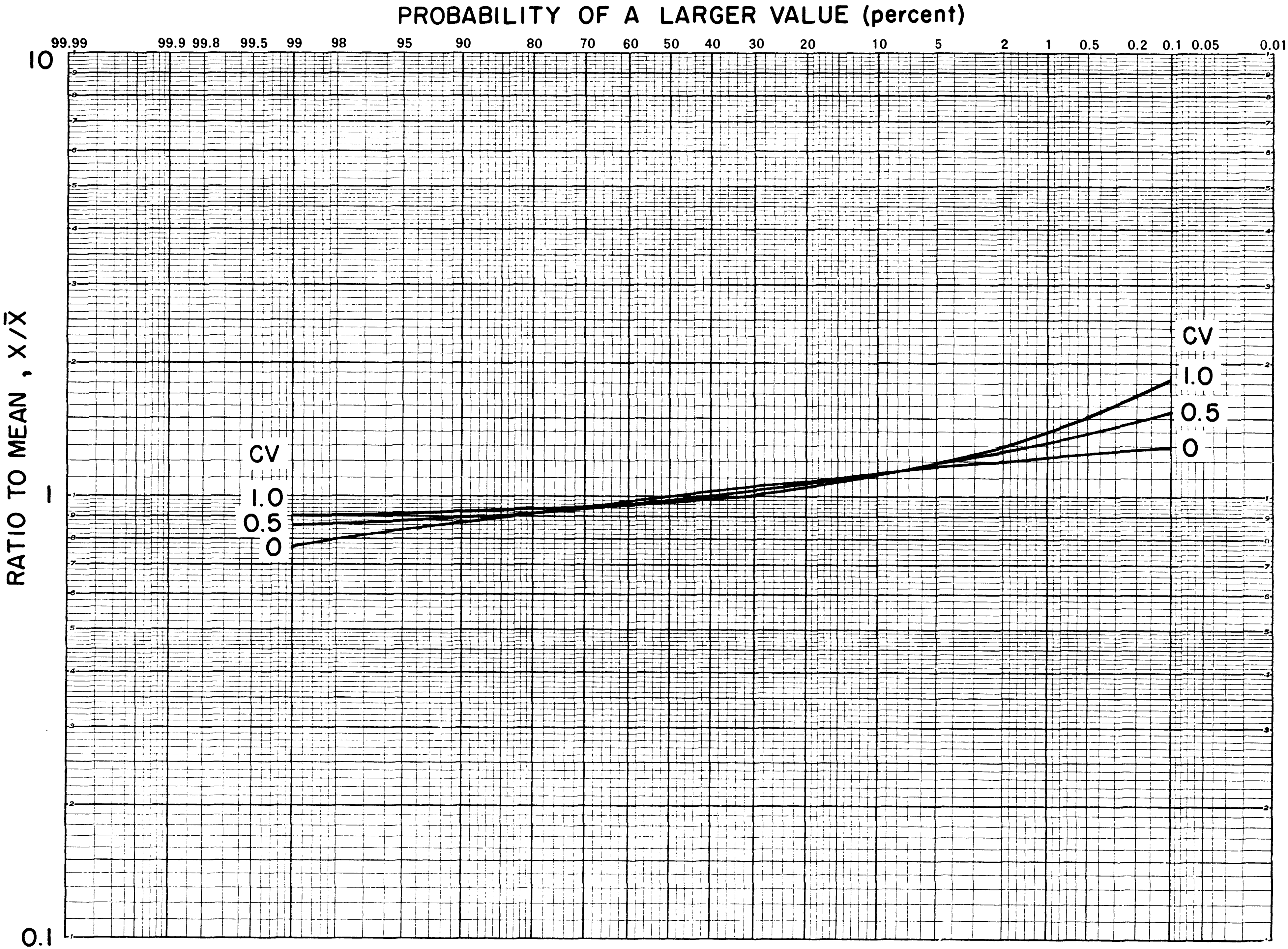

FIGURE 2.—Modified log-normal curves for C_v=0.1.

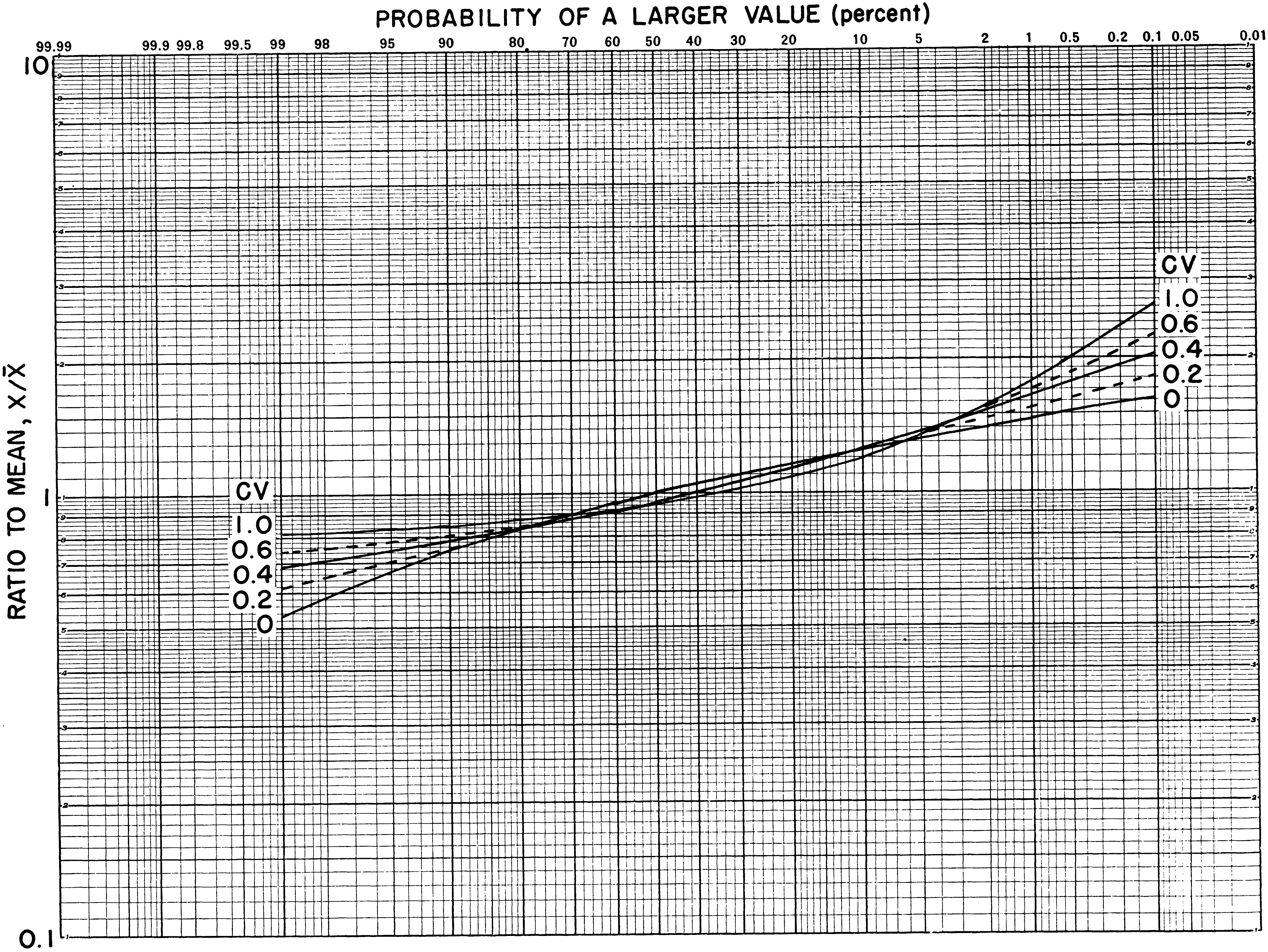

FIGURE 3.—Modified log-normal curves for C_v=0.2.

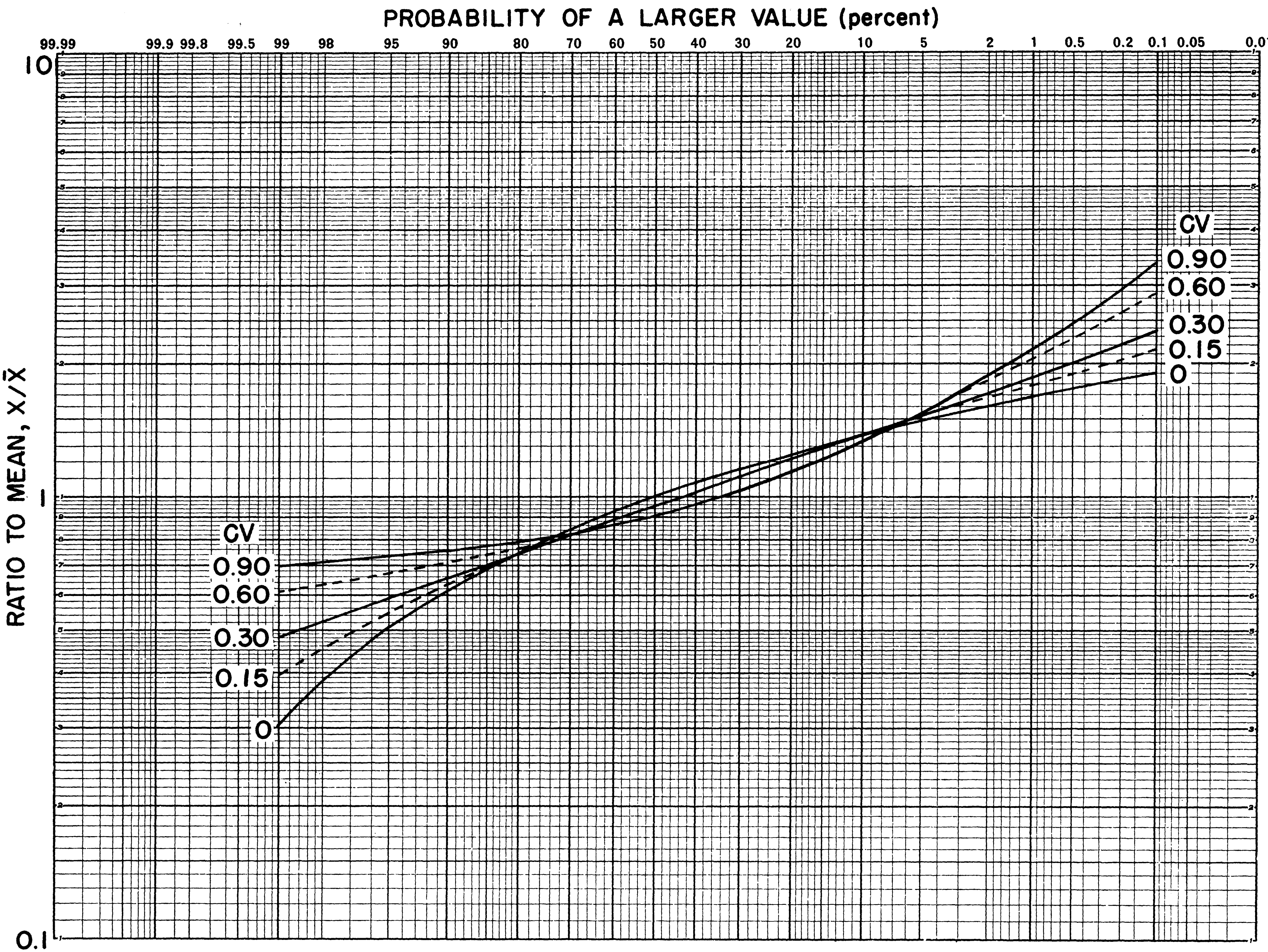

FIGURE 4.—Modified log-normal curves for C_v=0.3.

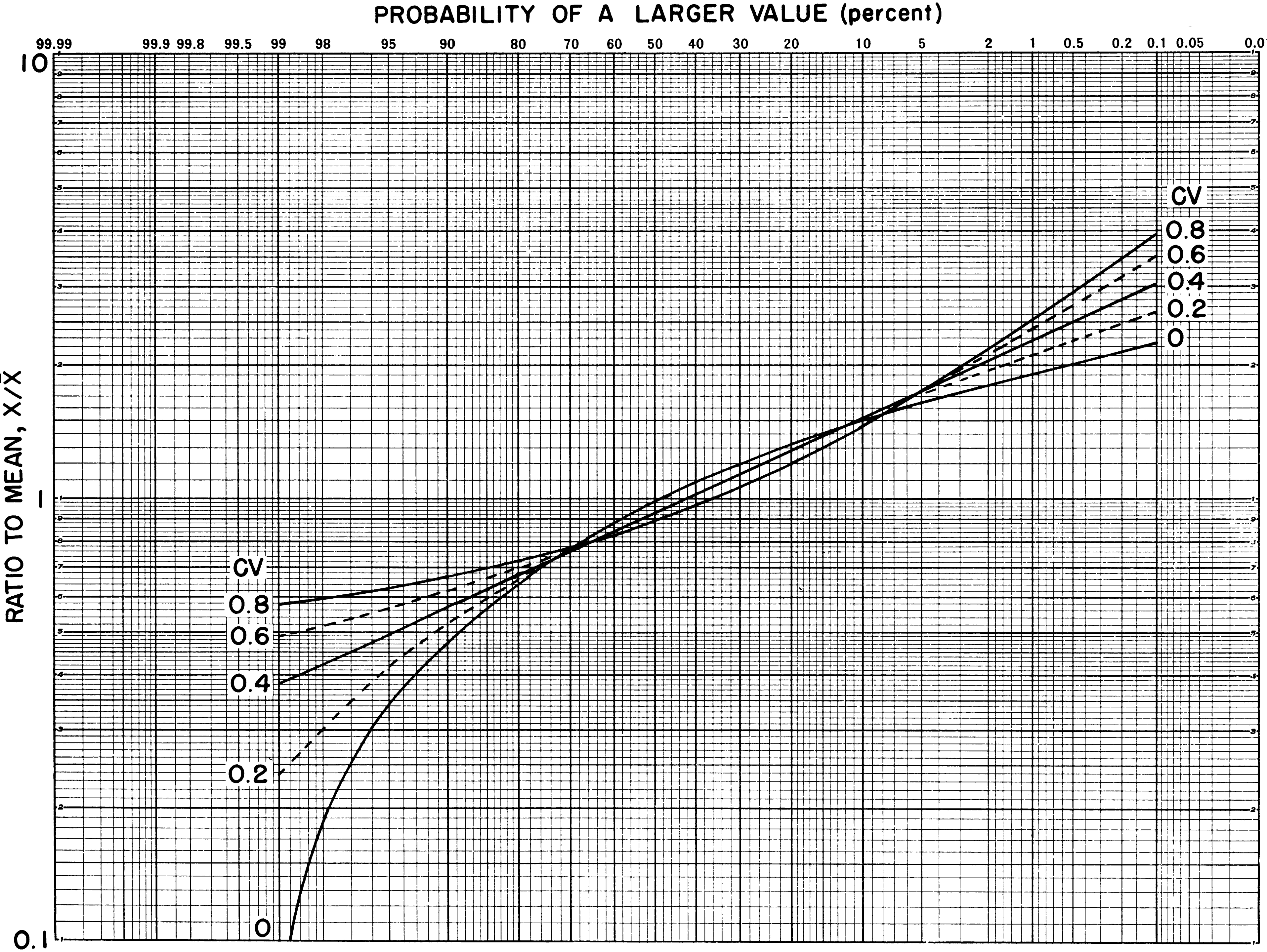

FIGURE 5.—Modified log-normal curves for C_v=0.4.

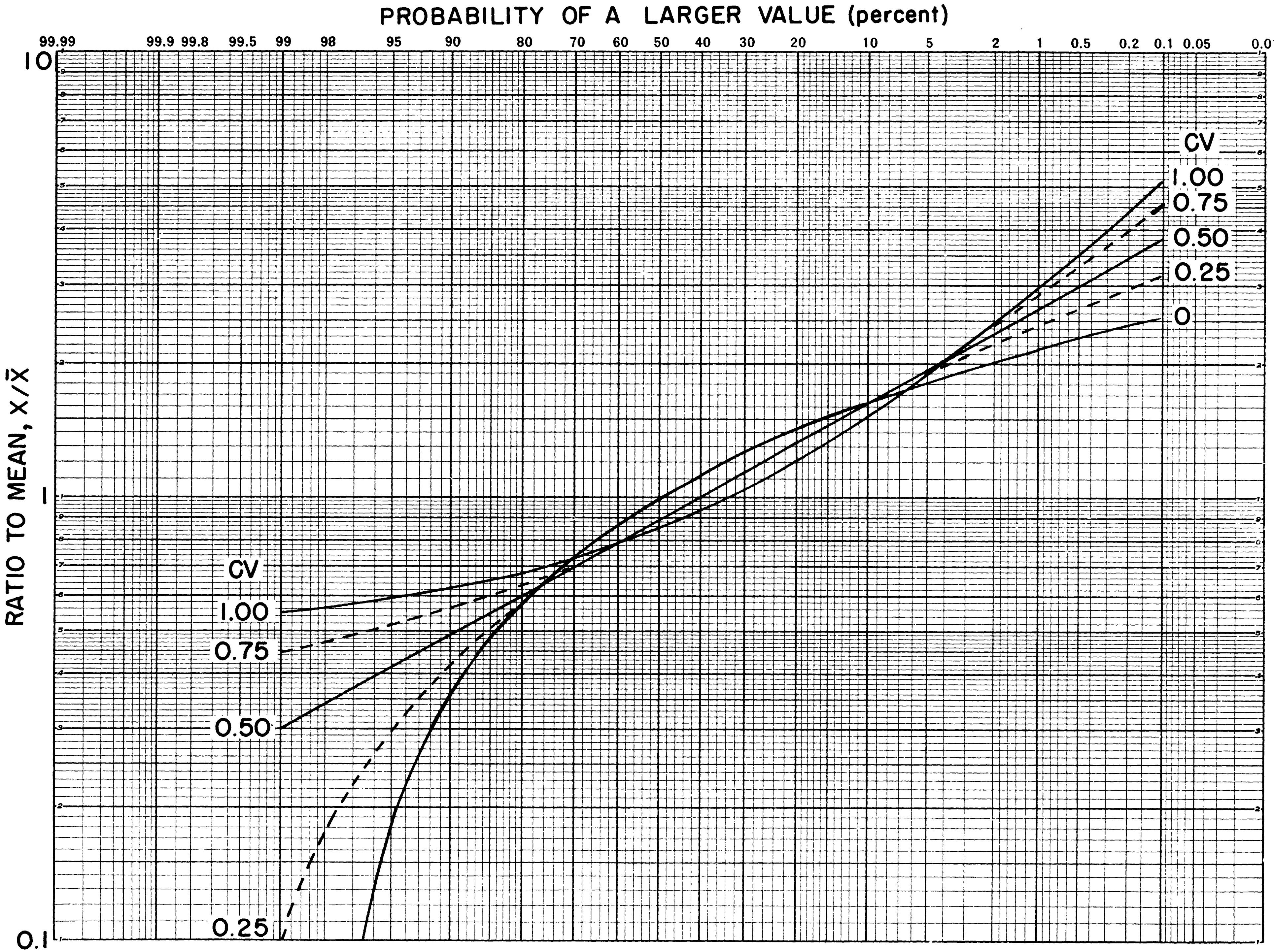

FIGURE 6.—Modified log-normal curves for C_v=0.5.

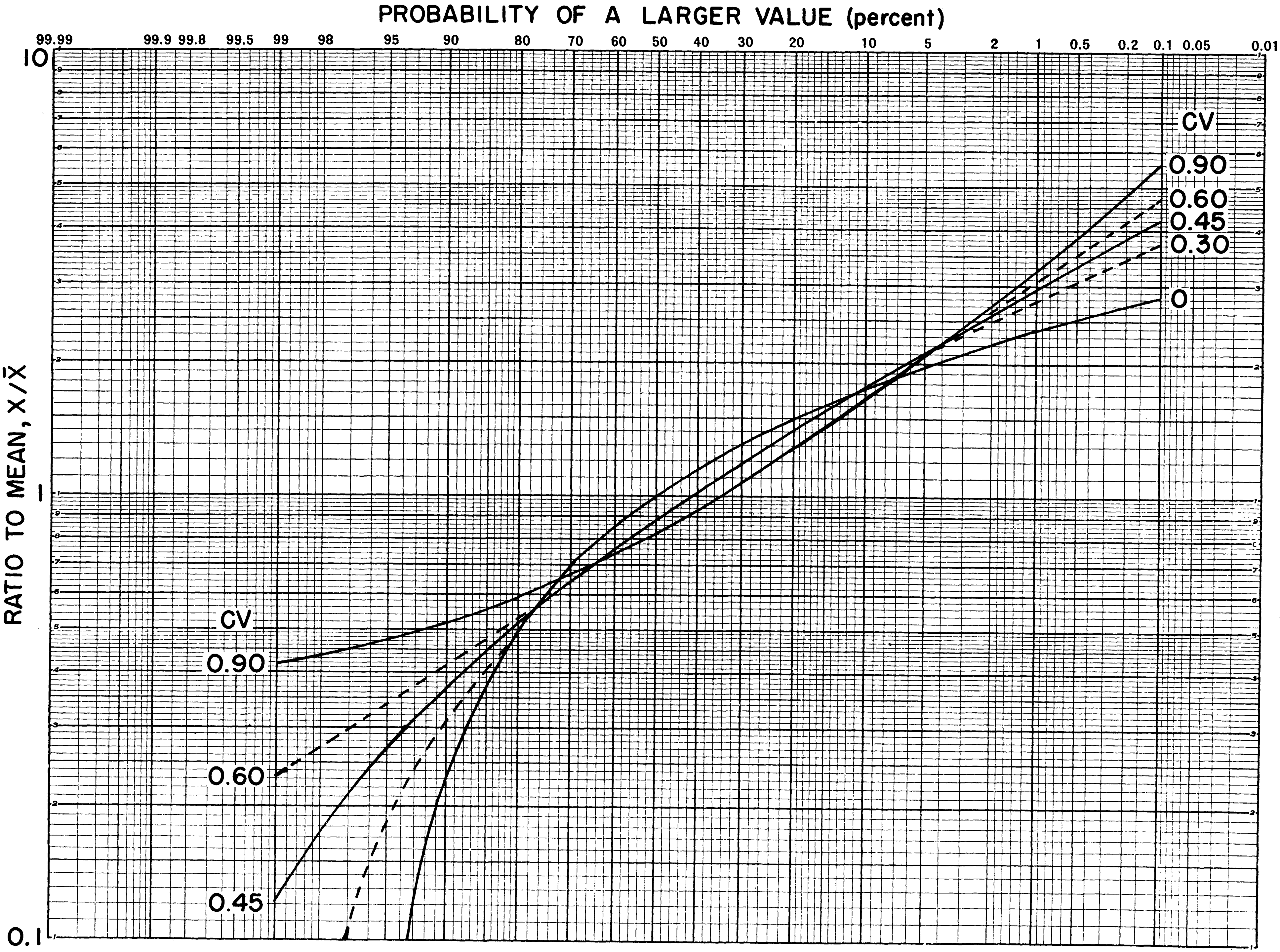

FIGURE 7.—Modified log-normal curves for C_v=0.6.

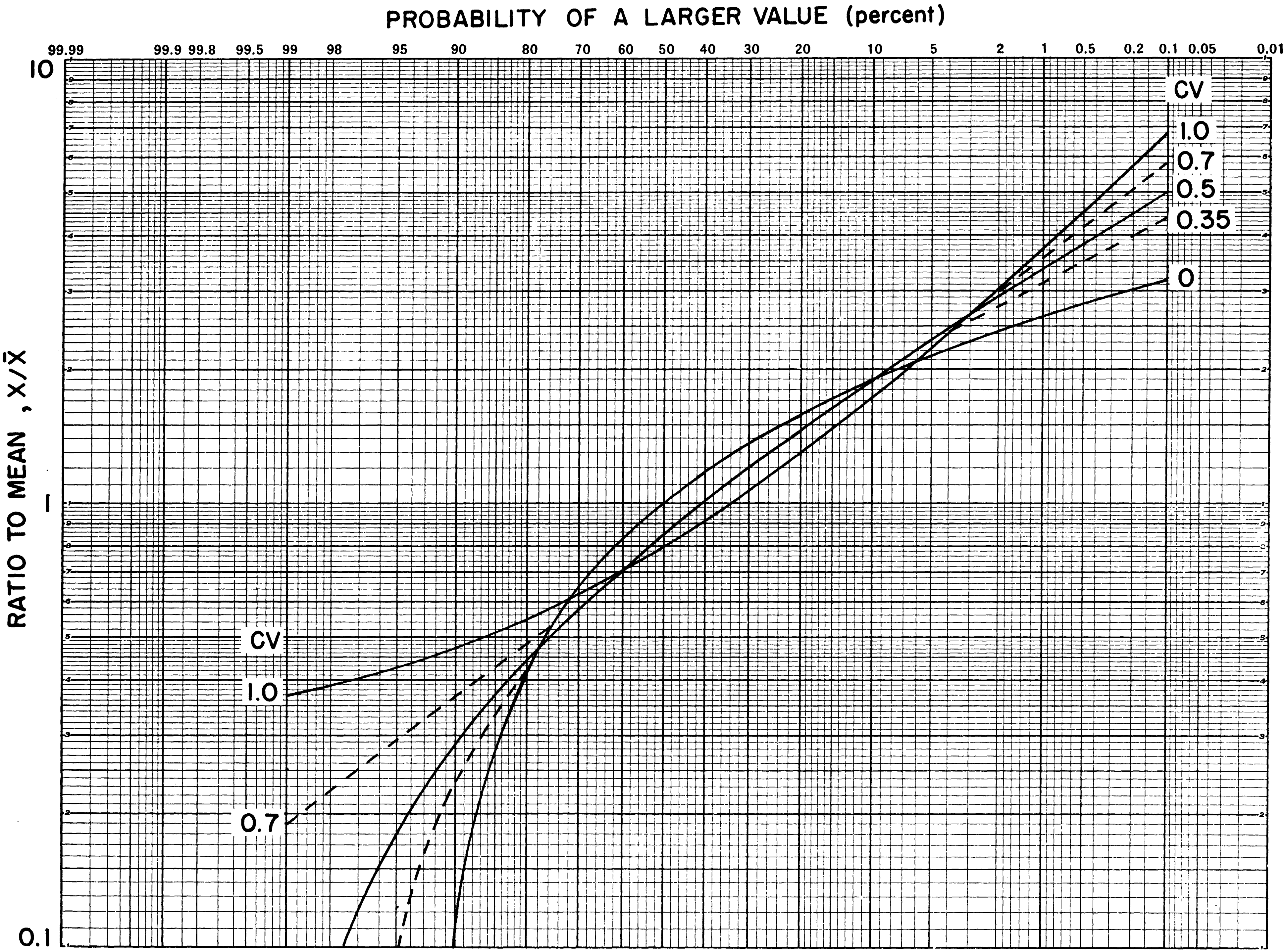

FIGURE 8.—Modified log-normal curves for C_v=0.7.

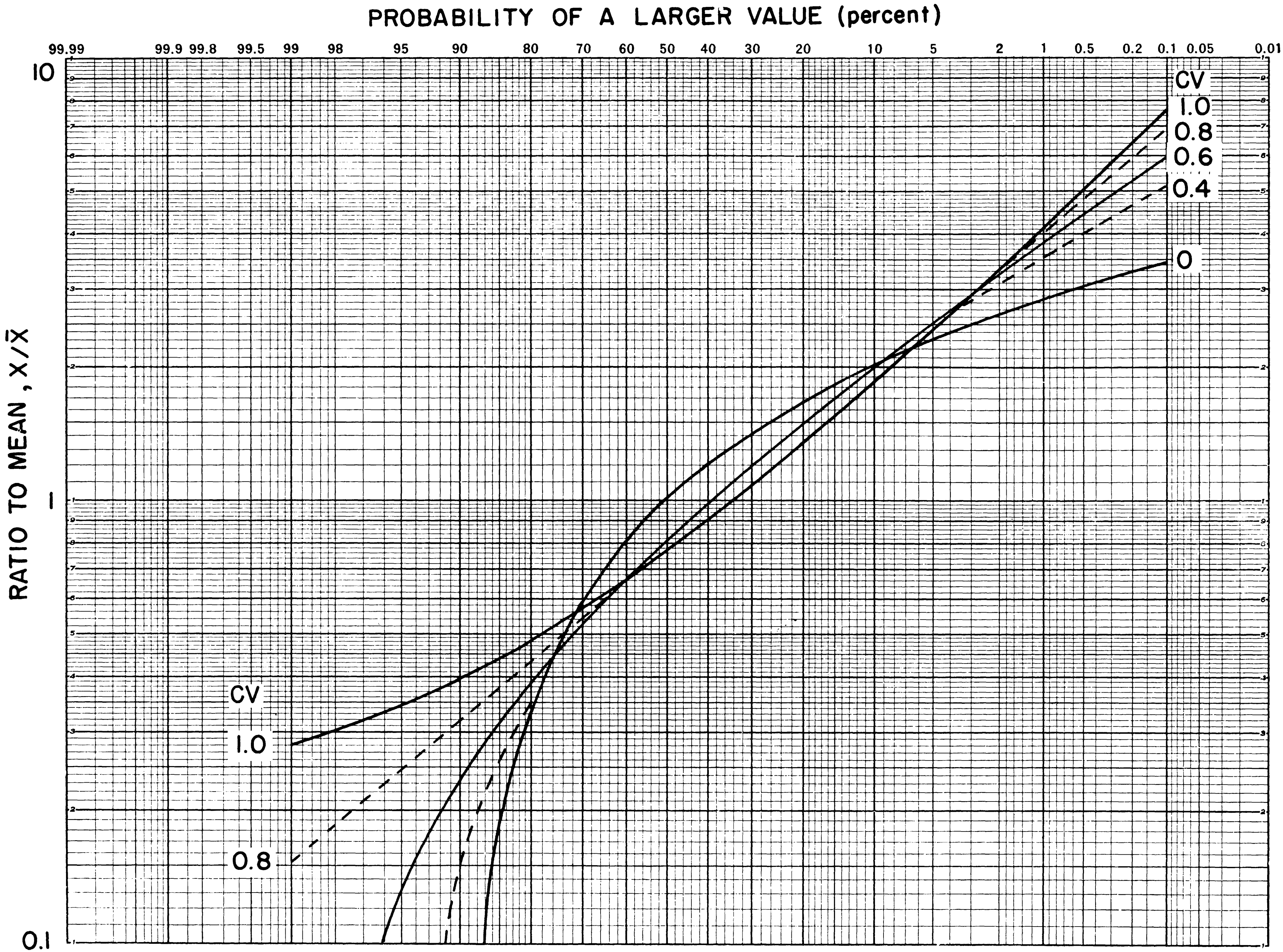

FIGURE 9.—Modified log-normal curves for $C_v=0.8$.

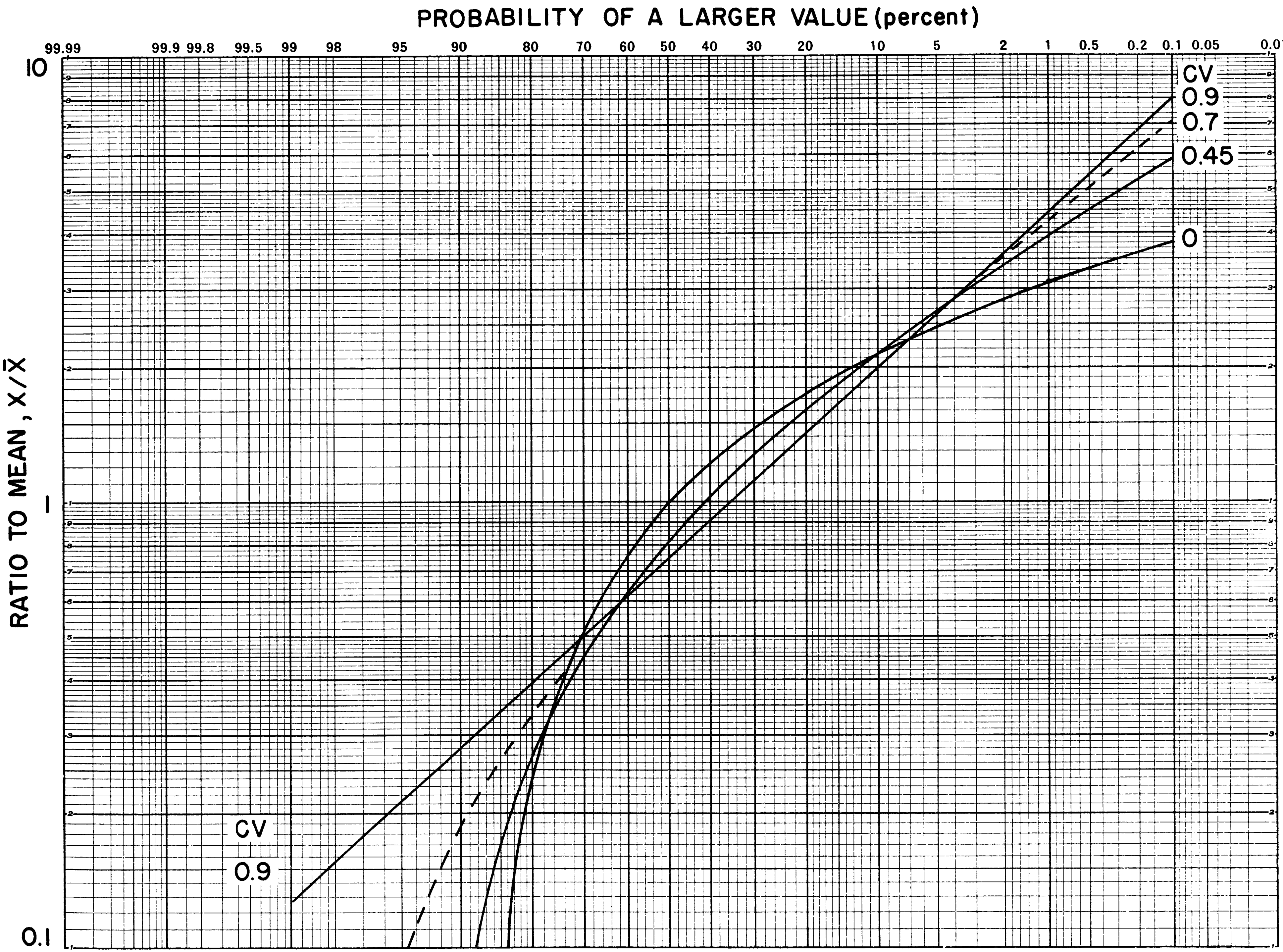

FIGURE 10.—Modified log-normal curves for C_v=0.9.

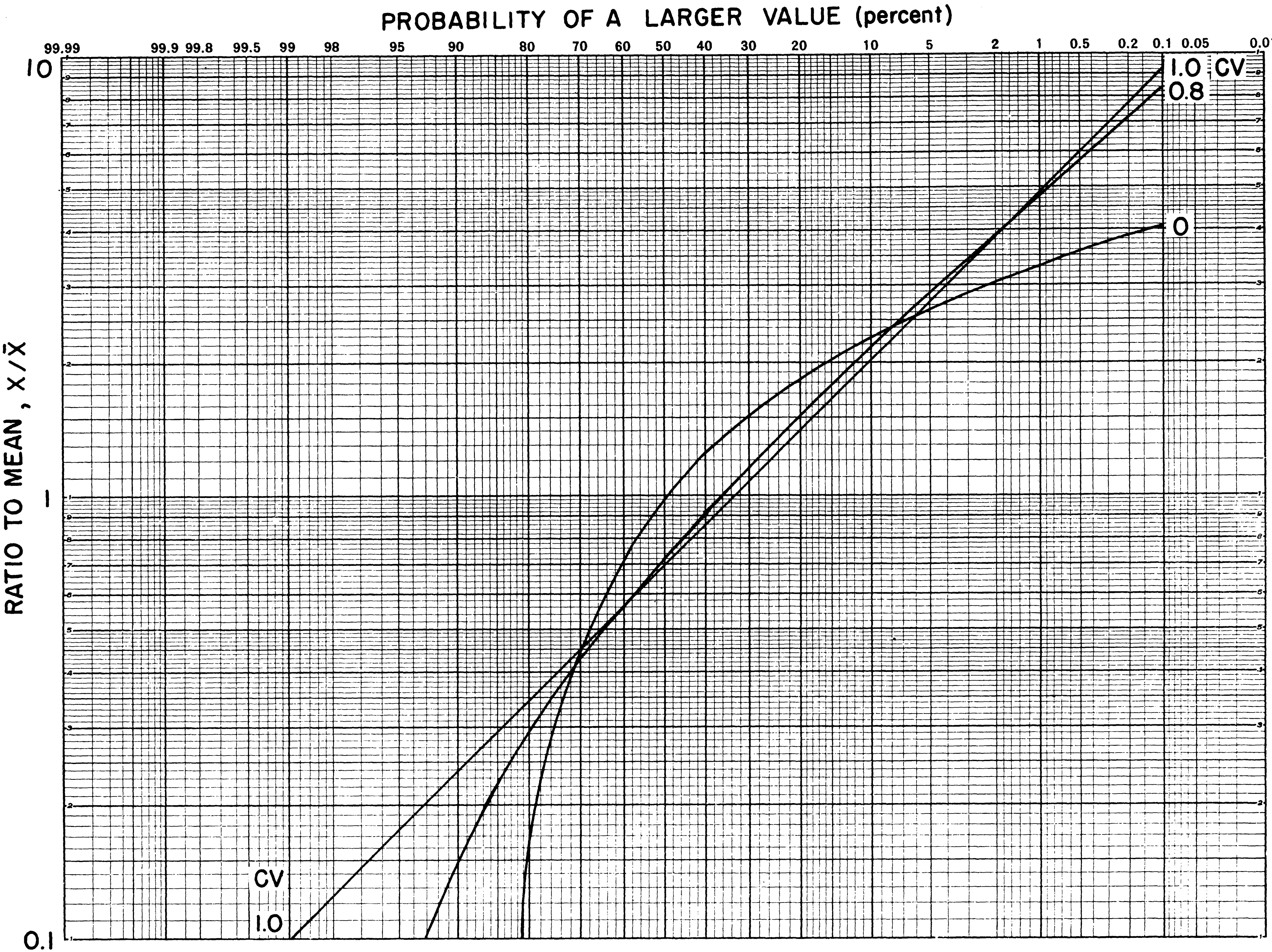

FIGURE 11.—Modified log-normal curves for C_v=1.0.

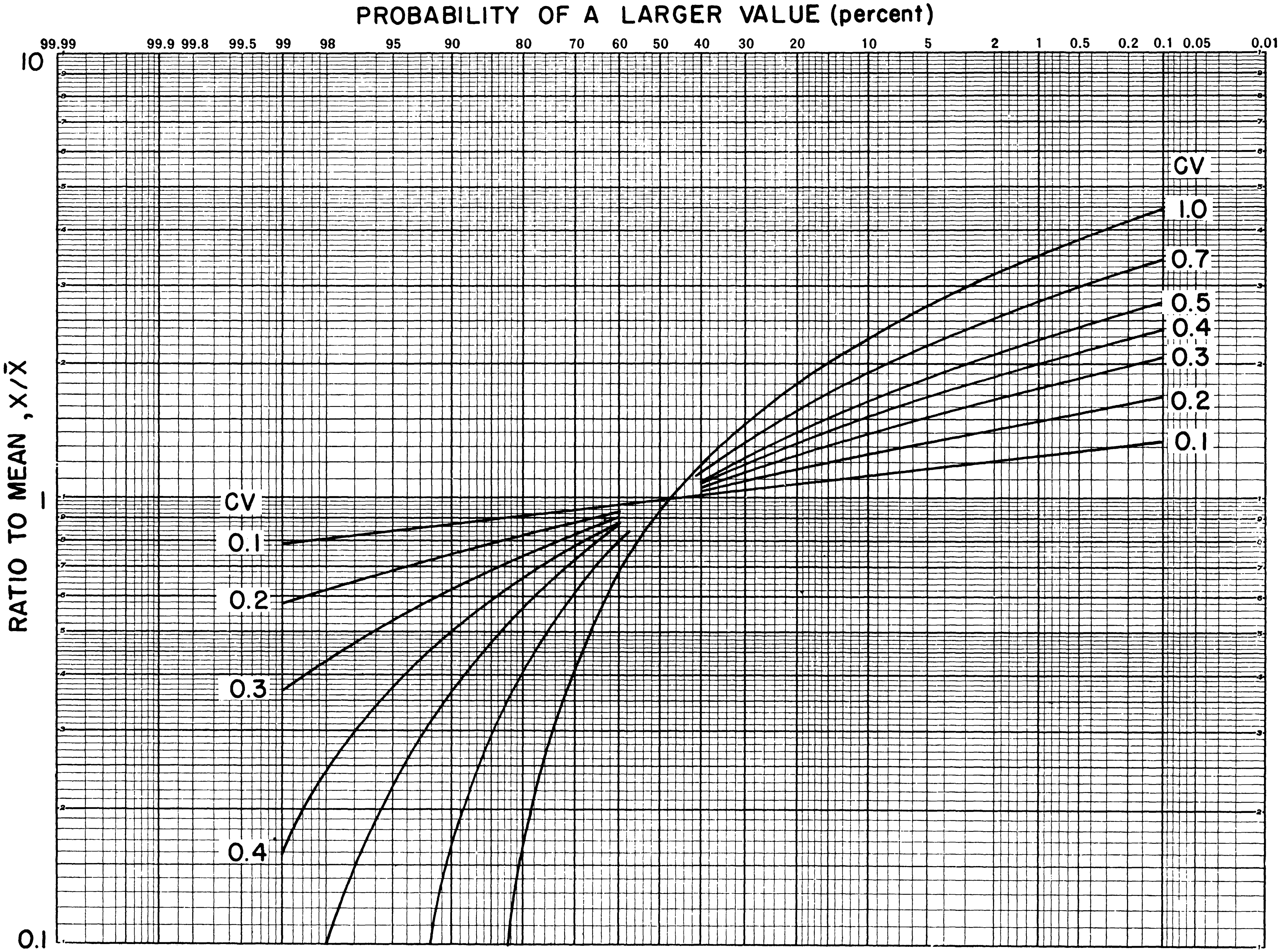

FIGURE 13.—Modified log-normal curves for $CV = 0.1$.

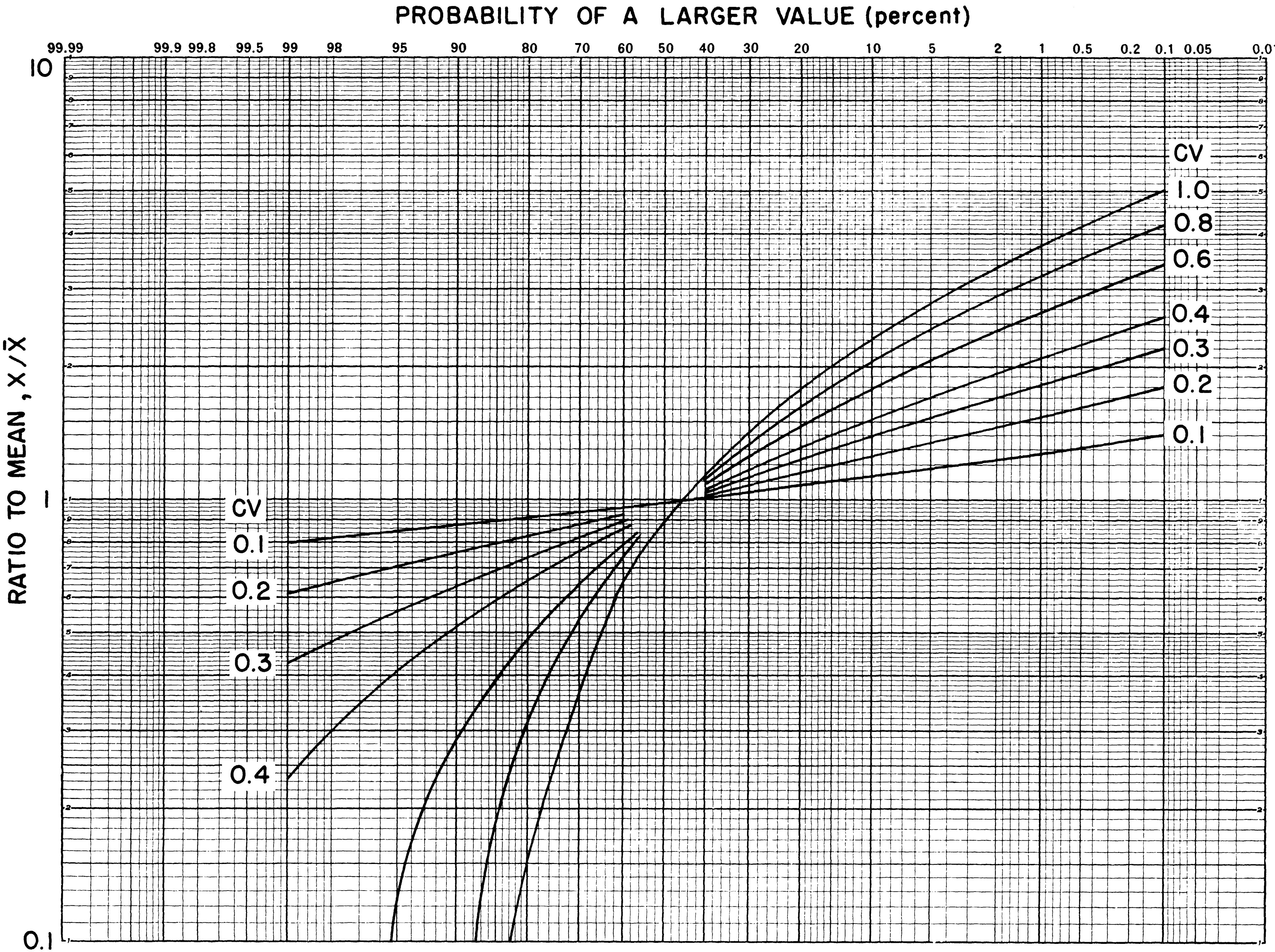

FIGURE 14.—Modified log-normal curves for $CV=0.2$.

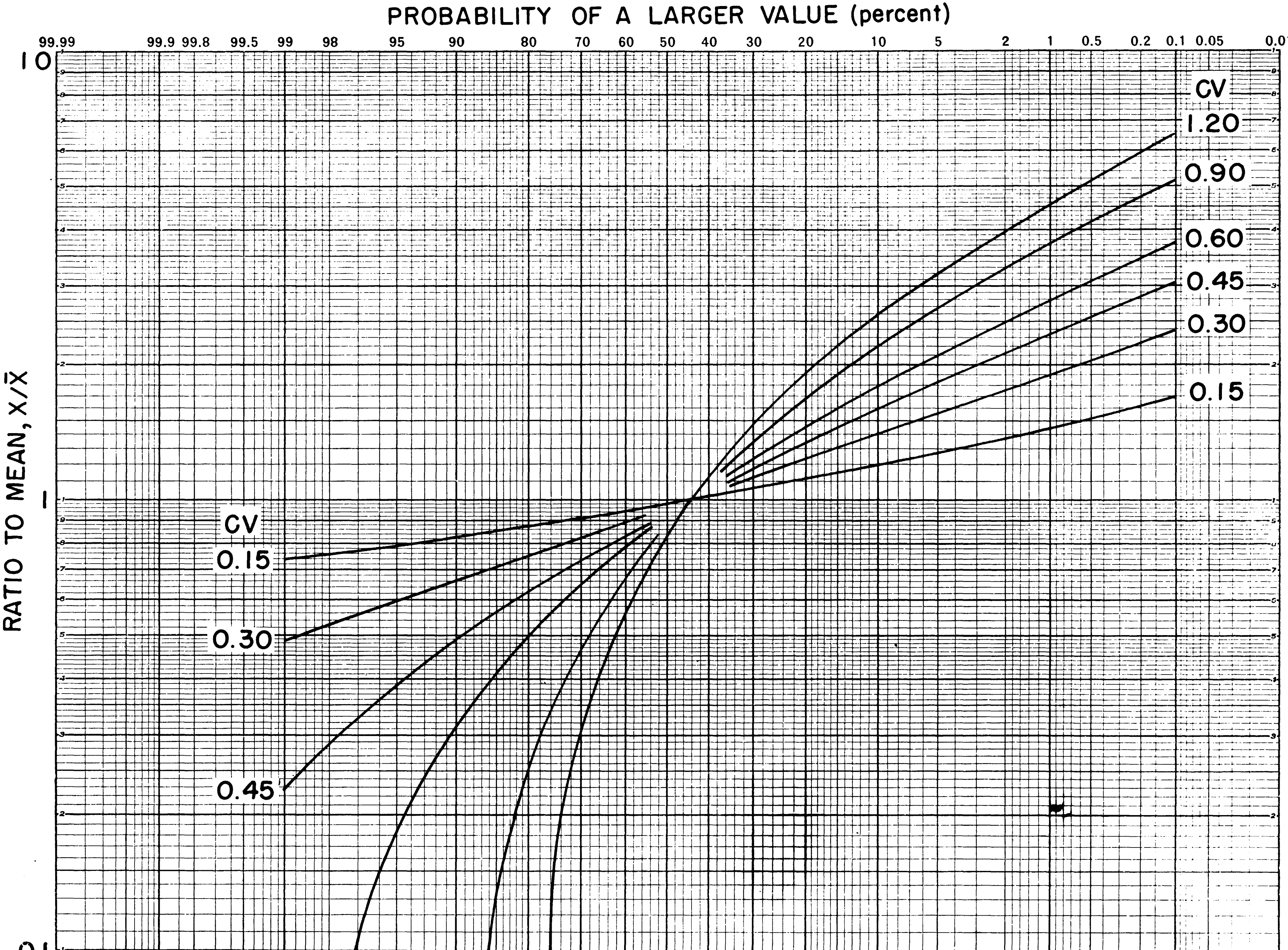

FIGURE 15.—Modified log-normal curves for $CV = 0.3$.

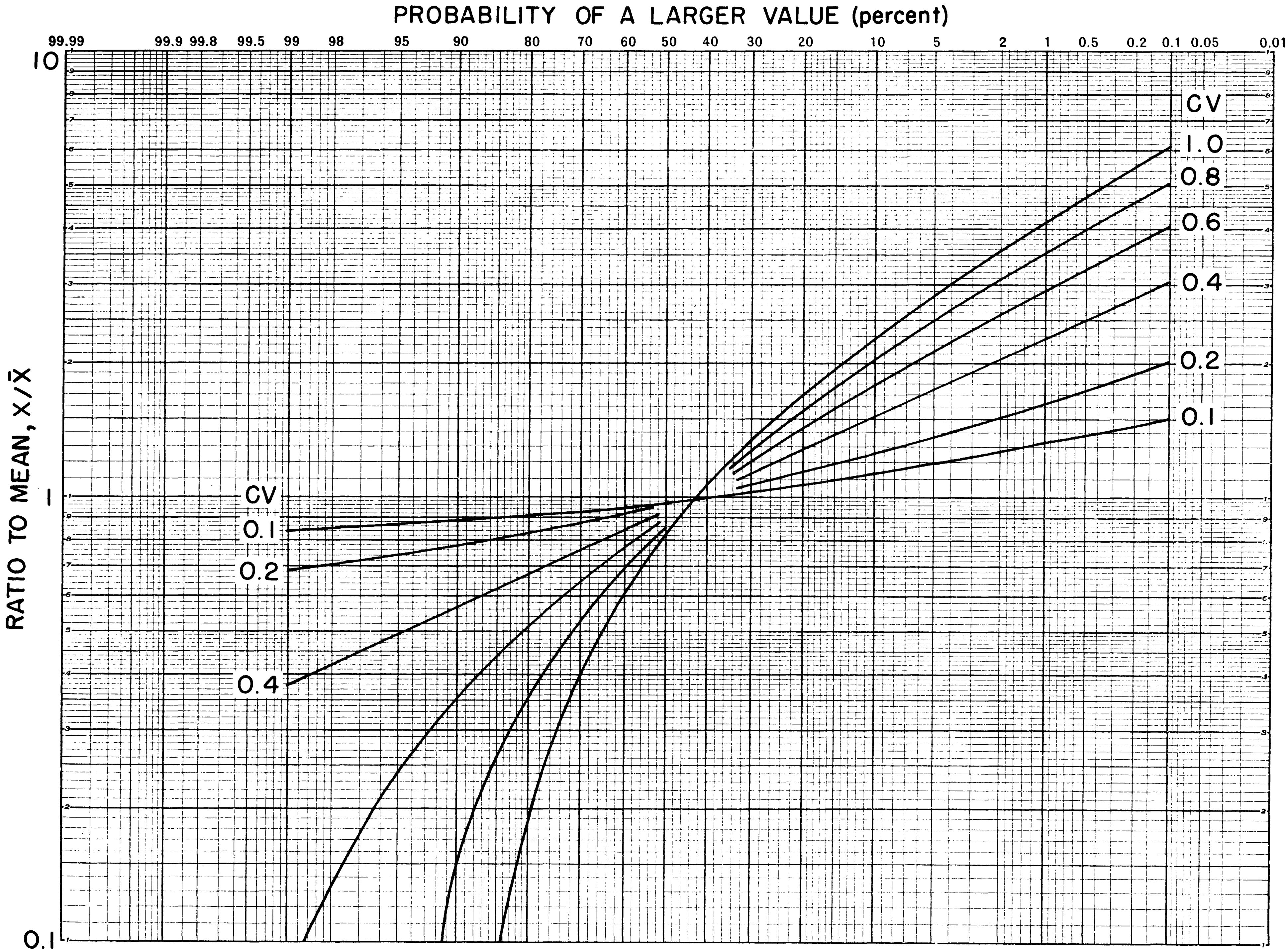

FIGURE 16.—Modified log-normal curves for $CV=0.4$.

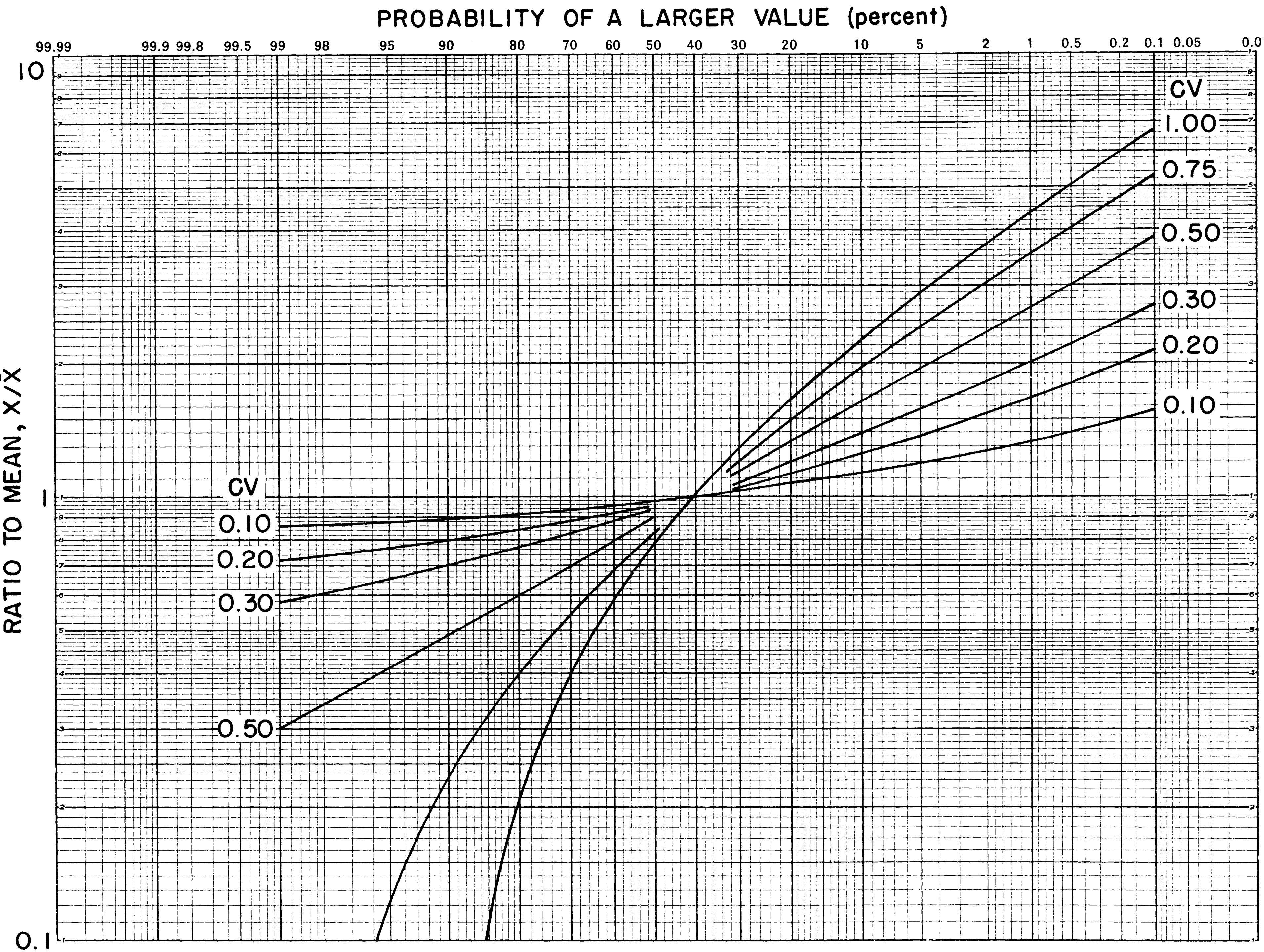

FIGURE 17.—Modified log-normal curves for $CV=0.5$.

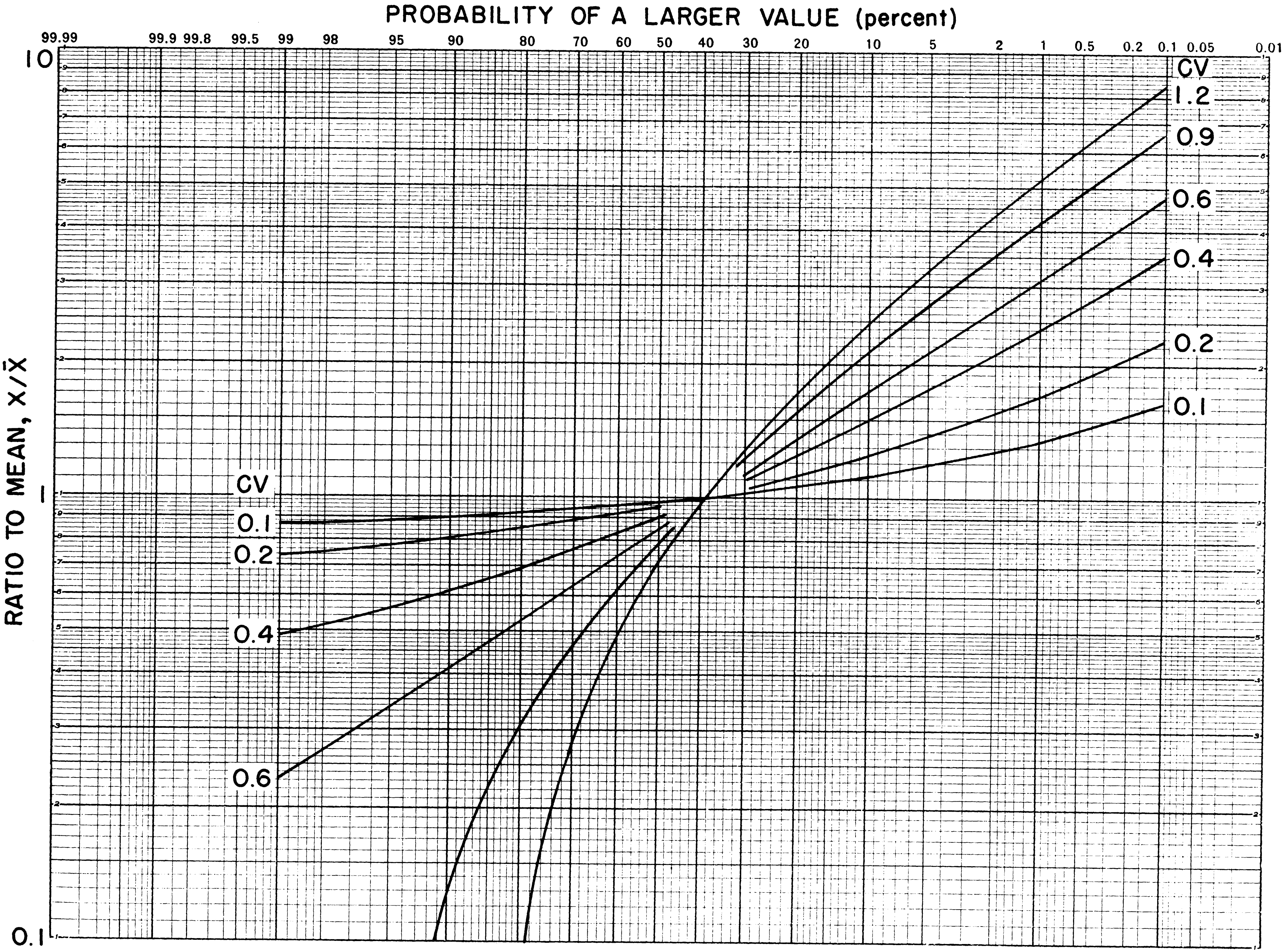

FIGURE 18.—Modified log-normal curves for $CV=0.6$.

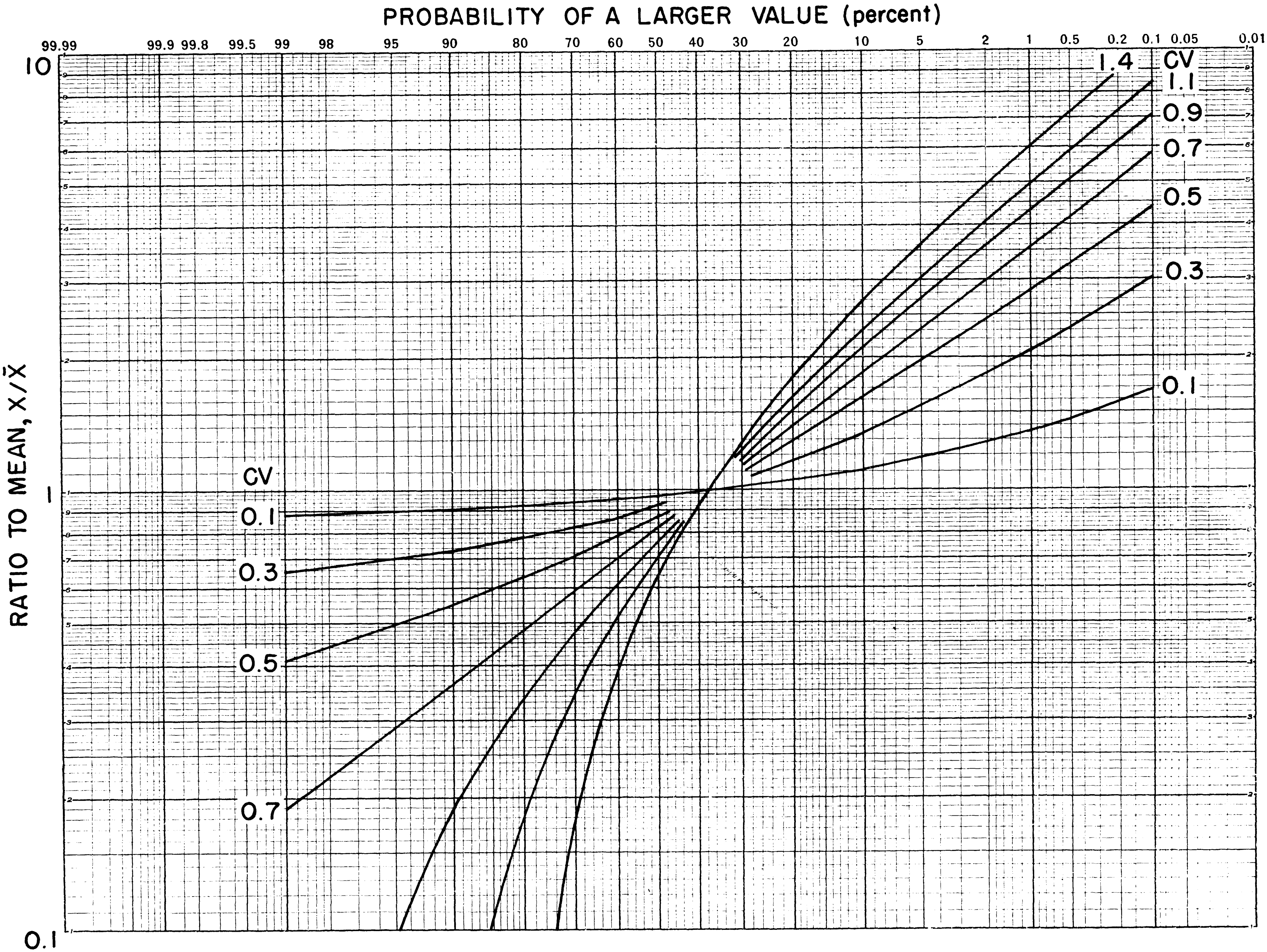

FIGURE 19.—Modified log-normal curves for $CV=0.7$.

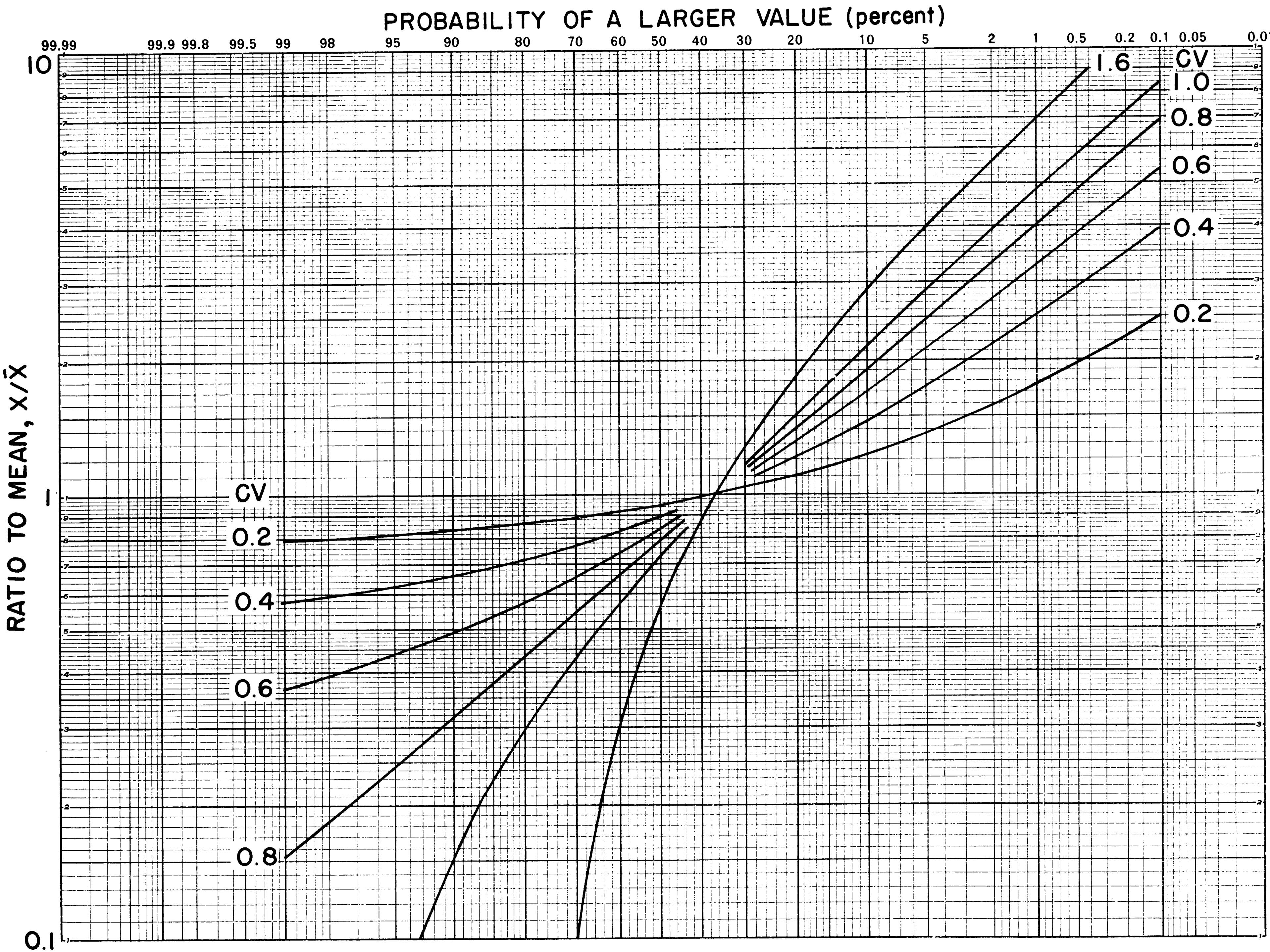

FIGURE 20.—Modified log-normal curves for $CV=0.8$.

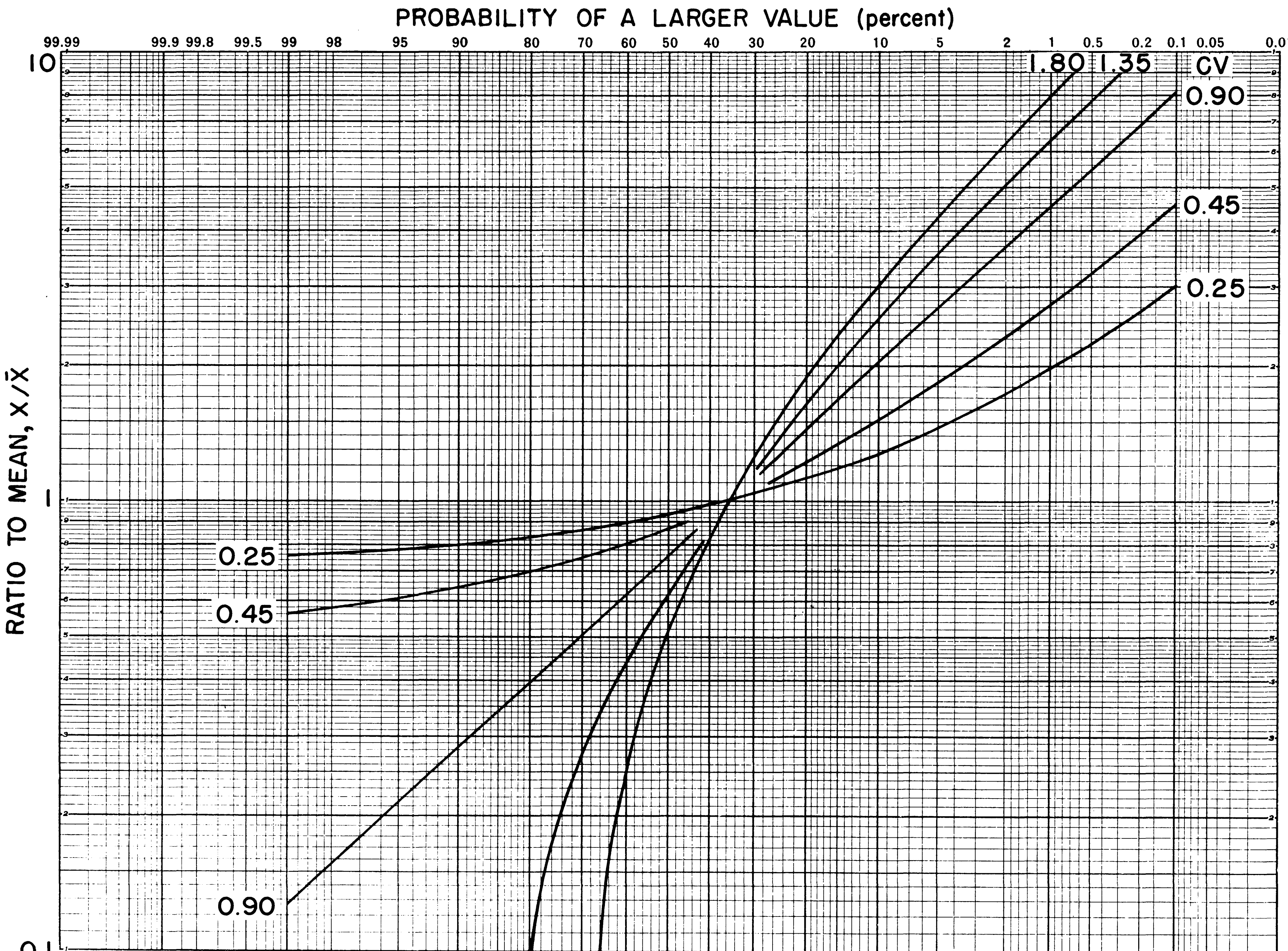

FIGURE 21.—Modified log-normal curves for $CV=0.9$.

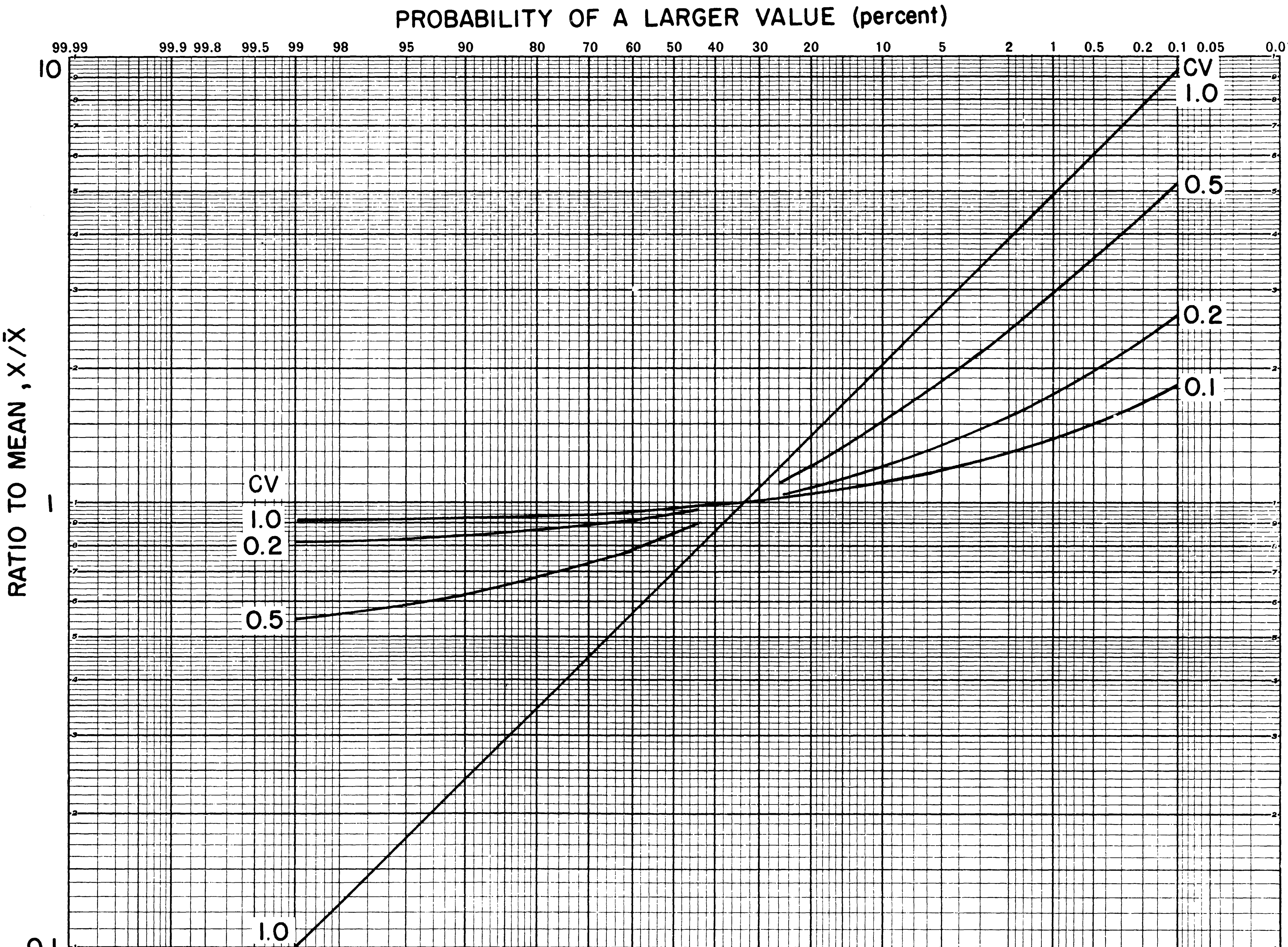

FIGURE 22.—Modified log-normal curves for $CV=1.0$.

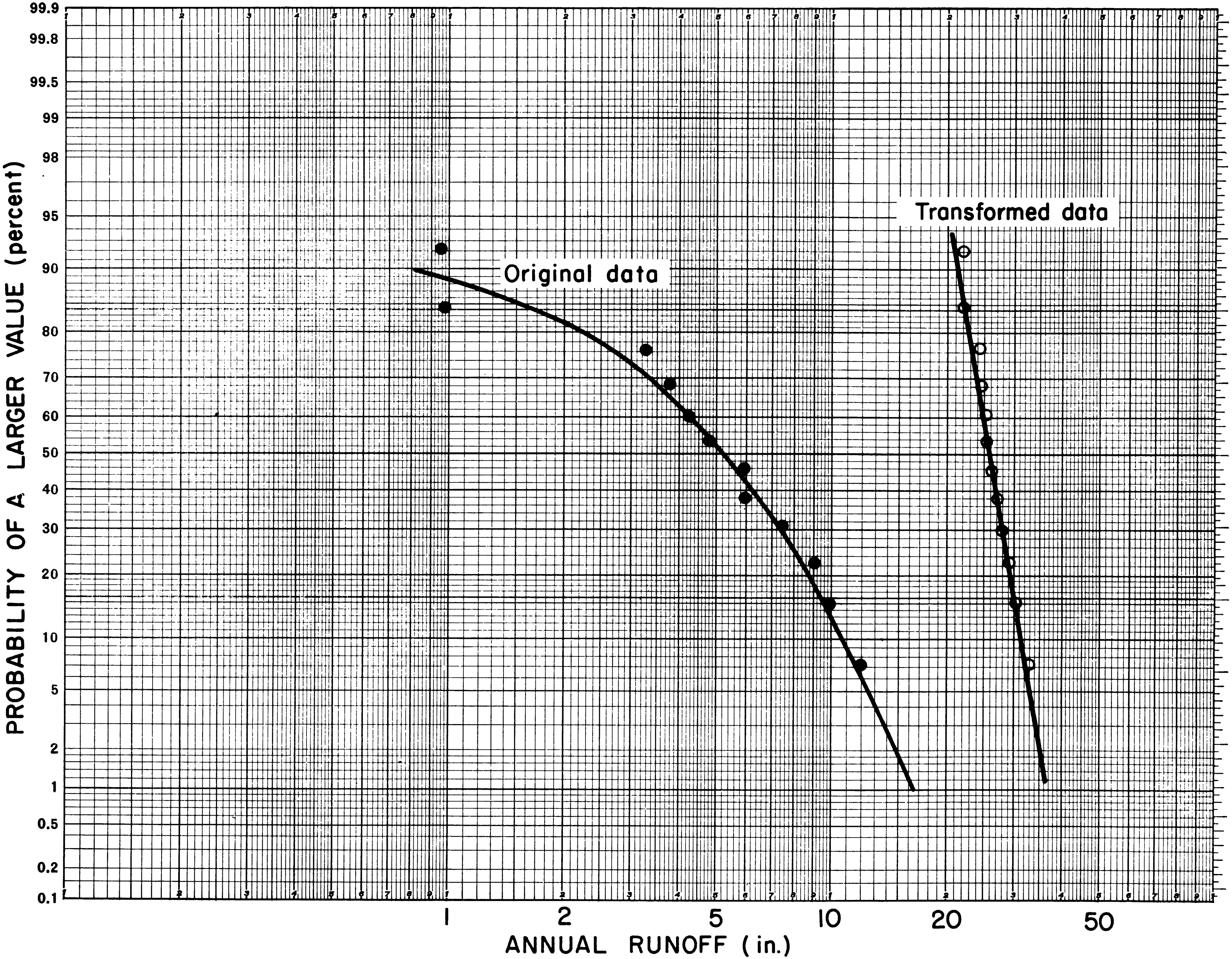

FIGURE 23.—Modified log-normal distribution fitted graphically to data of table 2.

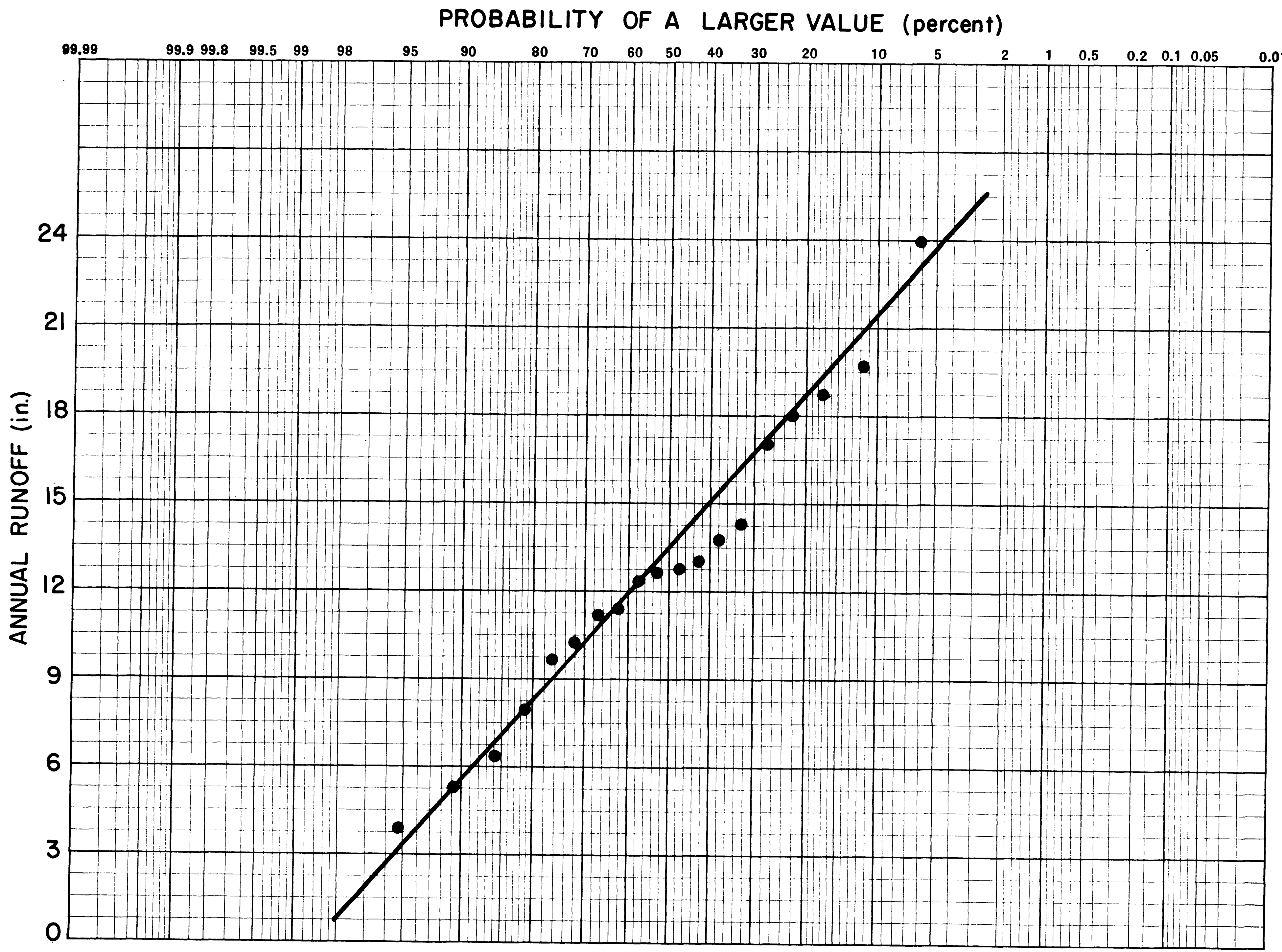

FIGURE [illegible]— Normal distribution fitted to data of table [illegible]

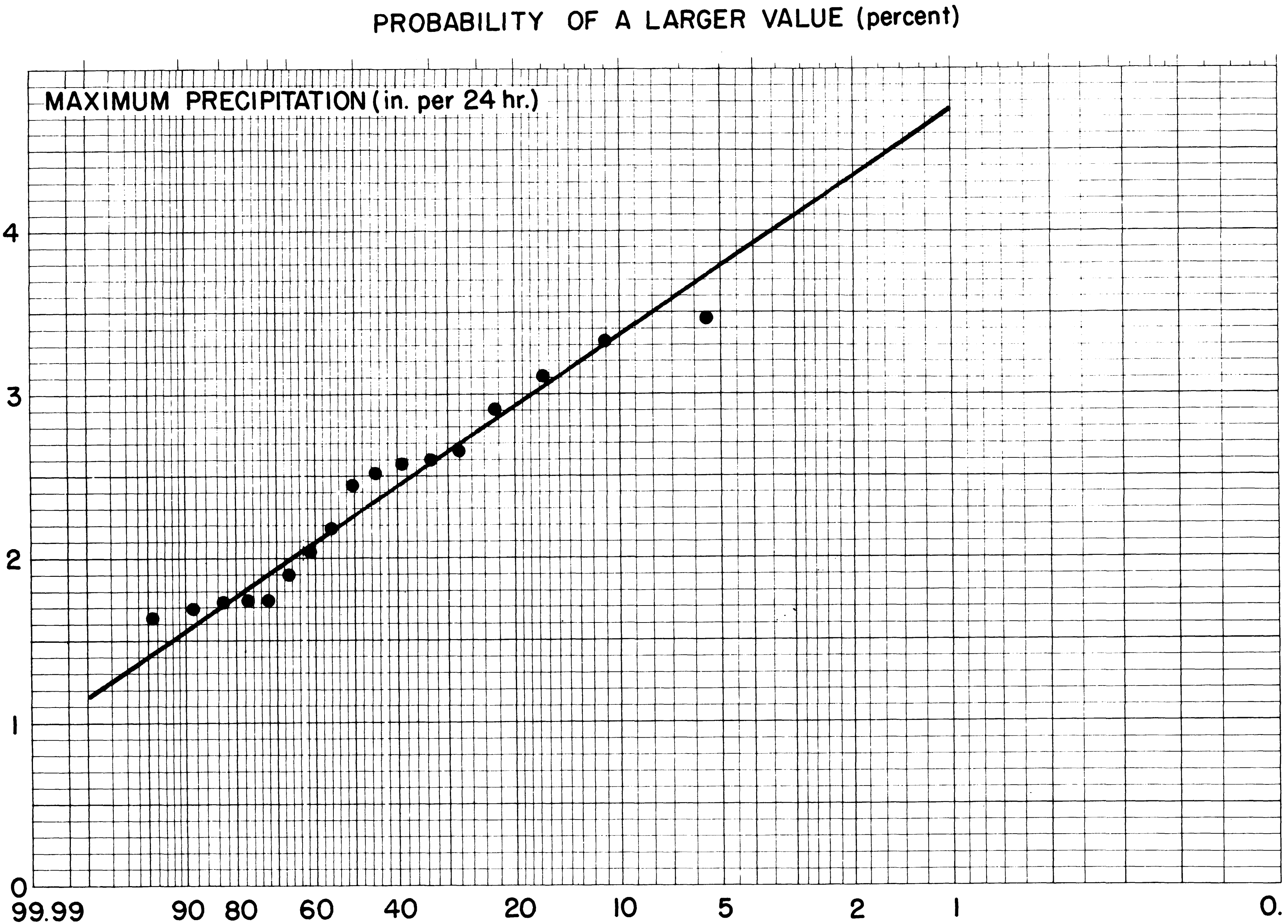

FIGURE 25.—Extreme-value distribution fitted to data of table 6.

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